direct reaction

- Projectile interacts mainly with one or a few nucleons at the surface of the target nucleus, transfer of mass and energy is small.
- Sufficient beam energy is needed to be sensitive to the individual nucleon.
- Example

$$\lambda \approx 1\text{fm} \rightarrow E = 20\text{MeV} \qquad E_d = 10\,\text{MeV/u}$$

numbers: deuteron beam

$$v = 4{,}4\,\frac{\text{cm}}{\mu\text{s}} \qquad \beta = 15\%$$

Interaction time is very short $\left(\approx 10^{-22}\text{s}\right)$

this time period corresponds to a ‚transit time' of the projectile through the target nucleus.

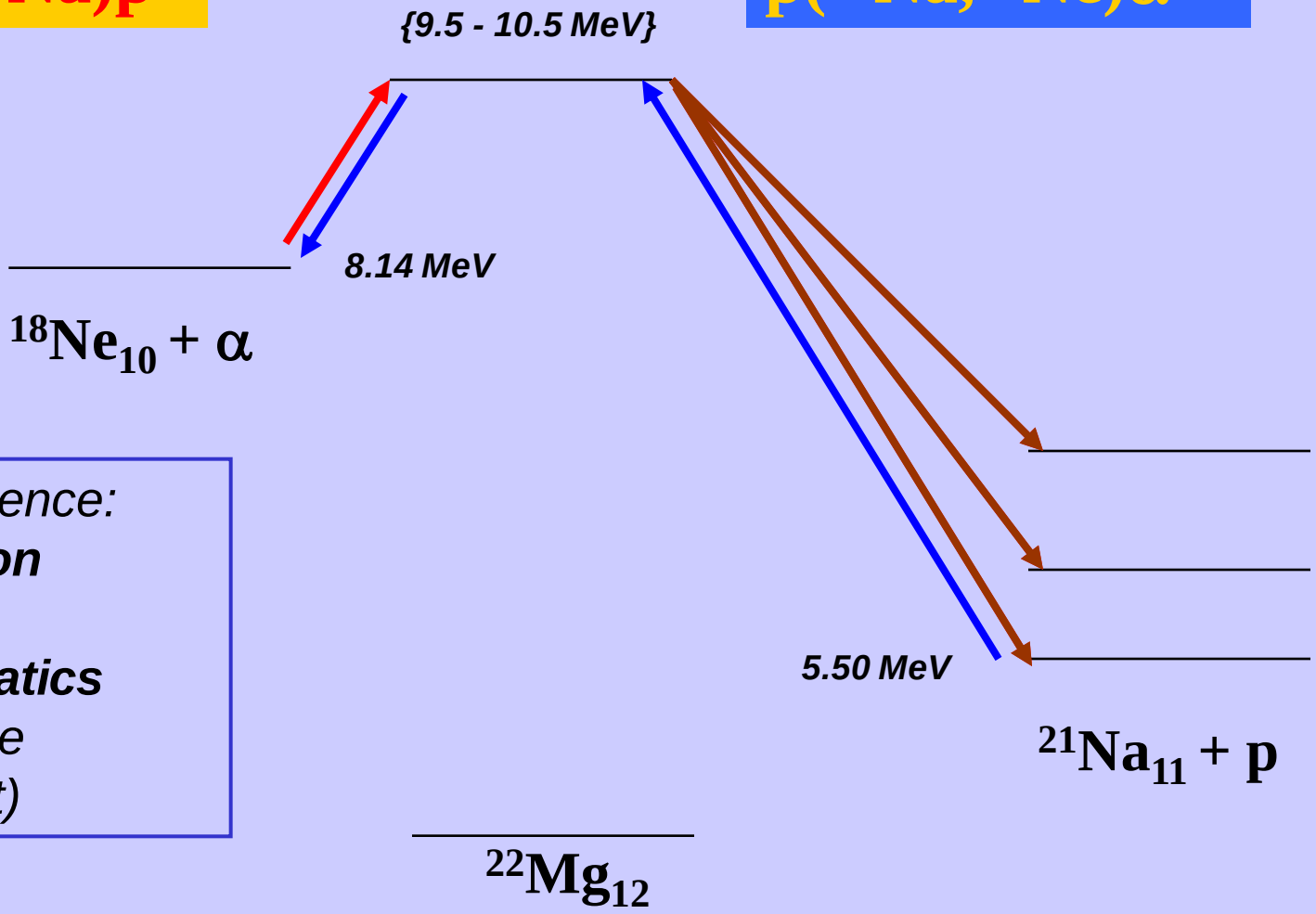
example CNO-cycle

^{18}Ne (α, p) ^{21}Na

α(^{18}Ne,^{21}Na)p

^{21}Na (p, α) ^{18}Ne

p(^{21}Na,^{18}Ne)α



Important difference:
inverse reaction

inverse kinematics
(heavy projectile onto light target)

Compound reaction

Projectile is absorbed by target nucleus.

Formation of a short lived intermediate state: compound nucleus

$$a + X \rightarrow Z^{*} \rightarrow Y + b$$

Course of compound nucleus reaction is in CM system symmetric.

With increasing energy of projectile also direct reactions or a so called pre-compound stage happens, compound formation is competing with these effects.

Bohr model

1. Formation

$$a + X \rightarrow Z^*$$

is enhanced when collision energy is close to the energy of an excited state of nucleus Z

intermediate state or resonance

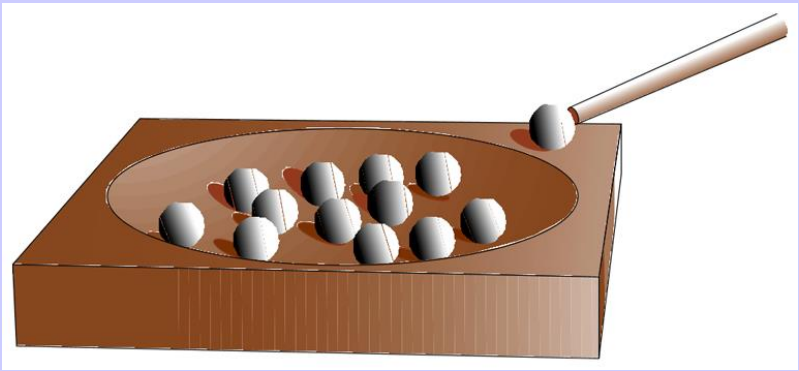
2. Decay

$$Z^* \rightarrow Y + b$$

Decay of state Z^* does not depend on the formation of the resonance Z^*. It depends only on the properties of the intermediate state.

illustration by N. Bohr from

Neutron capture and nuclear constitution,
Nature, 137 (1936) 344



Compound nucleus

Compound nucleus reaction

Reaction model is based on idea that atomic nuclei are treated as statistical ensemble (grand canonical ensemble).

Kinetic energy of projectile and the energy which is released e.g. by a capture into the target nucleus, are distributed in a statistical way (similar to heat dissipation in gas) onto all nuclear degrees of freedom.

There is the formation of a heated **Compound** nucleus. Temperature is gievn by the sum of bombarding energy and binding energy differences between projectile-target and compound system.

Compound nucleus

Formation und decay of compound nucleus

Formation probability is largely enhanced when excitation energy of reaction is close to the energy of an excited state.

Live time of a compound state is typically 10^{-16} s.

This is orders of magnitude slower then the time scale of fast reactions of typically 10^{-22} s like direct reactions.

example: After capture of slow neutrons with an energy of a few eV in nuclei of medium mass A~100-150 there is an energy release of typically 8 MeV from neutron binding energy.

In heavy nuclei the following resonance parameters are observed:

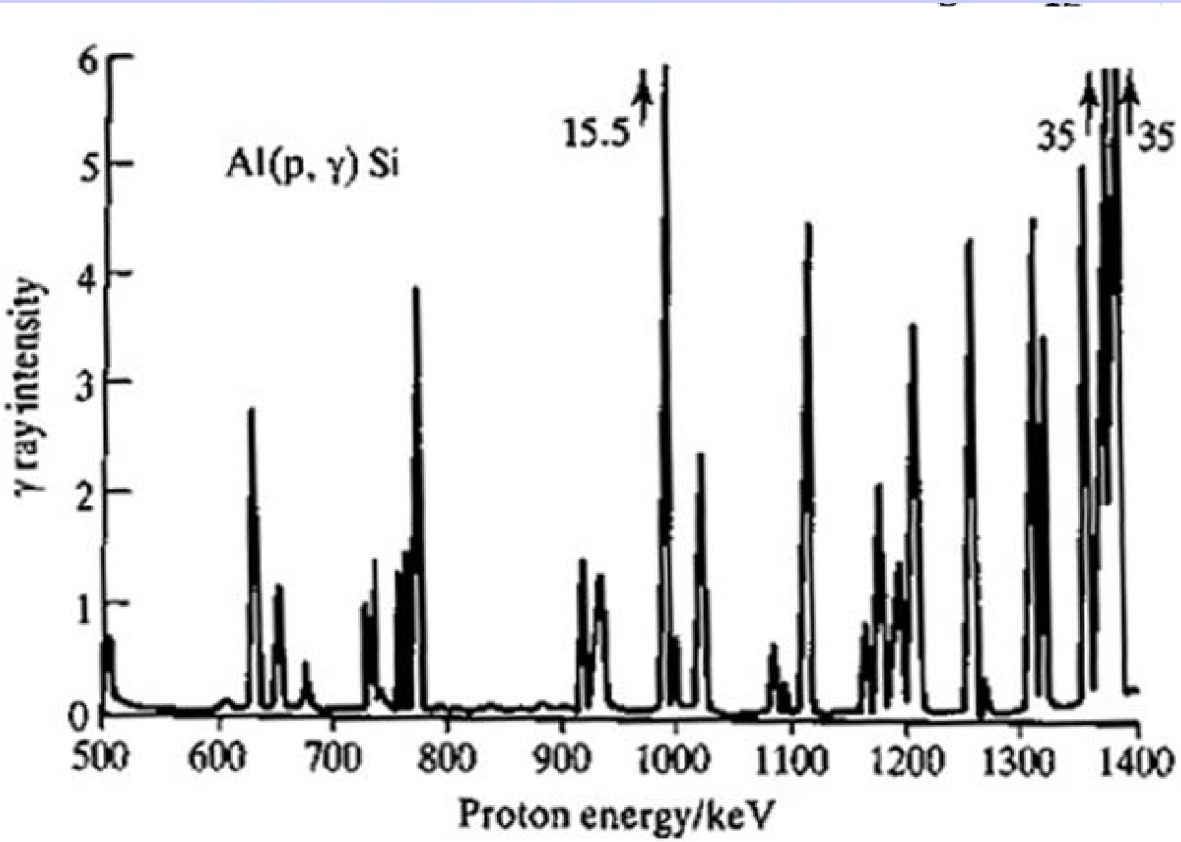
Width of neutron resonance: ~ 10^{-2} eV

Average distance between resonances: ~ 50 keV.

Compound nucleus

Formation and decay of compound nucleus are independent processes.

Cross section of nuclear reaction via an excited long lived compound state shows at low energies narrow resonances.



$^{27}Al(p,\gamma)\,^{28}Si$

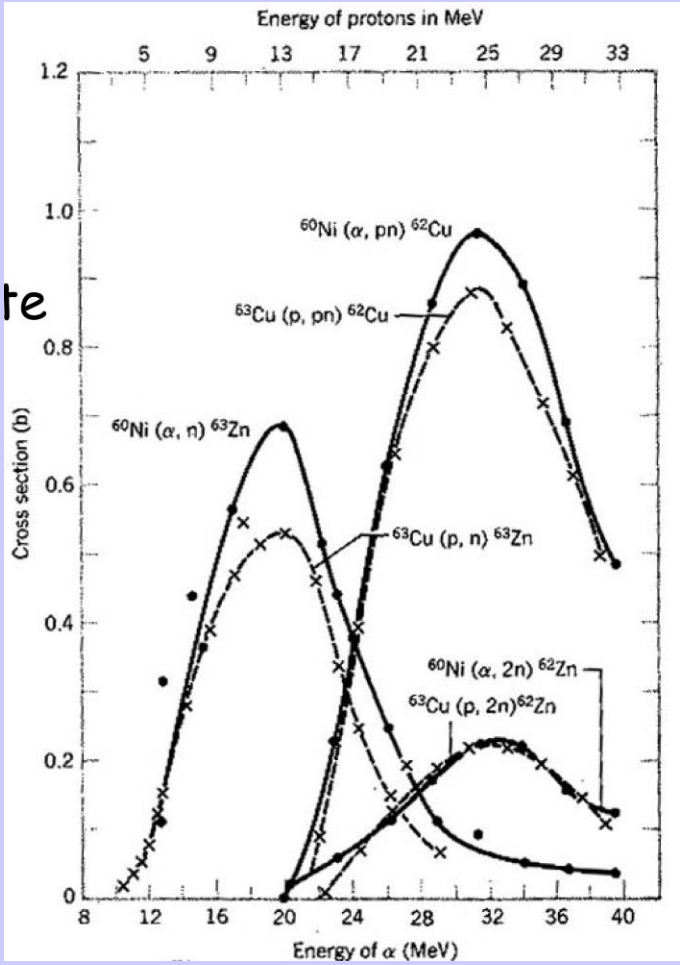
Compound nucleus

Compound nucleus formation:

$$p + {}^{63}_{29}Cu \rightarrow {}^{64}_{30}Zn^{*} \rightarrow {}^{63}Zn + n$$

$$\alpha + {}^{60}_{28}Ni \rightarrow {}^{64}_{30}Zn^{*} \rightarrow {}^{62}Cu + n + p$$

$$\sigma\left({}^{60}Ni(\alpha,n)^{63}Zn\right) \approx \sigma\left({}^{63}Cu(p,n)^{63}Zn\right)$$



Two different entrance channels create the same compound nucleus $^{64}Zn^{*}$ with very similar cross sections.

Resonance curve

Wave function for a decaying intermediate state Z*

with energy E_0 and live time τ

$$\psi(t) = \psi(0)\, e^{-i\omega_0 t}\, e^{-t/2\tau} \qquad E = \hbar\omega \qquad \hbar = 1$$

$$|\psi(t)|^2 = |\psi(0)|^2 e^{-t/\tau}$$

Exponential decay
τ decay constant

Frequency- or energy dependence is obtained from wave function via Fourier transformation of $\psi(t)$

$$f(\omega) = \int_0^\infty \psi(t) e^{i\omega t} dt$$

resonance curve

Fourier transformation:

$$f(\omega) = \int_0^\infty \psi(t) e^{i\omega t} dt$$

$$f(E) = \int_0^\infty \psi(t) e^{iEt} dt$$

$$= \int_0^\infty \psi(0) e^{-iE_0 t} e^{-t/2\tau} e^{iEt} dt$$

$$= \int_0^\infty \psi(0) e^{-t\left(i(E_0 - E) + \frac{1}{2\tau}\right)} dt$$

$$= \frac{\psi(0)}{(E_0 - E) - i/2\tau}$$

Resonance curve

Probability to find a state with energy E

$$E = f^* f$$

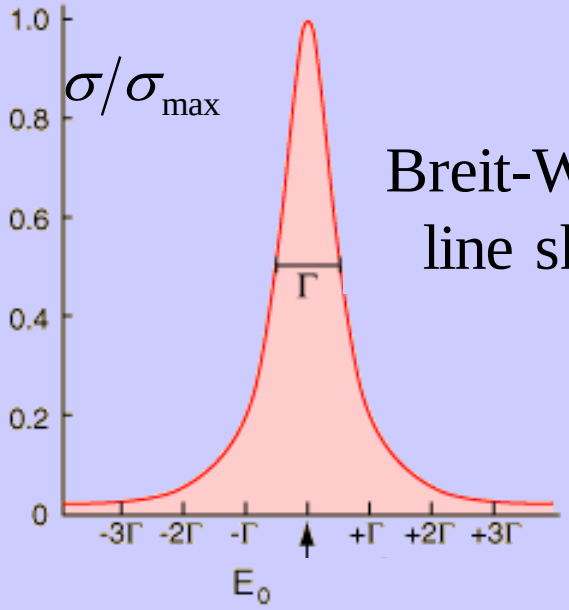
$$P(E) = f^*(E) \cdot f(E) = \frac{\psi(0)}{(E_0 - E) - i/2\tau} \cdot \frac{\psi(0)}{(E_0 - E) + i/2\tau}$$

$$= \frac{|\psi(0)|^2}{(E_0 - E)^2 + 1/4\tau^2}$$

Energy dependence – line shape

probability for Z* to have energy E

Full Width at Half Maximum FWHM=Γ



Breit-Wigner line shape

$$\frac{1}{\left(E_0 - E_0 + \Gamma/2\right)^2 + \frac{1}{4\tau^2}} = \frac{1}{2} 4\tau^2$$

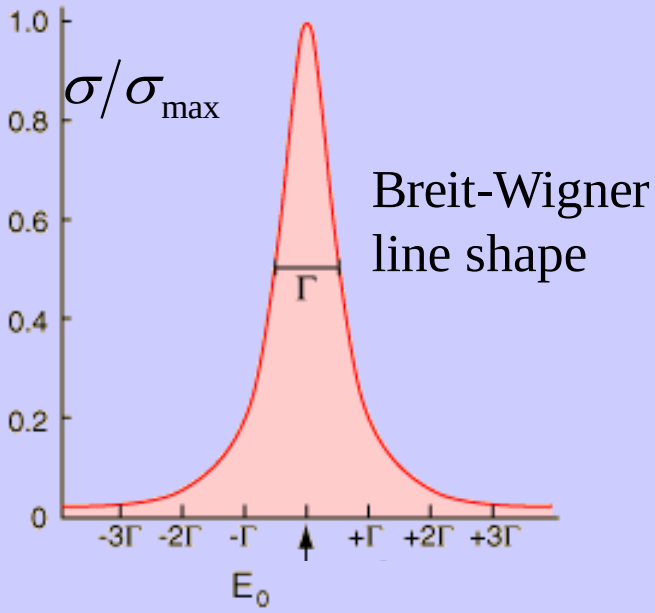
$$\frac{1}{\frac{\Gamma^2}{4} + \frac{1}{4\tau^2}} = 2\tau^2$$

$$\Rightarrow \frac{1}{\frac{\Gamma^2\tau^2 + 1}{4\tau^2}} = 2\tau^2$$

Energy width vs live time

$$\Rightarrow \frac{4\tau^2}{\Gamma^2\tau^2+1} = 2\tau^2$$

$$\Rightarrow \frac{2}{\Gamma^2\tau^2+1} = 1$$



Breit-Wigner line shape

$$\tau \cdot \Gamma = 1$$

$$\left(\Delta E \Delta t \approx 1 \qquad \hbar = 1\right)$$

Heisenberg's uncertainty principle

limited live time ⟺ uncertainty in energy

Breit-Wigner cross section

For Compound nucleus reaction

$$i \rightarrow Z \rightarrow f$$

The transition probability is given for

$$W_{\mathrm{if}} = \frac{2\pi}{\hbar} \left| M_{\mathrm{if}} \right|^2 \rho\left(E_{\mathrm{f}}\right)$$

$\left|M_{\mathrm{if}}\right|$ *matrix element is deduced from second order perturbation theory*

$$M_{\mathrm{if}} = \sum_{\mathrm{Z}} \frac{M_{\mathrm{iZ}} M_{\mathrm{Zf}}}{E\text{-}E_Z}$$

sum takes all intermediate states into account

Breit-Wigner cross section

assumption only one intermediate state with:

$$\psi(t) = \psi(0)\, e^{-\frac{i}{\hbar}\left(E_0 - i\frac{\Gamma}{2}\right)t}$$

This is an excited state with energy:

$$E_Z = E_0 - i\Gamma/2$$

$$\left|M_{if}\right|^2 = \frac{\left|M_{iZ}\right|^2 \left|M_{Zf}\right|^2}{\left(E\text{-}E_0\right)^2 + \Gamma^2/4}$$

Absolute value of complex number

$$E_Z = E_0 - i\frac{\Gamma}{2}$$

$$\left|E_Z\right| = \sqrt{E_0^2 + \frac{\Gamma^2}{4}}$$

Total cross section & phase space

$$\sigma_{if} = \frac{2\pi}{\hbar}\left|M_{if}\right|^2\left(\frac{4\pi}{h^3}\frac{p_f^2}{v_f}\right)\frac{1}{v_i}$$

One particle in normalisation volume

decay of intermediate state

$$\Delta E \Delta t = \hbar$$

rate:

$$\Gamma \cdot \tau = \hbar$$

$$\tau = \hbar / \Gamma$$

Heisenberg‘s uncertainty principle:
width (energy) vs life time (decay rate)

decay into a specific final state f with width Γ_f is called partial decay width.

Total decay rate

Decay into a specific final state f with width Γ_f is called partial decay width.

$$\Gamma = \sum_{j=1}^{n} \Gamma_j$$

Total width

partial width

Probability for decay into a specific state j:

$$\Gamma_j / \Gamma$$

branching ratio

Decay and formation of compound state

decay of intermediate state Z

transition probability ~ decay rate ~ energy width

$$\Gamma_f = 2\pi \left| M_{Zf} \right|^2 \frac{4\pi}{h^3} \frac{p_f^2}{v_f} \qquad \Gamma(Z \to f)$$

formation of state Z

transition probability ~ cross section ~ energy width

$$\Gamma_i = 2\pi \left| M_{iZ} \right|^2 \frac{4\pi}{h^3} \frac{p_i^2}{v_i} \qquad \Gamma(Z \to i)$$

Detailed Balance

Erinnerung: Einheiten

$$\Gamma(Z \to f) = \sigma \cdot \hbar \cdot n \cdot v_i = \left[m^2 \cdot MeVs \cdot \frac{1}{m^3} \cdot \frac{m}{s} \right]$$

cross section compound reaction

$$\sigma_{if} = \frac{2\pi}{\hbar} \frac{|M_{iZ}|^2 |M_{Zf}|^2}{(E - E_0)^2 + \Gamma^2/4} \cdot \left(\frac{4\pi}{h^3} \frac{p_f^2}{v_f} \right) \frac{1}{v_i}$$

$$|M_{Zf}|^2 = \Gamma_f \cdot \frac{1}{2\pi} \left(\frac{4\pi}{h^3} \frac{p_f^2}{v_f} \right)^{-1} = \Gamma_f \frac{v_f}{p_f^2} \pi\hbar^3$$

$$|M_{iZ}|^2 = \Gamma_i \cdot \frac{1}{2\pi} \left(\frac{4\pi}{h^3} \frac{p_i^2}{v_i} \right)^{-1} = \Gamma_i \frac{v_i}{p_i^2} \pi\hbar^3$$

mit $\frac{h^3}{8\pi^2} = \frac{h^3\pi}{(2\pi)^3} = \pi\hbar^3$

cross section

$$\sigma_{if} = \frac{2\pi}{\hbar}\frac{\Gamma_i\Gamma_f}{2\pi\cdot 2\pi}\cdot\frac{1}{(E-E_0)^2+\frac{\Gamma^2}{4}}\cdot\frac{1}{v_i}\left(\frac{4\pi}{h^3}\frac{p_f^2}{v_f}\right)^{-1}\left(\frac{4\pi}{h^3}\frac{p_f^2}{v_f}\right)\left(\frac{4\pi}{h^3}\frac{p_i^2}{v_i}\right)^{-1}$$

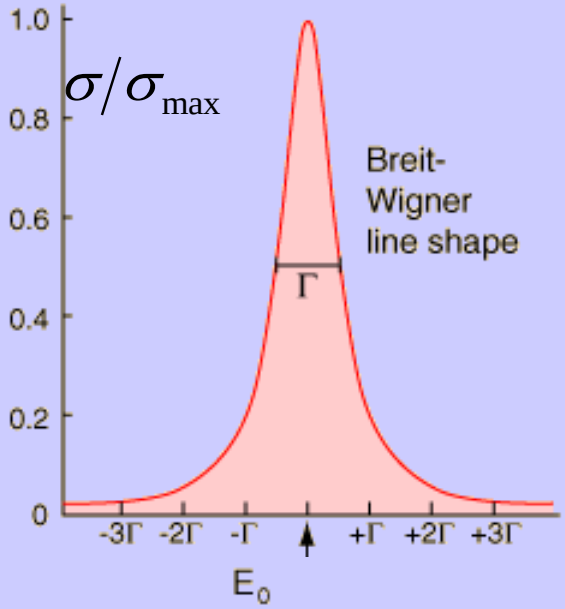
$$= \frac{2\pi}{\hbar}\frac{\Gamma_i\Gamma_f}{2\pi}\cdot\frac{1}{(E-E_0)^2+\frac{\Gamma^2}{4}}\cdot\frac{1}{v_i}\frac{v_i}{p_i^2}\pi\hbar^3$$

$$= \pi\hbar^2\frac{\Gamma_i\Gamma_f}{p_i^2}\cdot\frac{1}{(E-E_0)^2+\frac{\Gamma^2}{4}}$$

with $p_i = \frac{\hbar}{\lambda}$

p_i = c.m. momentum

Breit-Wigner cross section



$$\sigma_{if} = g \cdot \pi \cdot \lambda\!\!\!^{-2} \frac{\Gamma_i \Gamma_f}{(E - E_0)^2 + \frac{\Gamma^2}{4}}$$

Breit-Wigner cross section

g takes spin values (spin degeneracy) of the m-substates into account; for initial state $a+X$ and compound state Z^*

$$g = \frac{(2J_z + 1)}{(2J_X + 1)(2J_a + 1)}$$

$$a + X \rightarrow Z^* \rightarrow Y + b$$

comments & consequences

- Total scattering cross section

$$\sigma_{\mathrm{T}} = \sum_{\mathrm{f}} \sigma_{\mathrm{i}\to\mathrm{f}}$$

replace Γ_{f} by Γ in Breit-Wigner cross section

- Elastic scattering: $\sigma_{\mathrm{el}} = \sigma_{\mathrm{i}\to\mathrm{i}}$

with $\Gamma_{\mathrm{f}} = \Gamma_{\mathrm{i}}$

$$\sigma_{\mathrm{if}} = \frac{g\pi\lambda\!\!\!^{-2}\Gamma_{\mathrm{i}}^2}{\left(E - E_0\right)^2 + \frac{\Gamma^2}{4}}$$

remarks

- Resonance scattering $E = E_0$

$$\sigma_{if} = 4\pi\lambda\!\!\!^{-2} g \frac{\Gamma_i \Gamma_f}{\Gamma^2}$$

elastic and total cross section

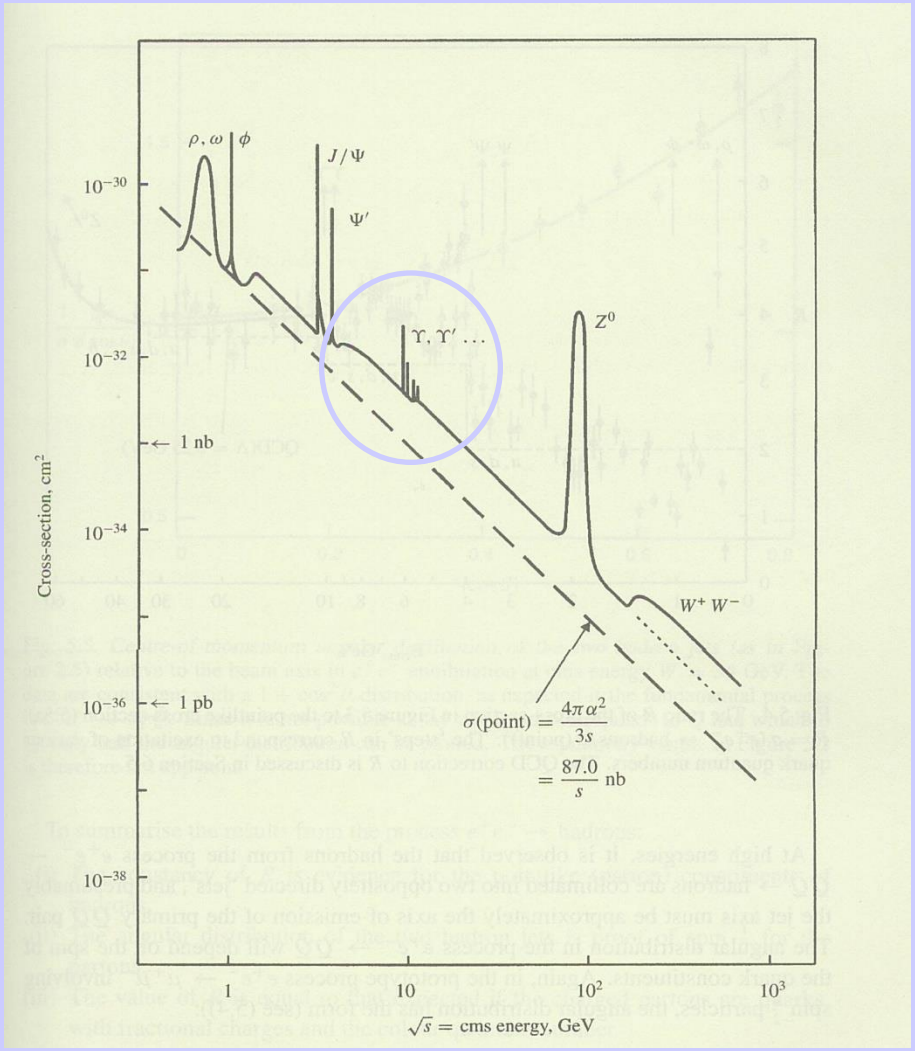
$$\sigma_{el} = 4\pi\lambda\!\!\!^{-2} g \frac{\Gamma_i^2}{\Gamma^2} = 4\pi\lambda\!\!\!^{-2} g \cdot B_i^2$$

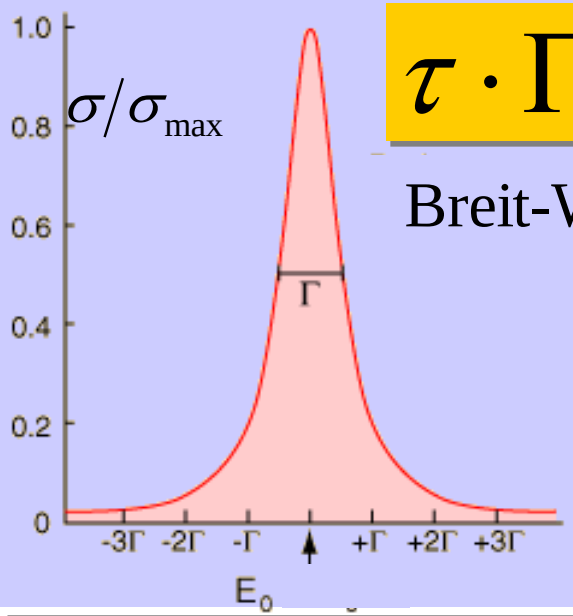
with $B_i = \frac{\Gamma_i}{\Gamma}$

$$\sigma_T = 4\pi\lambda\!\!\!^{-2} g \frac{\Gamma_i \Gamma}{\Gamma^2} = 4\pi\lambda\!\!\!^{-2} g \cdot B_i$$

$$\frac{\sigma_{el}}{\sigma_T} = \frac{\Gamma_i^2}{\Gamma^2} \cdot \frac{\Gamma}{\Gamma_i} = \frac{\Gamma_i}{\Gamma} = B_i$$

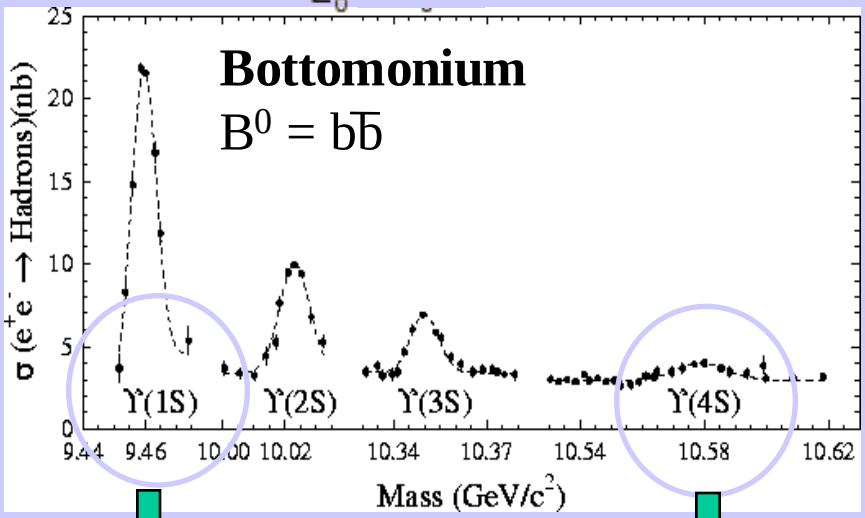
resonance cross section & bottomonium





$$\tau \cdot \Gamma = 1$$

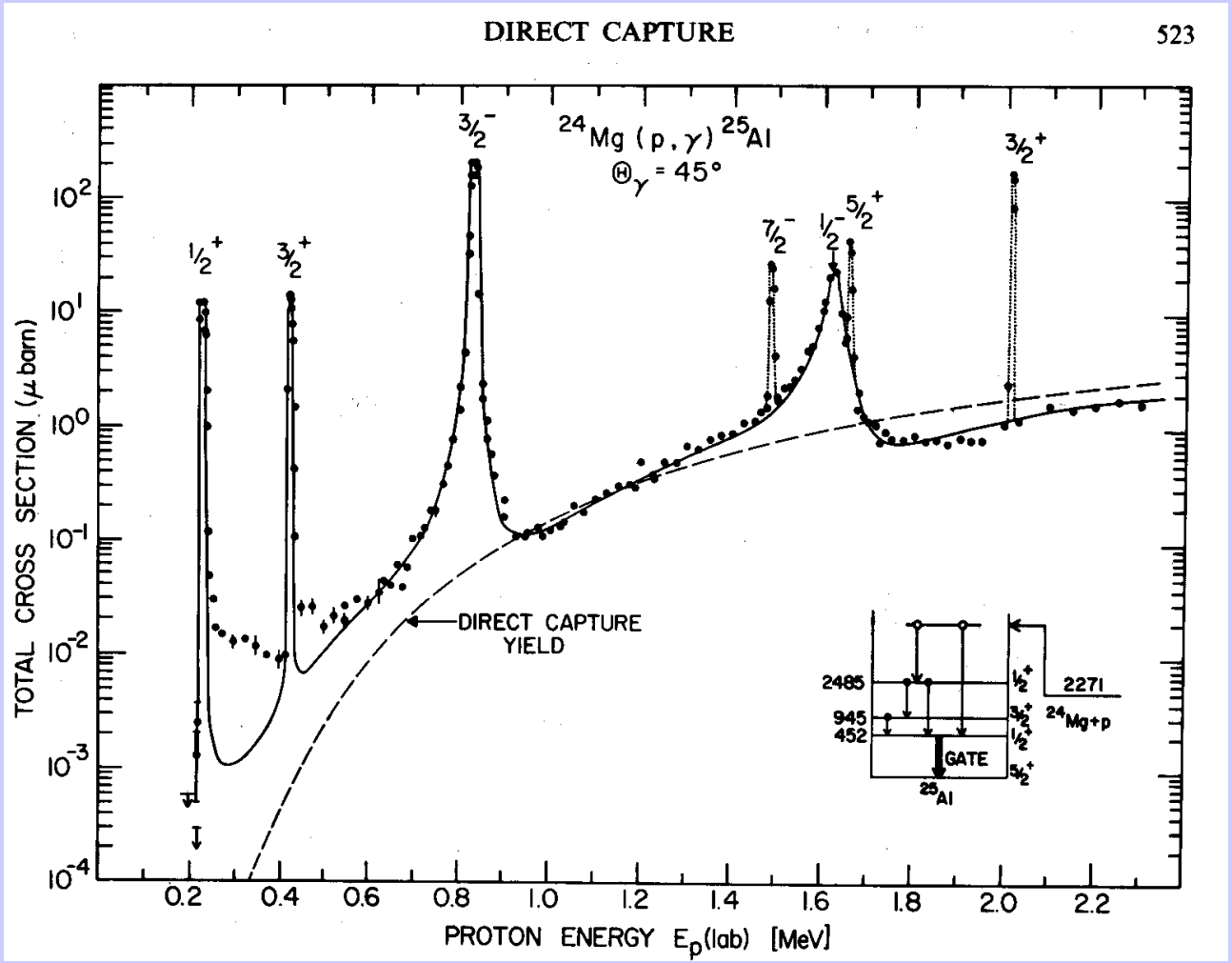
Breit-Wigner



width: 53,0 keV

width: 20,5 MeV

resonance cross section



remarks

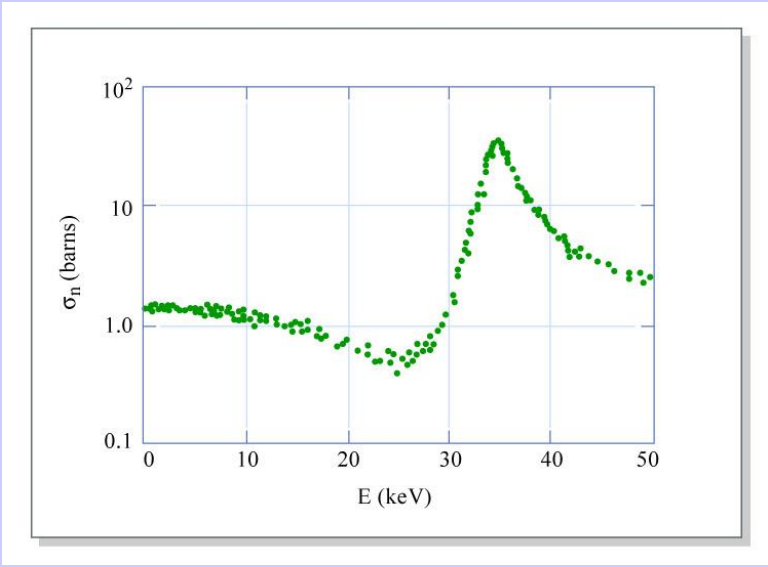
- There was only one resonance. There can be several overlapping resonaces.
- Resonance amplitude $\sim \frac{1}{i(E_0 - E) + \Gamma/2}$

fast change of phase while crossing the resonance energy

Typically there are two processes:

- non resonant scattering
  e.g. Rutherford scattering
- resonant scattering

interferences changes the shape of the resonances, typical asymmetric lines respectively interferences in cross section



example: n scattering with ^{27}Al