Fermis Golden Rule

\[ W_{fi} = \frac{2\pi}{\hbar} |H_{fi}|^2 \cdot \frac{dn}{dE_f} = \frac{2\pi}{\hbar} |M|^2 \cdot \rho(E_f) \]

- \( W_{fi} \) - transition probability from initial state to final state
- \( H_{fi}, M \) - matrix element - information about nuclear states
- \( \rho(E_f) \) - density of final states
- phase space factor
• If the Hamilton operator $\hat{H}$ is hermitian:

$$M_{if} = \langle f | \hat{H} | i \rangle = \langle i | \hat{H} | f \rangle^*$$

$$|M_{a \rightarrow b}|^2 = |M_{b \rightarrow a}|^2 = |M|^2 \quad \text{"Detailed Balance"}$$

• Density of states

$$\rho(E_f) = \frac{4\pi}{h^3} \cdot p_f^2 \cdot \frac{dp_f}{dE_f} \cdot g_f$$

because $T(p, e)R$ is a two body problem with two body kinematics in the centre of mass system (CM) for momenta in final state is only one independent momenta $p_f$
\[ E_f = \frac{P_f^2}{2m_x} + \frac{P_f^2}{2m_y} \]

\[ \frac{dE_f}{dP_f} = \frac{P_f}{m_x} + \frac{P_f}{m_y} = P_f \left( \frac{m_x + m_y}{m_x m_y} \right) \]

\[ = P_f \cdot \frac{1}{m_f} = v_f \frac{m_f}{m_f} \]

\[ m_f \] - reduced mass in final channel

\[ v_f \] - relative velocity of final particles after reaction
Particle flux = particle density × velocity

\[ = n_a \cdot v_a \]

\[ = \frac{N_a}{V} \cdot v_a \]

\[ = \frac{N_a}{A} \cdot \frac{1}{s} = \frac{\dot{N}_a}{A} = \text{particle number per area} \cdot \text{time} \]

For one particle per unit volume (considering only 1 incident particle)

\[ = \frac{1}{V} \cdot v_a = v = \text{flux} \]

\[ \hat{=} \text{relative velocity in initial state} \]
Cross section

\[ \sigma_{a \to b} = \frac{W}{n_a v_a} = \frac{16 \pi^3}{h^4} \cdot |M_{if}|^2 \cdot \frac{p_b^2}{v_a v_b} \cdot (2J_b + 1)(2J_B + 1) \]

- matrix element
- phase space and particle flux
- spin degeneracy factor

\[ g_f = (2J_x + 1)(2J_y + 1) \]

- Normalized volume from particle current (phase space)
  Cancels out with normalized volume from matrix element.
Cross section
Principle of Detailed Balance

\[
\sigma_{a \to b} = \frac{16\pi^3}{h^4} |M_{if}|^2 \cdot \frac{p_b^2}{v_a v_b} (2j_b + 1)(2J_B + 1)
\]

\[
\sigma_{b \to a} = \frac{16\pi^3}{h^4} |M_{if}|^2 \cdot \frac{p_a^2}{v_b v_a} (2j_a + 1)(2J_A + 1)
\]

\[
\frac{\sigma_{a \to b}}{\sigma_{b \to a}} = \frac{\sigma_{fi}}{\sigma_{if}} = \frac{p_b^2}{p_a^2} \cdot \frac{(2j_b + 1)(2J_B + 1)}{(2j_a + 1)(2J_A + 1)}
\]

Principle of Detailed Balance
→ time reversal invariance
→ example: study of astro physics reactions
Inverse reaction comparison of cross sections

\[ ^{27}\text{Al}(p,\alpha) \quad E_p[\text{MeV}] \]

\[ ^{27}\text{Al}(p,\alpha) \xrightleftharpoons{} ^{24}\text{Mg} \]

\[ \theta_{\text{C.M.}} = 168.1^\circ \]

\[ ^{24}\text{Mg}(\alpha,p) \quad E_\alpha[\text{MeV}] \]

\[ <\theta_{\text{C.M.}}> = 177.77^\circ \]
inverse reactions

A + a ⇌ B + b

or

B + b ⇌ A + a
example CNO-cycle

Reaction network:
- \((p,\gamma)\)
- \((\alpha,\gamma)\)
- \((\alpha,p)\)
- \((p,\alpha)\)

\(\beta^+-\text{decay}\)
three breakout reactions from hot CNO cycle into rp-process:

\[ ^{18}\text{F}(p,\gamma)^{19}\text{Ne}, \]
\[ ^{15}\text{O}(\alpha,\gamma)^{19}\text{Ne} \]
\[ ^{18}\text{Ne}(\alpha,p)^{21}\text{Na} \]

\[^{18}\text{Ne}(\alpha,p)^{21}\text{Na} \] reaction is relevant at high temperature.

Two actual experimental approaches:
- Measure cross section of \[^{18}\text{Ne}(\alpha,p)^{21}\text{Na} \] with \(^4\text{He}\) target and radioactive \(^{18}\text{Ne}\) beam

- Measure cross section of inverse reaction: \[^{21}\text{Na}(p,\alpha)^{18}\text{Ne} \]

‘Principle of Detailed Balance‘ yields cross section for \[^{18}\text{Ne}(\alpha,p)^{21}\text{Na} \] reaction rate.

Problem: \[^{21}\text{Na} \] is also instable, half life: \(t_{1/2} \sim 22\) sec

solution: (1) in situ production of \[^{21}\text{Na} \] via \(^d(^{20}\text{Ne},^{21}\text{Na})n\) reaction
(2) then use weak secondary \(^{21}\text{Na}\) beam for \(p(^{21}\text{Na},^{18}\text{Ne})\alpha\) reaction
Important difference: inverse reaction

inverse kinematics (heavy projectile onto light target)