Fermis Golden Rule

$$W_{fi} = \frac{2\pi}{\hbar} \left| H_{fi} \right|^2 \cdot \frac{dn}{dE_f} = \frac{2\pi}{\hbar} \left| M \right|^2 \cdot \rho\left(E_f\right)$$

W_{fi} - transition probability from initial state to final state

H_{fi} , M - matrix element

- -information about nuclear states
- -interaction mechanism

$\rho\left(E_f\right)$ - density of final states
phase space factor

- If the Hamilton operator $\hat{H}$ is hermitian:

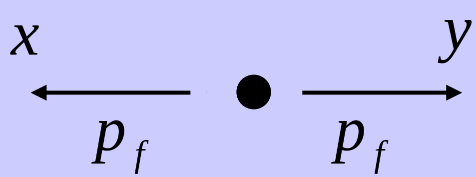
$$M_{if} = \left\langle f \middle| \hat{H} \middle| i \right\rangle = \left\langle i \middle| \hat{H} \middle| f \right\rangle^*$$

$$\left|M_{a\to b}\right|^2 = \left|M_{b\to a}\right|^2 = \left|M\right|^2$$ „Detailed Balance“

- Density of states $\rho\left(E_f\right) = \frac{4\pi}{h^3} \cdot p_f^2 \cdot \frac{dp_f}{dE_f} \cdot g_f$

because $T\left(p,e\right)R$ is a two body problem with two body kinematics in the centre of mass system (CM) for momenta in final state is only one independent momenta p_f

$\vec{p}_e = -\vec{p}_R$



$$E_f = \frac{P_f^2}{2m_x} + \frac{P_f^2}{2m_y}$$

$$\frac{dE_f}{dP_f} = \frac{P_f}{m_x} + \frac{P_f}{m_y} = P_f\left(\frac{m_x + m_y}{m_x m_y}\right)$$

$$= P_f \cdot \frac{1}{m_f} = v_f \frac{m_f}{m_f}$$

$$= v_f$$

m_f - reduced mass in final channel

v_f - relative velocity of final particles after reaction

$$\text{particle flux} = \text{particle density} \times \text{velocity}$$

$$= n_a \cdot v_a$$

$$= \frac{N_a}{V} \cdot v_a$$

$$= \frac{N_a}{A} \cdot \frac{1}{s} = \frac{\dot{N}_a}{A} = \frac{\text{particle number}}{\text{area} \cdot \text{time}}$$

For one particle per unit volume (considering only 1 incident particle)

$$= \frac{1}{V} \cdot v_a = v = \text{flux}$$

$$\hat{=} \text{ relative velocity in initial state}$$

Cross section

$$\sigma_{a\to b} = \frac{W}{n_a v_a} = \frac{16\pi^3}{h^4} \cdot \left|M_{if}\right|^2 \cdot \frac{p_b^2}{v_a v_b} \cdot (2j_b+1)(2J_B+1)$$

- matrix element
- phase space and particle flux
- spin degeneracy factor

$$g_f = (2J_x+1)(2J_y+1)$$

- Normalized volume from particle current (phase space)

Cancels out with normalized volume from matrix element.

Cross section
Principle of Detailed Balance

$$\sigma_{a\to b} = \frac{16\pi^3}{h^4}\left|M_{if}\right|^2 \cdot \frac{p_b^2}{v_a v_b}(2j_b+1)(2J_B+1)$$

$$\sigma_{b\to a} = \frac{16\pi^3}{h^4}\left|M_{if}\right|^2 \cdot \frac{p_a^2}{v_b v_a}(2j_a+1)(2J_A+1)$$

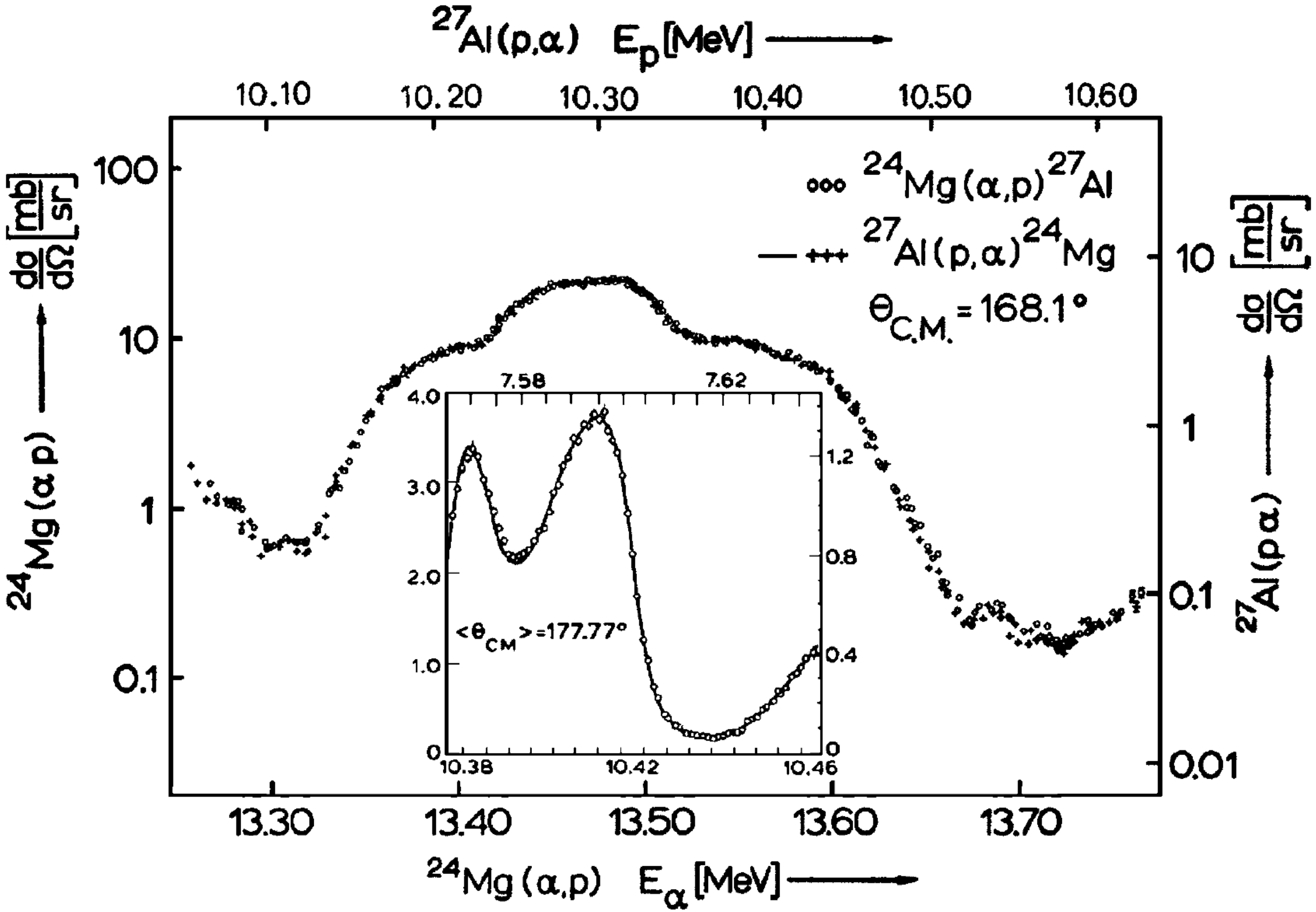
$$\frac{\sigma_{a\to b}}{\sigma_{b\to a}} = \frac{\sigma_{fi}}{\sigma_{if}} = \frac{p_b^2}{p_a^2} \cdot \frac{(2j_b+1)(2J_B+1)}{(2j_a+1)(2J_A+1)}$$

Principle of Detailed Balance

→ time reversal invariance

→ example: study of astro physics reactions

Inverse reaction comparison of cross sections



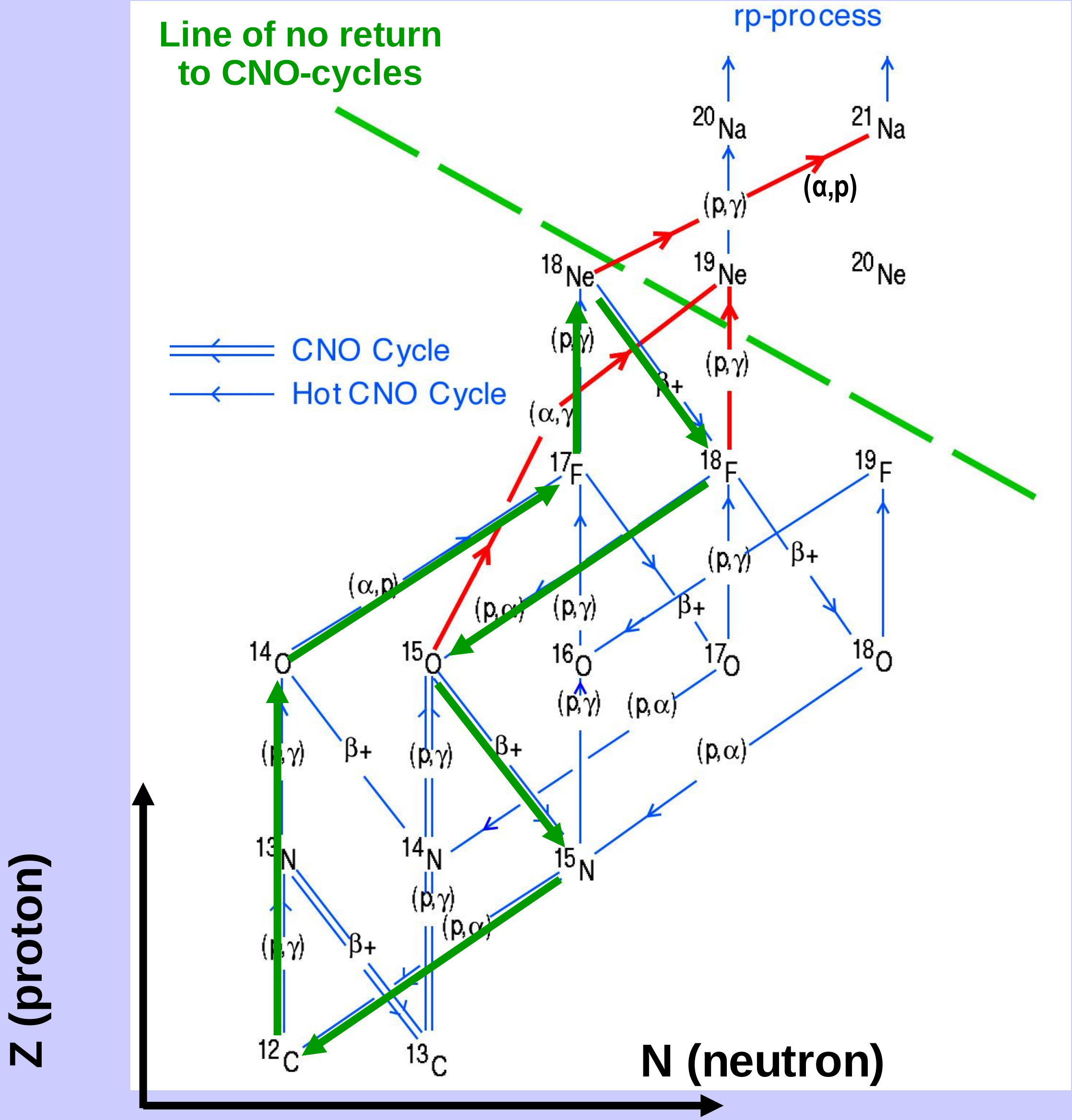
inverse reactions

$$A + a \Rightarrow B + b$$

or

$$B + b \Rightarrow A + a$$

example CNO-cycle



Reaction network:

- (p,γ)
- (α,γ)
- (α,p)
- (p,α)

β^+-decay

example CNO-cycle

three breakout reactions from hot CNO cycle into rp-process:

$^{18}F(p,\gamma)^{19}Ne$,

$^{15}O(\alpha,\gamma)^{19}Ne$

$^{18}Ne(\alpha,p)^{21}Na$

$^{18}Ne(\alpha,p)^{21}Na$ reaction is relevant at high temperature.

Two actual experimental approaches:

- Measure cross section of **$^{18}Ne(\alpha,p)^{21}Na$** with ^{4}He target and radioactive ^{18}Ne beam

- Measure cross section of inverse reaction: **$^{21}Na(p,\alpha)$ ^{18}Ne**

‚Principle of Detailed Balance' yields cross section for **$^{18}Ne(\alpha,p)^{21}Na$** reaction rate.

Problem: **^{21}Na** is also instable, half life: $t_{1/2}$ ~ 22 sec

solution: (1) in situ production of **^{21}Na** via **$d(^{20}Ne, ^{21}Na)n$** reaction

(2) then use weak secondary **^{21}Na** beam for **$p(^{21}Na,^{18}Ne)\alpha$** reaction

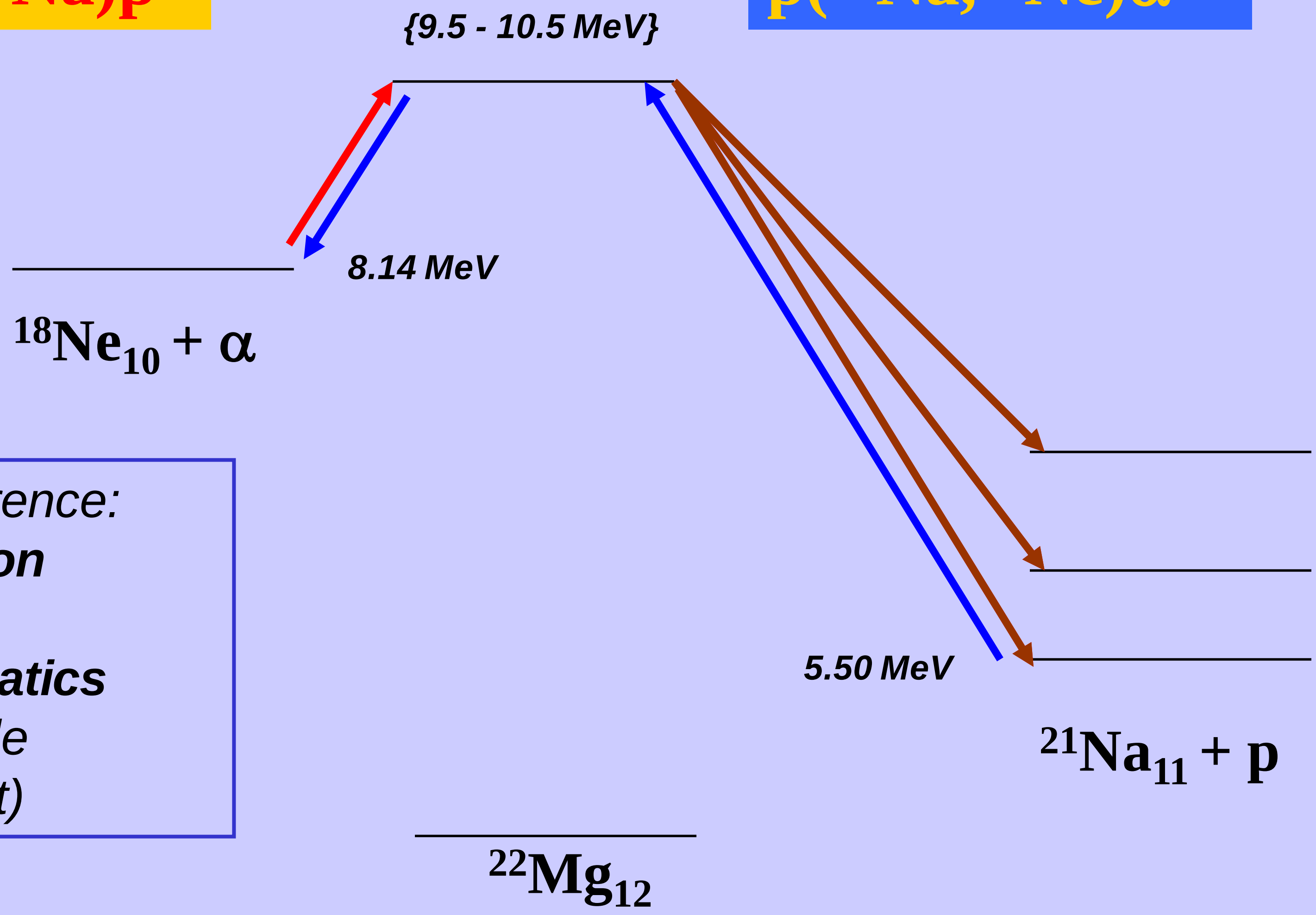
example CNO-cycle

$^{18}Ne\ (\alpha, p)\ ^{21}Na$

$\alpha(^{18}Ne, ^{21}Na)p$

$^{21}Na\ (p, \alpha)\ ^{18}Ne$

$p(^{21}Na, ^{18}Ne)\alpha$



Important difference:
inverse reaction

inverse kinematics
(heavy projectile
onto light target)