Chapter 1:
Conserved quantities and nuclear reactions

Kinematics & conservation laws

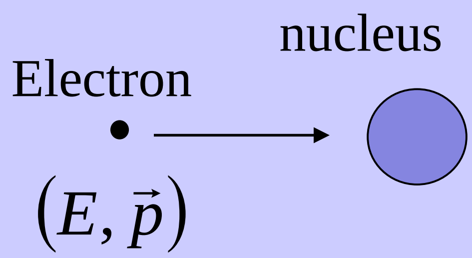
$$\underbrace{p}_{\text{projectil}} + \underbrace{T}_{\text{target}} \longrightarrow \underbrace{R}_{\text{recoil nucleus}} + \underbrace{e}_{\text{ejectile}}$$

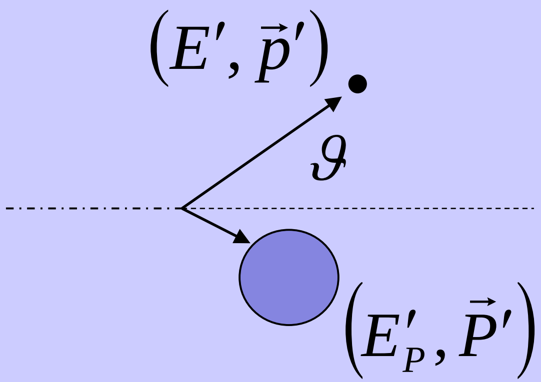
Notation for reaction:

T (p,e) R

(p,e)

example: kinematics of electron scattering





$$\underbrace{p(electron)}_{\text{projectil}} + \underbrace{P(nucleus)}_{\text{target}} \rightarrow \underbrace{P'(nucleus)}_{\text{recoil}} + \underbrace{p'(electron)}_{\text{ejectile}}$$

$$\left(E/c, \vec{p}\right) \qquad \left(Mc, 0\right) \qquad \left(E'/c, \vec{p}\,'\right) \qquad \left(E'_P/c, \vec{P}\right)$$

Energy- and momentum conservation: $p + P = p' + P'$

or $$p^2 + 2pP + P^2 = p'^2 + 2p'P' + P'^2$$

invariant mass: $$p^2 = p'^2 = m_e^2 c^2 \text{ and } P^2 = P'^2 = M^2 c^2$$

$$p \cdot P = p' \cdot P'$$

in most cases recoil cannot be measured: $p \cdot P = p' \cdot \left(p + P - p'\right)$

example: kinematics of electron scattering

$$p = \left(E/c , \vec{p} \right) \quad P = \left(Mc, 0 \right) \qquad\qquad p' = \left(E'/c , \vec{p}\,' \right) \quad P' = \left(E'_P/c , \vec{P} \right)$$

energie- and momentum conservation: $p \cdot P = p' \cdot \left(p + P - p' \right)$

insert relativistic 4-momentum: $E \cdot Mc^2 = E'E - \vec{p} \cdot \vec{p}\,' c^2 + E' \cdot Mc^2 - m_e^2 c^4$

at relativictic energies: $m_e^2 c^4 \approx 0$ and $E \approx \left| \vec{p} \right| \cdot c$

Energy of scattered electrons: $$E' = \frac{E}{1 + E/Mc^2 \cdot \left(1 - \cos\vartheta \right)}$$

nuclear reactions at low energies

example:

$$\alpha + {}^{14}N \rightarrow {}^{17}O + p$$

$${}^{14}N(\alpha, p){}^{17}O$$

$$(\alpha, p)$$

The reaction ^{14}N(α,p)^{17}O performed by Rutherford in 1917 (reported 1919), is generally regarded as the first nuclear transmutation experiment.

Conservation laws and nuclear reactions at low energies

- Charge
- (energy, momentum) four momentum
- Angular momentum

  (*orbital angular momentum and spin*)
- Parity
- Number of nucleons

  only at low energies: $E < mc^2$

are related, orbital angular momentum of nucleus wave function determines parity

$$P = (-1)^L$$

- Baryon number
- Lepton number

Parity of wave function

Definition: The wave function of a nucleus $\psi(\vec{r})$ has a parity. After parity transformation the wave function may be the same function (even parity) or inverted (odd parity).

even: $\psi(-\vec{r}) = +\psi(\vec{r}) \Leftrightarrow$ Parity $= +$ or $+1$ or even

odd: $\psi(-\vec{r}) = -\psi(\vec{r}) \Leftrightarrow$ Parity $= -$ or -1 or odd

$$\vec{r} \rightarrow -\vec{r} \quad \Leftrightarrow \quad \vartheta \rightarrow \pi - \vartheta, \quad \varphi \rightarrow \pi + \varphi$$

$$\cos\vartheta \rightarrow -\cos\vartheta$$

$$\psi_{n\ell m}(\vec{r}) = \psi_{n\ell m}(r, \vartheta, \varphi) = R_{n\ell}(r) \cdot Y_{\ell m}(\vartheta, \varphi) \propto P_\ell^m(\cos\vartheta) \cdot e^{im\varphi}$$

$$\psi_{n\ell m}(-\vec{r}) = \psi_{n\ell m}(r, \pi-\vartheta, \pi+\varphi) \propto \underbrace{P_\ell^m(-\cos\vartheta)}_{(-1)^{\ell+m} P_\ell^m(\cos\vartheta)} \cdot e^{im\varphi} \cdot \underbrace{e^{im\pi}}_{(-1)^m}$$

Parity of particle wave function in central potential is $(-1)^\ell$.

Therefore, parity is determined by angular momentum.

Angular momentum and parity

In general:

$$1 + 2 \longrightarrow 3 + 4$$

$$P_1 \quad P_2 \qquad P_3 \quad P_4$$

$$\underbrace{J_1 \quad J_2}_{L} \qquad \underbrace{J_3 \quad J_4}_{l}$$

Ang. momentum:

$$\vec{J}_1 \oplus \vec{J}_2 \oplus \vec{L} = \vec{J}_3 \oplus \vec{J}_4 \oplus \vec{l}$$

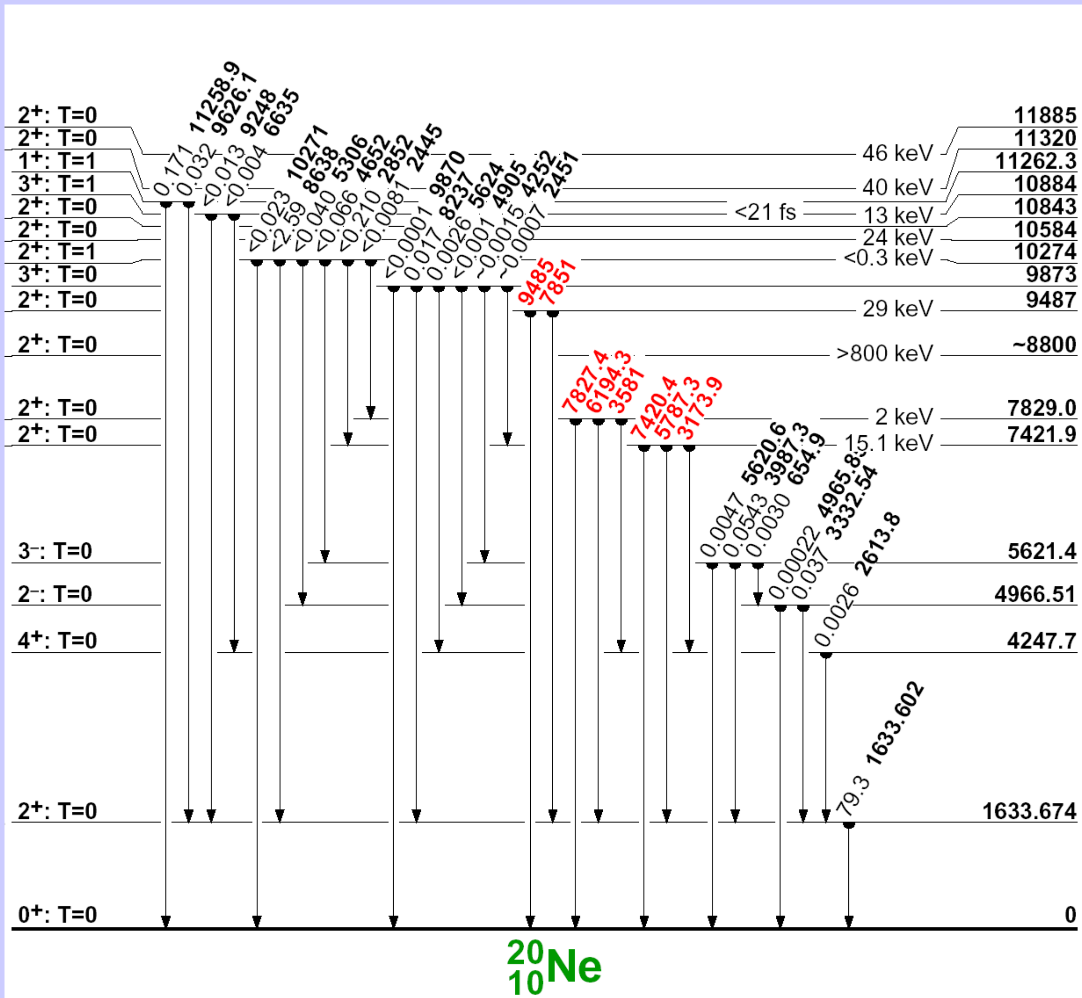
$$\left|\vec{J}_{in}\right| = \left|\vec{J}_{out}\right| \qquad J_{in,z} = J_{out,z}$$

Parity:

$$P_1 P_2 (-1)^L = P_3 P_4 (-1)^l$$

Angular momentum and Parity

example: $\alpha + {}^{16}O \rightarrow {}^{20}Ne^{*} \rightarrow {}^{20}Ne + \gamma$



Angular momentum and Parity

example:

1 $$\alpha + {}^{16}O \rightarrow {}^{20}Ne^* \rightarrow {}^{20}Ne + \gamma$$

$0^+ \quad 0^+ \qquad\qquad 0^+ \quad 1^-$

possible intermediate excited states in ${}^{20}Ne^*$

$$J^P = 0^+, \quad 1^-, \quad 2^+, \; ...$$

0^+

L: $\mathbf{0 + 0 = 0}$ (but no γ!)

P: $(+1)(+1)(-1)^L = (+1)$

$P = (-1)^L$

Rel. angular momentum $L = 0, \; 2, \; 4$

s , d , ...

Angular momentum and Parity

1^+

$$L = 1$$

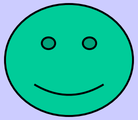
$$\mathbf{0} \oplus \mathbf{0} \oplus \boldsymbol{L} = \mathbf{1} = \mathbf{0} \oplus \mathbf{1}$$
$$= 1$$

$$(+1)(+1)(-1) = +1 = (+1)(-1)$$
$$-1 \neq +1 \neq -1$$

Parity violation

1^-

$$\mathbf{0} \oplus \mathbf{0} \oplus \mathbf{1} = \mathbf{1} = \mathbf{0} \oplus \mathbf{1}$$

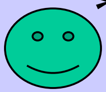


$$(+1)(+1)(-1) = -1 = -1$$
$$-1 = -1 = -1$$

Angular momentum and Parity

2^{+}

ang.: $0 \oplus 0 \oplus 2 = 2 = 0 \oplus 1 \oplus \begin{matrix} 1 \\ 3 \end{matrix}$ *



Par.: $(+1)(+1)(-1)^{2} = (+1) = (+1)(-1)(-1)^{1}$

$(+1) = (+1) = (+1)$

γ ray with higher multipolarity carries away angular momentum

2^{-}

ang.: $0 \oplus 0 \oplus 2 = 2 = 0 \oplus 1 \oplus \begin{matrix} 1 \\ 3 \end{matrix}$ *

Par.: $(+1)(+1)(-1)^{2} \neq (-1) \neq (+1)(-1)(-1)^{1}$

$(+1) \neq (-1) \neq (+1)$

Parity violation

Angular momentum, Parity, Symmetry

2

$$p + {}^{7}Li \rightarrow \alpha + \alpha$$

$$\underbrace{\tfrac{1}{2}^{+} \quad \tfrac{3}{2}^{-}}_{L} \qquad \underbrace{0^{+} \quad 0^{+}}_{l}$$

α-particles (${}^{4}He$ -nuclei) are identical bosons.

$\Rightarrow$ wave function has to be symmetric

$$\psi(1, 2) = \psi(2, 1)$$

$\Rightarrow$ angular momentum l is even

$$P_{\substack{\text{right}\\ side}} = +1$$

$\Rightarrow L$ has to be odd due to parity conservation

Angular momentum, Parity, Symmetry

Angular momentum:

$$\tfrac{1}{2} \oplus \tfrac{3}{2} \oplus L = 0 \oplus 0 \oplus l$$

$\rightarrow l$ is even

$$(-1)^l = +1$$

Parity:

$$\underbrace{(+1)(-1)}_{(-1)}\underbrace{(-1)^L}_{(-1)} = (+1)$$

$\rightarrow L$ is odd