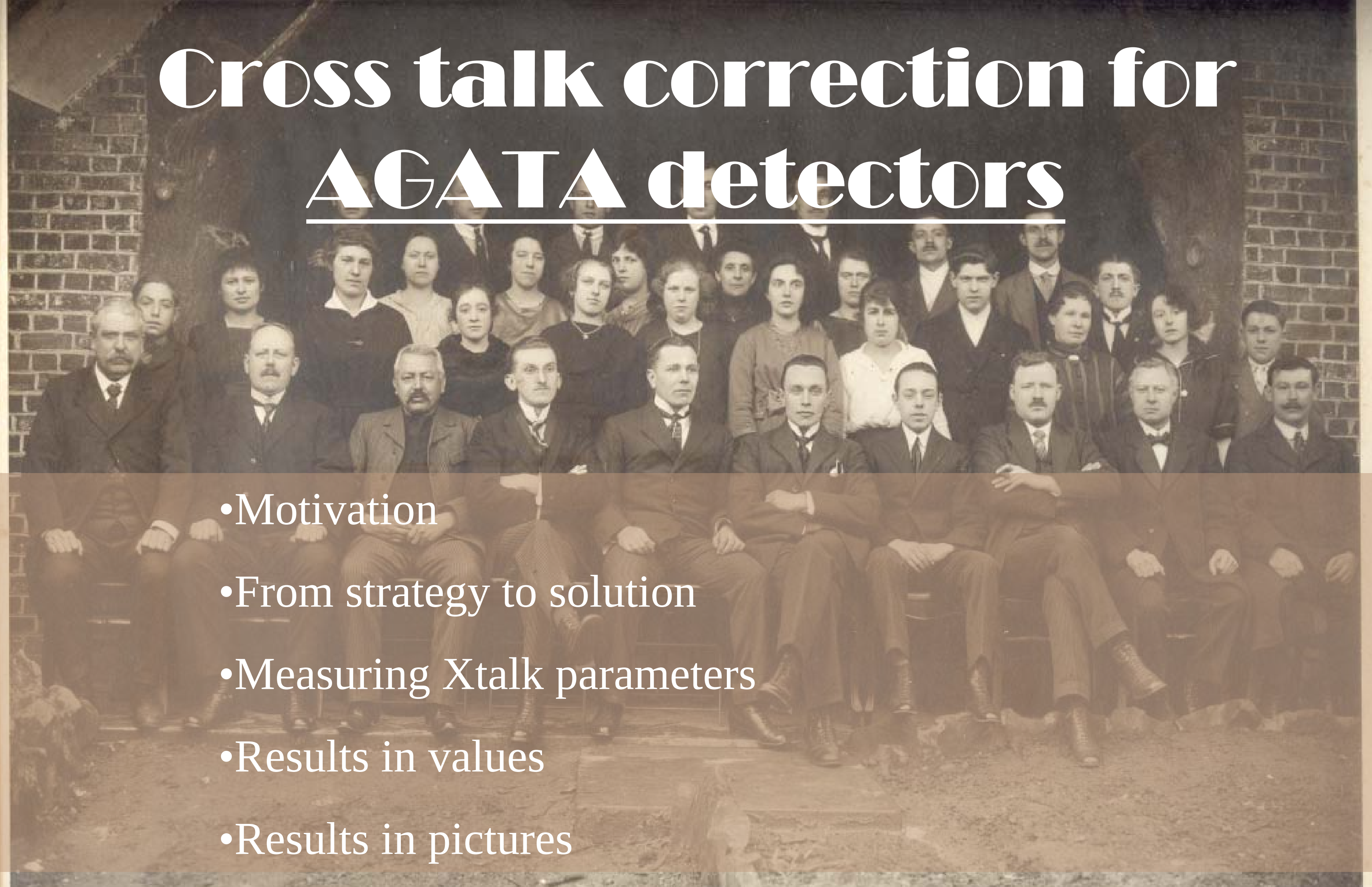

Cross talk correction for AGATA detectors

- Motivation
- From strategy to solution
- Measuring Xtalk parameters
- Results in values
- Results in pictures

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At the 6th AGATA week, Padua Nov. 2007

Motivation

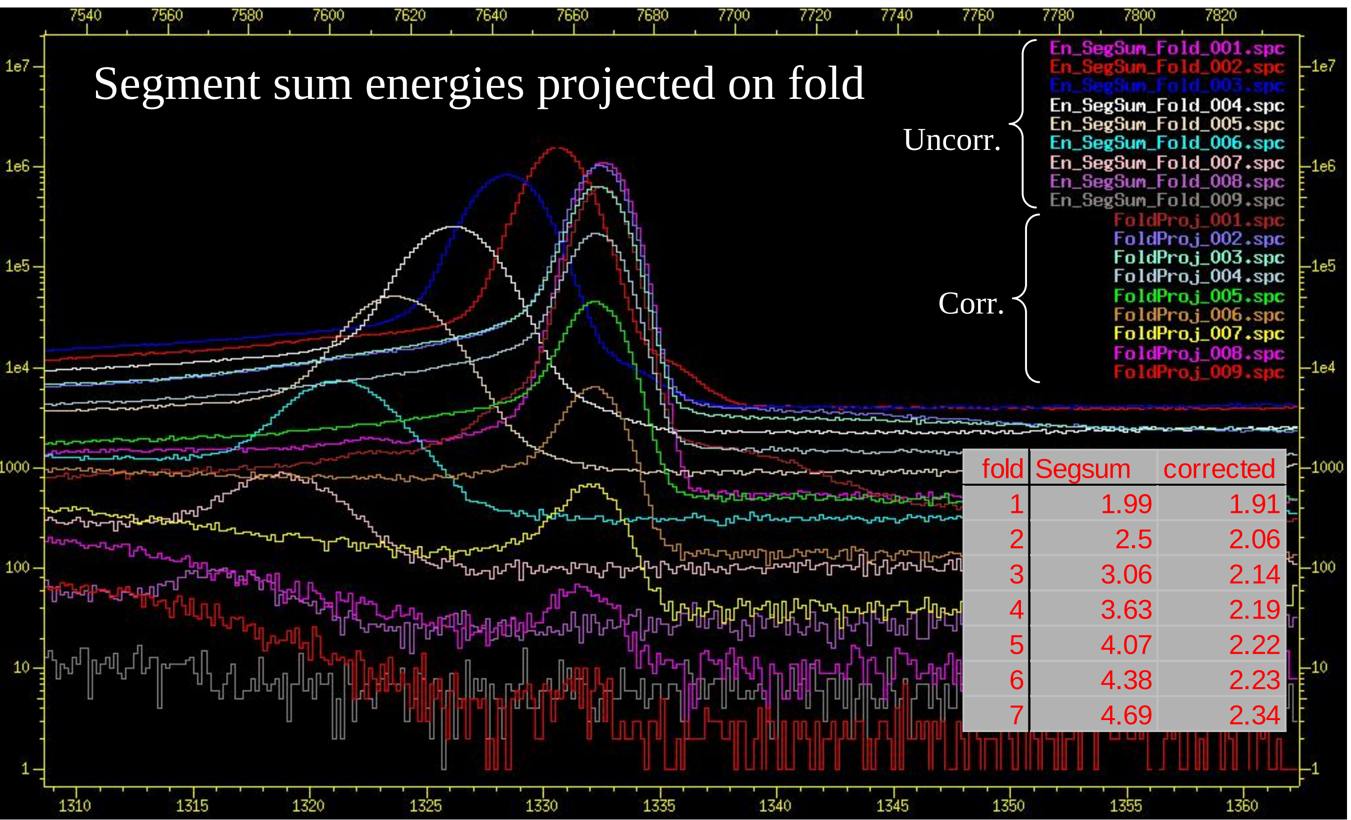
Cross talk correction is crucial for AGATA:

- Crosstalk is present in any segmented detector
- Creates strong energy shifts proportional to fold

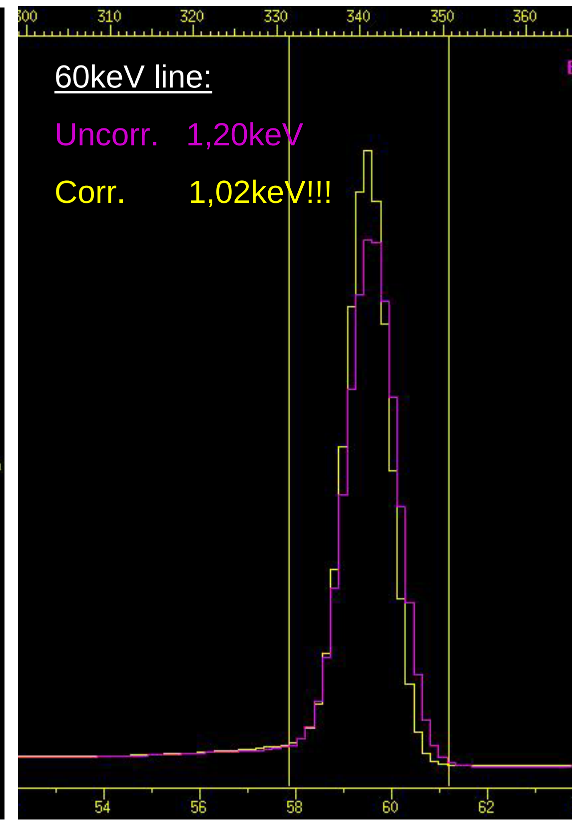
  (origin: see last AGATA week)
- Tracking needs <u>segment energies</u> !

Results preview (S001):

- ALL folds aligned
- BONUS:
  improved resolution!



fold	Segsum	corrected
1	1.99	1.91
2	2.5	2.06
3	3.06	2.14
4	3.63	2.19
5	4.07	2.22
6	4.38	2.23
7	4.69	2.34



Cross talk correction: strategy

<u>Step 1: setting up a practical model:</u>

•*Cross talk propagation is dominantly a linear process:*

⇒ use matrices : $E_{meas} = \mathbf{B} \cdot E_{true}$

•*We want a „practical approach“: want to <u>measure</u> all the matrix elements involved*

⇒ Remark true core energy is linear dependent on true segment energy

⇒ True core energy is not a parameter to solve for!

⇒ **B** is a 37x36 matrix.

•*Practical limitation: Energies below treshold often returned as zero´s*

⇒ Take out (projection) segments that are not hit

example : event with hit segments 1,2 and 3.
Model to set up is:

$$\begin{bmatrix} E_{core} \\ \hline E_{seg1} \\ E_{seg2} \\ E_{seg3} \end{bmatrix}_{meas} = B \cdot \begin{bmatrix} E_{seg1} \\ E_{seg2} \\ E_{seg3} \end{bmatrix}_{true}$$

Every possible hitpattern yields a different model matrix **B** !

Cross talk correction: strategy

Step 2: Identification of the matrix elements B_{ij}:

•*Imagine first that there is no cross talk.*

⇒ measured segment energy is true segment energy, measured core energy is segment sum

⇒ in matrix form:

$$\begin{bmatrix} E_{core} \\ \hline E_{seg1} \\ E_{seg2} \\ E_{seg3} \end{bmatrix}_{meas} = \begin{bmatrix} 1 & 1 & 1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} E_{seg1} \\ E_{seg2} \\ E_{seg3} \end{bmatrix}_{true}$$

identity

•*In practice, segments are calibrated on singles. Conservation of calibration implies:*

⇒ measured segment energies of 1folds is true segment energy.

⇒ matrix form including cross talk:

$$\begin{bmatrix} E_{core} \\ E_{seg1} \\ E_{seg2} \\ E_{seg3} \end{bmatrix}_{meas} = \begin{bmatrix} 1+\delta_{01}^* & 1+\delta_{02}^* & 1+\delta_{03}^* \\ 1 & \delta_{12}^* & \delta_{13}^* \\ \delta_{21}^* & 1 & \delta_{23}^* \\ \delta_{31}^* & \delta_{32}^* & 1 \end{bmatrix} \cdot \begin{bmatrix} E_{seg1} \\ E_{seg2} \\ E_{seg3} \end{bmatrix}_{true}$$

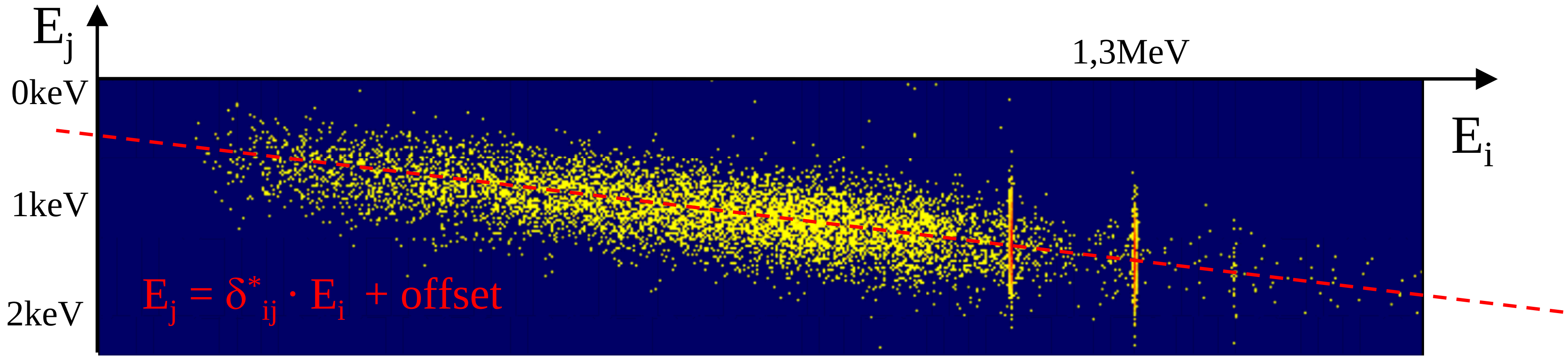
⇒ (Note: only 36x36 effective matrix elements δ_{ij}^* measurable, true crosstalk matrix is 37x37!)

Measuring the cross talk parameters

$$B = \begin{pmatrix} 1+\delta^*_{01} & 1+\delta^*_{02} & 1+\delta^*_{03} & \cdots \\ 1 & \delta^*_{12} & \delta^*_{13} & \cdots \\ \delta^*_{21} & 1 & \delta^*_{23} & \cdots \\ \delta^*_{31} & \delta^*_{32} & 1 & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

a) From singles:

δ^*_{ij} = shift observed in segment j when only segment i is hit.



+ Fast collection of data (<u>every</u> single event yields <u>35 entries</u> in the 35x36 matrices)

+ Simple analysis – no gates required

- Needs special care digitizer settings not to suppress low energies

- Typical correction for digitizer overflow necessary (values < 0)

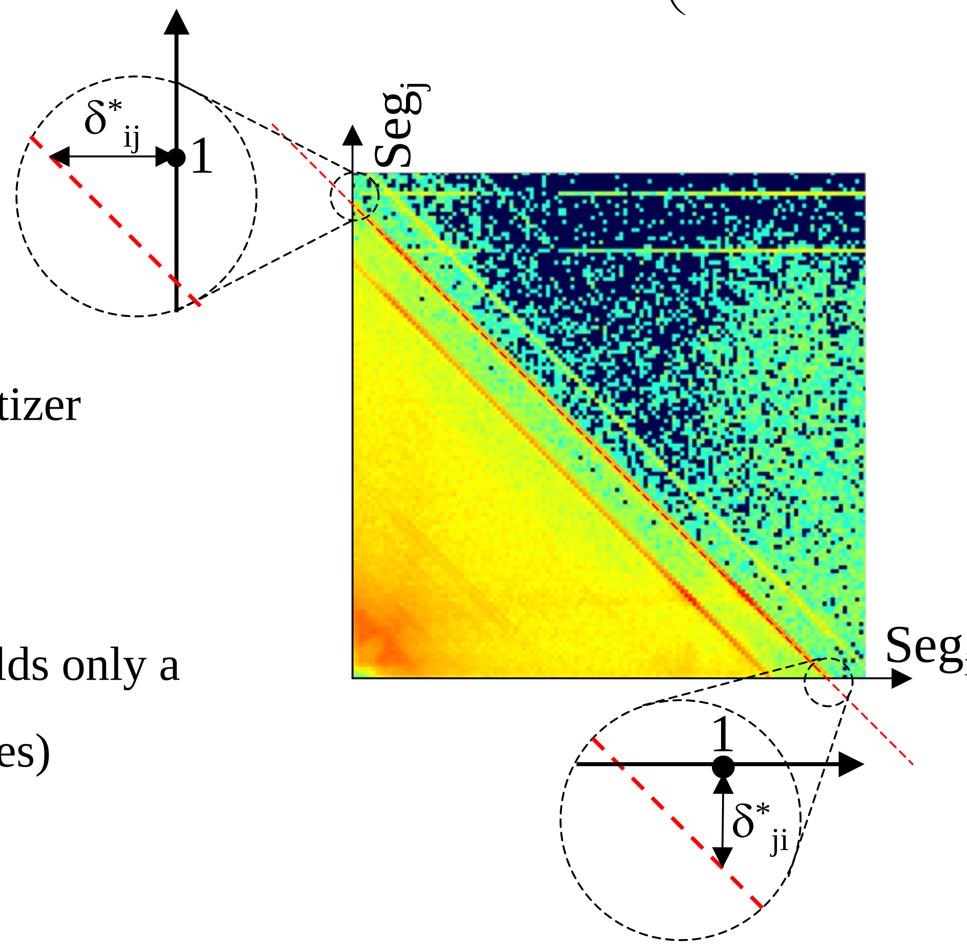
Measuring the cross talk parameters

$$B = \begin{pmatrix} 1+\delta^*_{01} & 1+\delta^*_{02} & 1+\delta^*_{03} & \cdots \\ 1 & \delta^*_{12} & \delta^*_{13} & \cdots \\ \delta^*_{21} & 1 & \delta^*_{23} & \cdots \\ \delta^*_{31} & \delta^*_{32} & 1 & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

b) From doubles:

δ^*_{ij} and δ^*_{ji} from 2folds between seg i and seg j

and calibrated to 1 on singles



+ Needs no special care from digitizer

+ Optimal data to correct 2folds

- Needs a lot of statistics

(every 2fold photopeak event yields only a single entry in the 35x36 matrices)

- Complex fitting

Cross talk correction: solution

Step 3: Inverting the model:

• *Inverting a non-square matrix = fitting.*

• *Pseudo matrix inverses are not unique*

⇒ guide choice of matrix by properties of noise

⇒ better description of noise = better result.

• Two models were compared: *OLS* and *GLS*

Differences between models:

OLS:

• Core and Seg all have same noise σ

⇒ low energy approximation

⇒ Simple (over-)estimated noise reduction:

fold	Core	Segsum	estimate Etrue
1	1	1.00	0.71
2	1	1.41	0.82
3	1	1.73	0.87
4	1	2.00	0.89
5	1	2.24	0.91
n	1	sqrt(n)	weighted avg

GLS:

• Core and Seg all have realistic, individual σ

⇒ For 1 channel: $\sigma^2(E) = \sigma^2_{elec.} + F\cdot E$

⇒ For whole detector (covariance matrix Σ):

$$\Sigma_{\text{det}}(E_1,E_2,\ldots) = \Sigma_{electronic} + \Sigma_{statistic}$$

$$\Sigma_{electronic} = \begin{pmatrix} \sigma^2_{core} & 0 & 0 & \ldots \\ 0 & \sigma^2_{seg1} & 0 & \ldots \\ 0 & 0 & \sigma^2_{seg2} & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

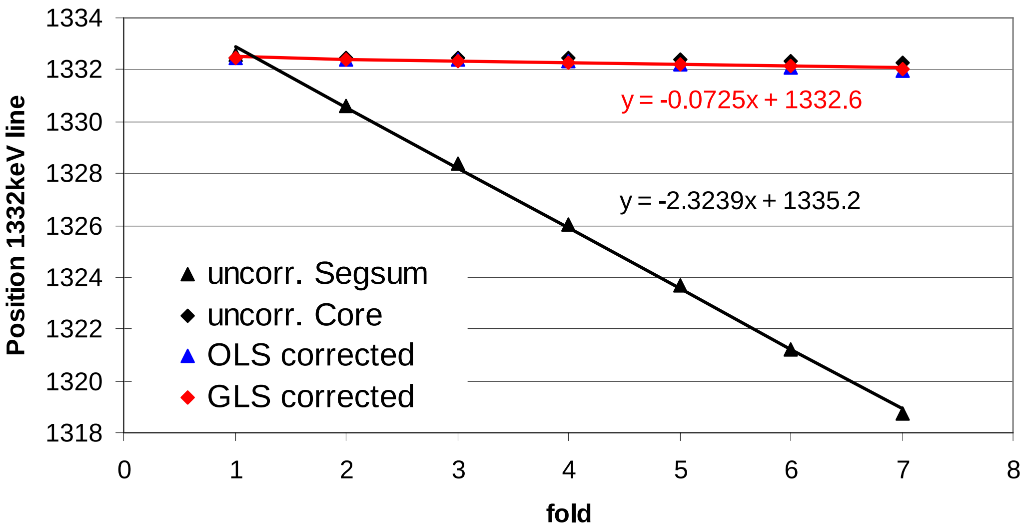
$$\Sigma_{statistic}(E_1,E_2,\ldots) = F\cdot\begin{pmatrix} (E_1+E_2+\ldots) & E_1 & E_2 & \ldots \\ E_1 & E_1 & 0 & \ldots \\ E_2 & 0 & E_2 & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

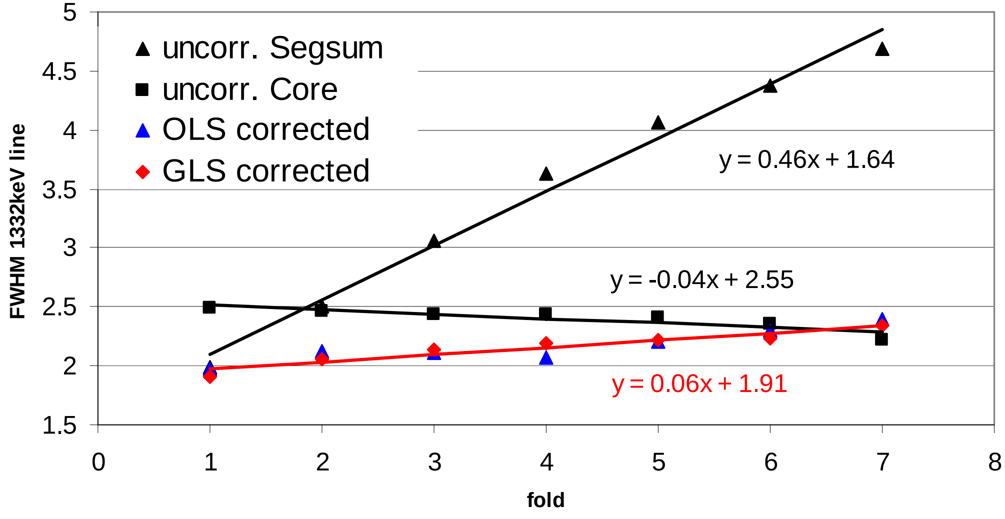
OUTCOME (upto 1,3MEV) :

OLS performs only slightly worse than GLS

Results in values

Note: All fits performed with „tail left free"
Due to „bow effects" (but only small effect)





Position [keV] of the 1332 keV line

fold	uncr. Seg	uncr. Core	OLS corr.	GLS corr.
1	1332.59	1332.47	1332.47	1332.48
2	1330.59	1332.47	1332.41	1332.42
3	1328.39	1332.44	1332.37	1332.36
4	1326.05	1332.44	1332.32	1332.29
5	1323.67	1332.41	1332.2	1332.21
6	1321.22	1332.35	1332.1	1332.14
7	1318.72	1332.24	1331.96	1332.04

Resolution [keV] of the 1332 keV line

fold	uncr. Seg	uncr. Core	OLS corr.	GLS corr.
1	1.99	2.49	1.97	1.91
2	2.5	2.46	2.12	2.06
3	3.06	2.43	2.11	2.14
4	3.63	2.44	2.07	2.19
5	4.07	2.41	2.21	2.22
6	4.38	2.35	2.3	2.23
7	4.69	2.22	2.39	2.34

Resolution [keV] of the low energy lines

Energy	fold	uncr. Seg	OLS corr.	GLS corr.
60keV	1	1.2	1.07	1.02
122keV	1	1.32	1.15	1.12
122keV	2	1.77	1.35	1.33
136keV	1	1.33	1.16	1.14
136keV	2	1.79	1.39	1.33

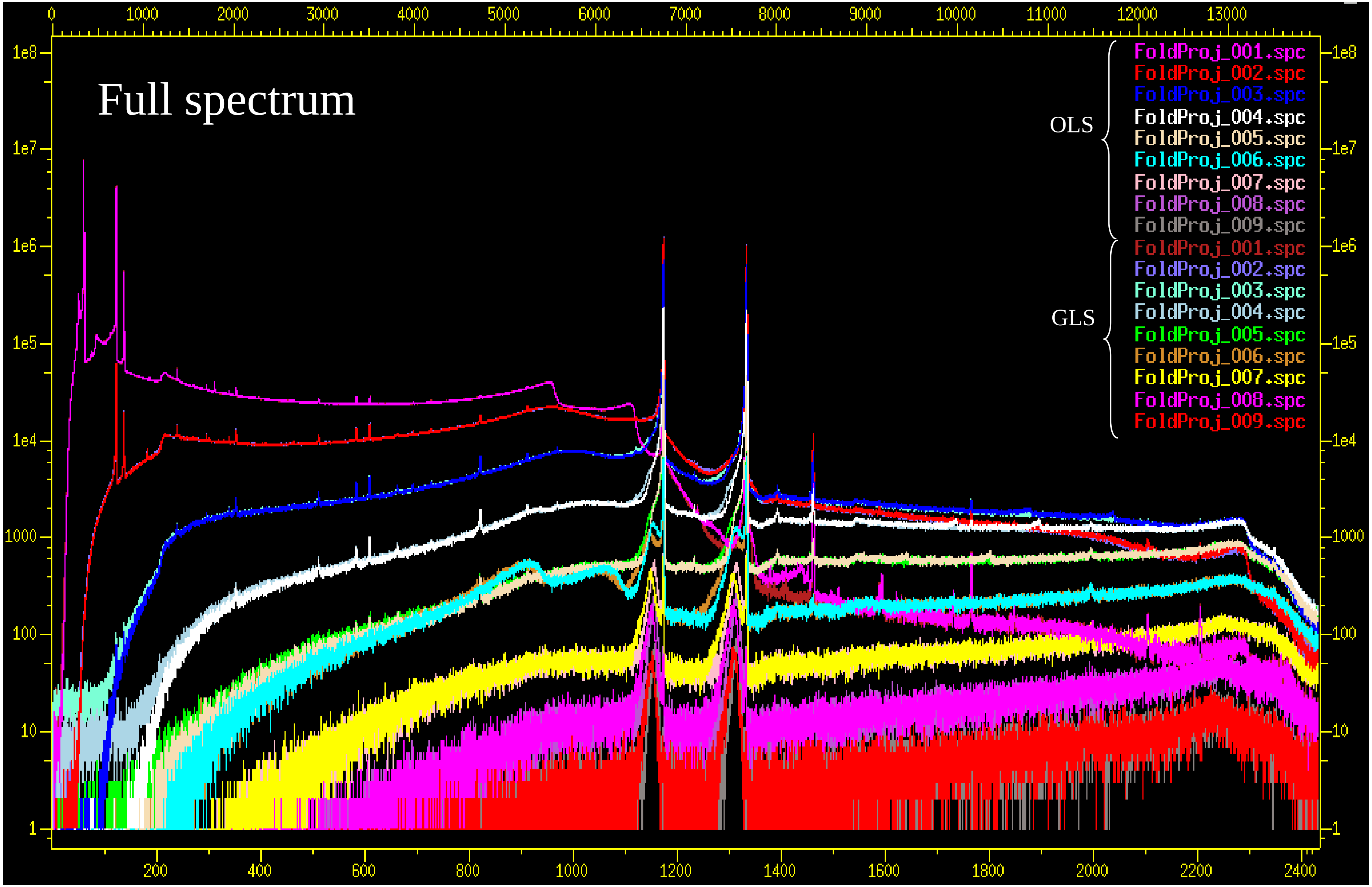
Comparison of resolution increase with theory

Energy	fold	theory	Ratio
60keV	1	0.825	0.85
122keV	1	0.841	0.85
122keV	2	0.73	0.75
136keV	1	0.845	0.86
136keV	2	0.73	0.74
1332keV	1	0.943	0.96

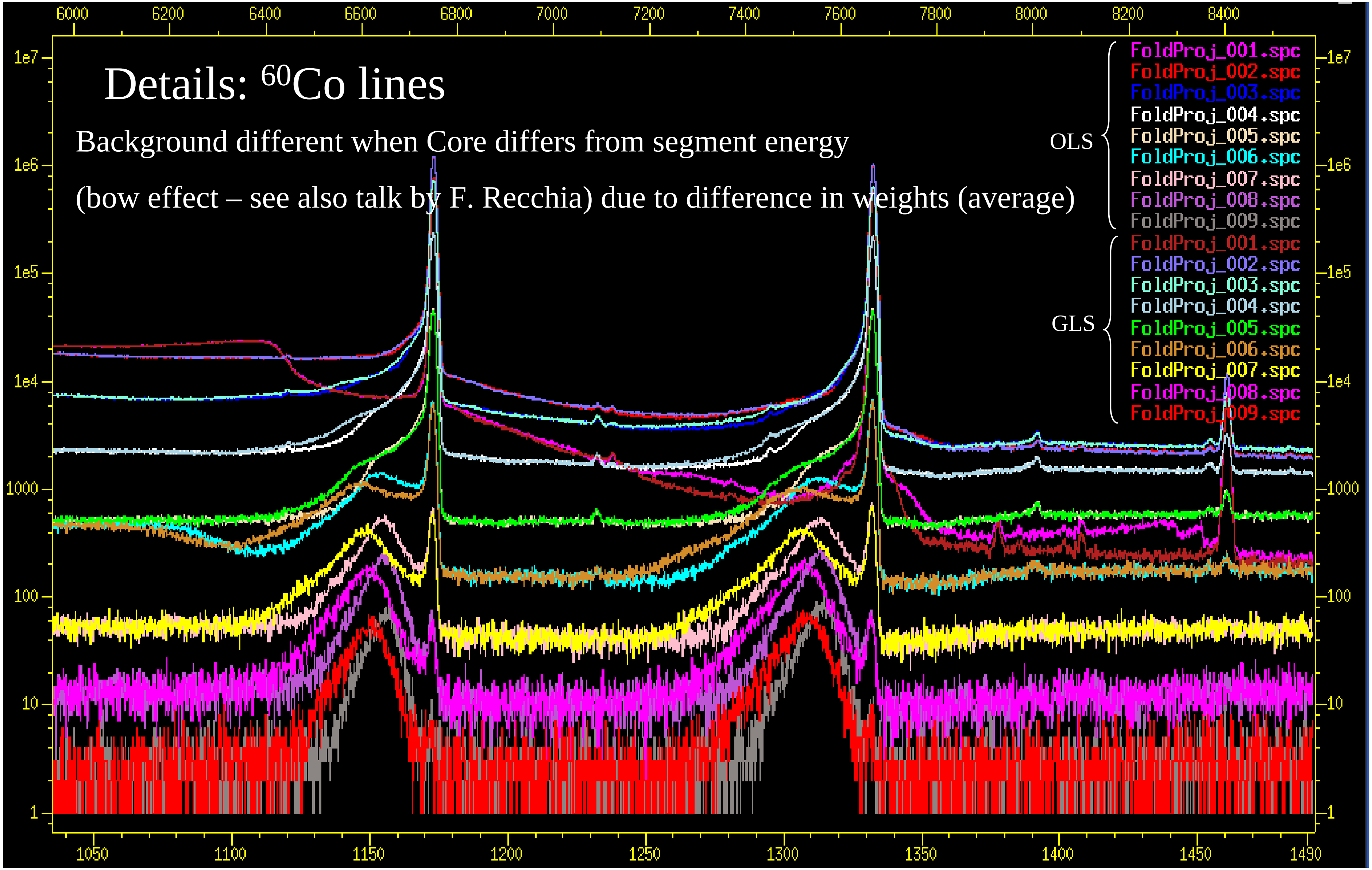
$$\text{Ratio} = \frac{GLS\ estimate}{Uncorr.\ Segsum}$$

Theory with
$\sigma_{core} = 0.64\text{keV}$
$\sigma_{seg} = 0.47\text{keV}$
$F = 3.55\ 10^{-4}\ \text{keV}$

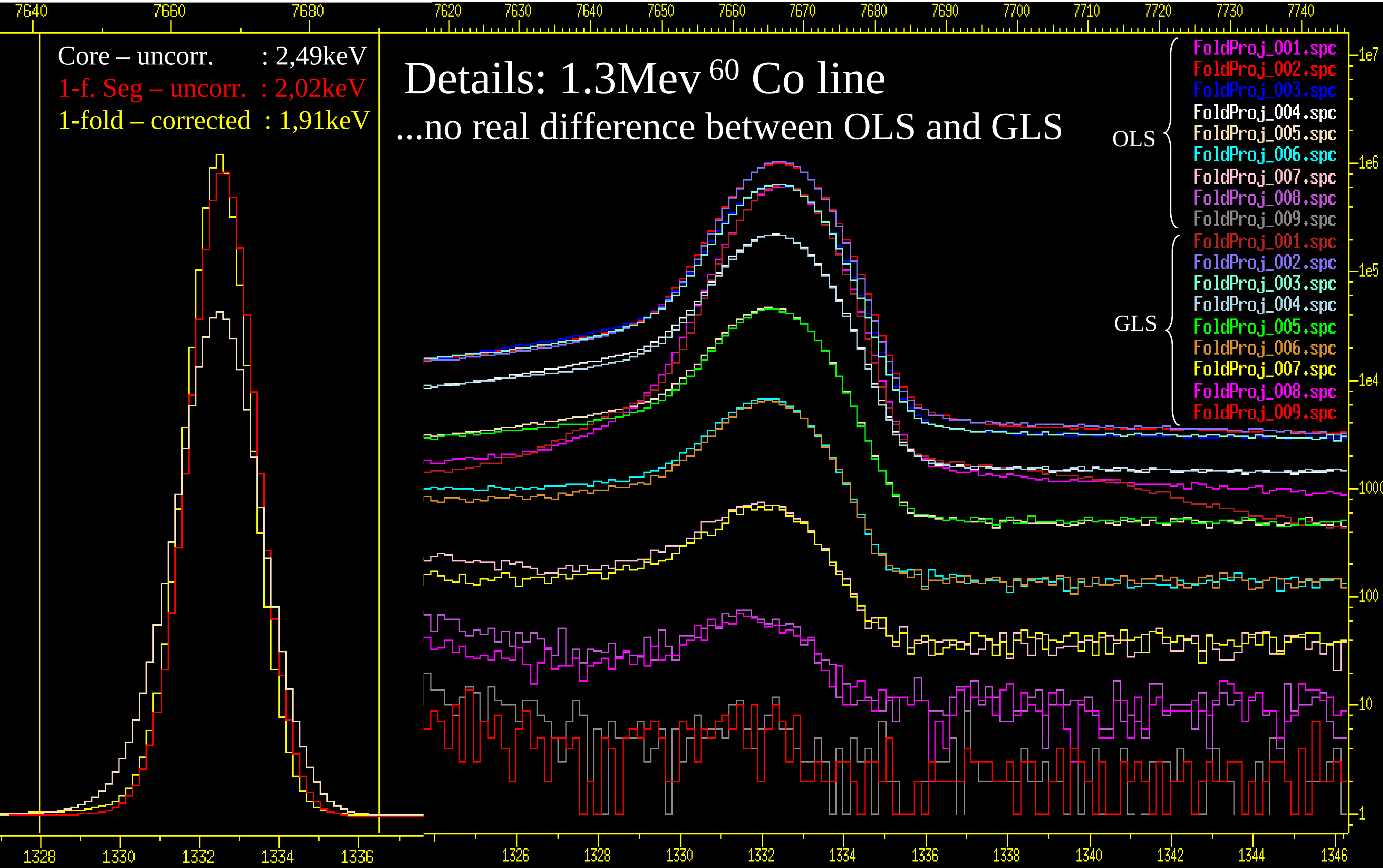
Results in pictures



Results in pictures



Results in pictures



Conclusion / Summary

• A practical method to correct for cross talk effects in segmented detectors was presented

• The method has as nice additional feature to increase energy resolution (through fitting).

• Simple (OLS) and more complex (GLS) fitting procedures were included. Both perform equally well (in the energy range upto 1.3MeV).

EXPERT SLIDES - About the method: a) model

- Measured Energies E_{meas} relate linearly to true energies E_{true}
- Core Energy is sum of Segment energies : dim E_{true} = 36

$$C = \begin{pmatrix} 1 & 1 & \cdots \\ \hdashline 1 & 0 & \cdots \\ 0 & 1 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

$$E_{meas}(37) = \left[\mathbf{N \cdot X \cdot C}\right](37\text{x}36) \cdot E_{true}(36)$$

- **C** : Creates core as segment sum (37 x 36)
- **X** : Adds Xtalk : **X** (37 x 37) = **1**+**Δ**
- **N** : Calibration : (37 x 37)

$$X = \begin{pmatrix} 1+\delta_{00} & \delta_{01} & \delta_{02} & \cdots \\ \delta_{10} & 1+\delta_{11} & \delta_{12} & \cdots \\ \delta_{20} & \delta_{21} & 1+\delta_{22} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$N = \begin{pmatrix} n_0 & 0 & 0 & \cdots \\ 0 & n_1 & 0 & \cdots \\ 0 & 0 & n_2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

⇒ Model matrix **B** (37 x 36) **= N·X·C**
with 36x36 observable Xtalk parameters:

- not all Xtalk matrix elements measurable
 ⇒ effective matrix elements δ^*
- $\delta^*_{ii} = 0$ from calibration **N**

$$B = \begin{pmatrix} 1+\delta^*_{01} & 1+\delta^*_{02} & 1+\delta^*_{03} & \cdots \\ 1 & \delta^*_{12} & \delta^*_{13} & \cdots \\ \delta^*_{21} & 1 & \delta^*_{23} & \cdots \\ \delta^*_{31} & \delta^*_{32} & 1 & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

EXPERT SLIDES - About the method: b) solution

<u>Problems "Inverting" **B**:</u>

• **B** not square - more measured values than unknown

⇒ Fitting : $E_{true} = (\mathbf{B}^T \mathbf{B})^{-1} \cdot \mathbf{B}^T \cdot E_{meas}$ — Ordinary Least Square Fitting (OLS)

• Usually $E_{meas} <$ treshold not completely measured!

⇒ $\mathbf{B}_n$ (n+1 x n) = $\mathbf{P}_n$(n+1 x 37)·**N**·**X**·**C**·$\mathbf{P}_n^*$(36 x n) — with $\mathbf{P}_n$, $\mathbf{P}_n^*$ projection on fold n

<u>Example : suppose 1fold without cross talk:</u>

$$B_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow E_{OLS} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} E_{core} \\ E_{seg} \end{pmatrix}$$

Better : including knowledge of noise : Generalized Least Square Fitting (GLS)

⇒Fitting : $E_{true} = (\mathbf{B}^T \Sigma^{-1} \mathbf{B})^{-1} \cdot \mathbf{B}^T \cdot \Sigma^{-1} \cdot E_{meas}$ — with Σ covariance matrix of noise

$$\Sigma = \begin{pmatrix} \sigma_{core}^2 & 0 \\ 0 & \sigma_{seg}^2 \end{pmatrix} \Rightarrow E_{OLS} = \frac{\sigma_{core}^2 \sigma_{seg}^2}{\sigma_{core}^2 + \sigma_{seg}^2} \cdot \begin{pmatrix} \frac{1}{\sigma_{core}^2} & \frac{1}{\sigma_{seg}^2} \end{pmatrix} \cdot \begin{pmatrix} E_{core} \\ E_{seg} \end{pmatrix}$$

weighted average !

For segmented detectors Σ = — with F = Fano factor x energy/e-h pair

$$\Sigma(E_1, E_2, \ldots) = \begin{pmatrix} \sigma_{core}^2 + F\cdot(E_1 + E_2 + \ldots) & F\cdot E_1 & F\cdot E_2 & \ldots \\ F\cdot E_1 & \sigma_{seg1}^2 + F\cdot E_1 & 0 & \ldots \\ F\cdot E_2 & 0 & \sigma_{seg2}^2 + F\cdot E_2 & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

DOUBLE WIN!!!

•Xtalk corrected

•Resolution optimized