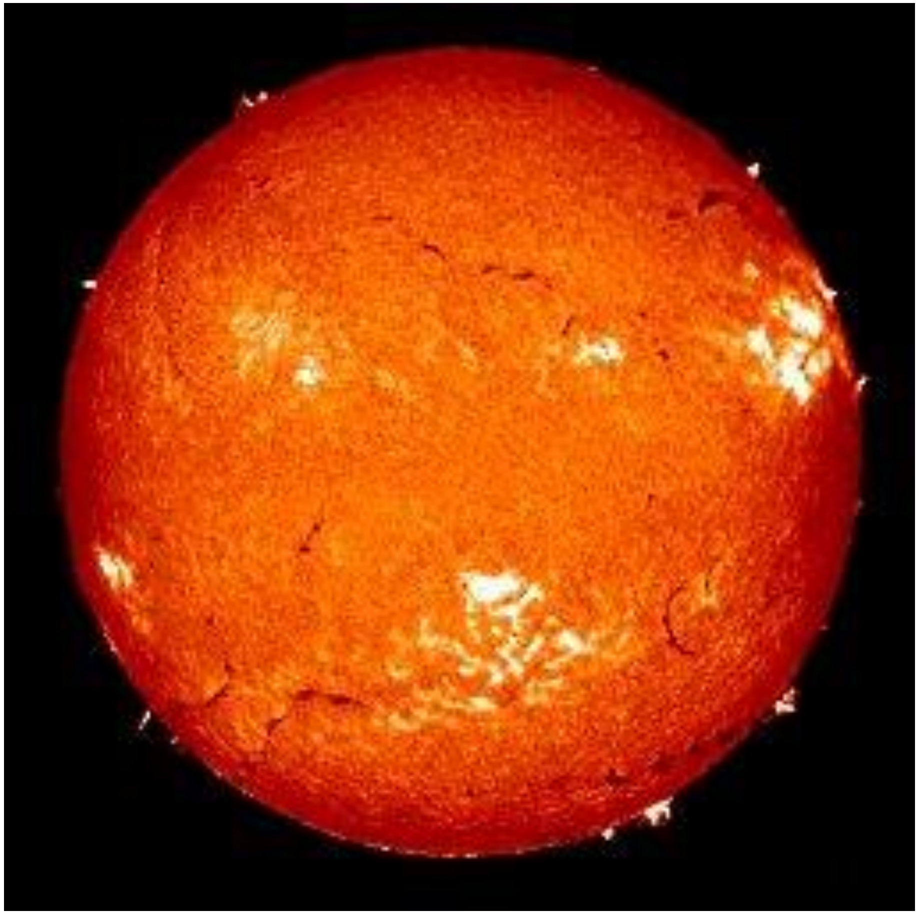
Hydrogen Burning

Interlude: the sun



From Rolfs & Rodney, Cauldrons in the Cosmos, Chapter 6

sun: observables

observable	Sym.	value	observation
average distance	a	149 Mio km	Venus radar
mass	$M_\odot$	$1.99 \cdot 10^{33}$g	earth orbit
diameter angle	d	31´59´´	direct measurment
diameter	D	$1.39 \cdot 10^{11}$cm	from d and a
average density	ρ	1.41 g/cm^3	from M and D
solar constant	S	$1.368 \cdot 10^{6}$ erg/s/cm^2	bolometer
Luminosity	$L_\odot$	$3.8 \cdot 10^{33}$ erg/s	from S and a
effective temperature	T_{eff}	5800K	from L and R
Rotation period at equator	$P_{Äqu}$	24d16h	observation of sunspots

Energy production in sun

- Suggestion: sun is powered by chemical process (e.g. coal burning)
  - $L = 3.86 \times 10^{33}$ erg/s ⇔ oxidation 1.4×10^{19} kg per second
  - age:

$$t = \frac{2 \times 10^{30}\,\text{kg}}{1.4 \times 10^{19}\,\text{kg/s}} = 4000\,\text{a}$$

- Far too short, insufficient energy production mechanism

Energy production in sun

- Suggestion: gravitational binding energy
  - Binding energy:

$$\left|E_{\mathrm{grav}}\right| \approx \frac{GM^2}{R} = 3.8 \times 10^{48}\,\mathrm{erg}$$

  - luminosity: $L = 3.86 \times 10^{33}$ erg/s
  - age:

$$t = \frac{\left|E_{\mathrm{grav}}\right|}{L} = 10^{15}\,\mathrm{s} = 30 \times 10^{6}\,\mathrm{a} << t_{\oplus}$$

  - Too short

Energy production in sun

- $\Rightarrow$ fusion of nuclei is possible energy source
- High temperature and high density is needed
  $\Rightarrow$ at the centre of the sun
- Sequence of nuclear reactions: proton-proton chain

1. $p + p \rightarrow {}^{2}D + e^{+} + \nu_e$
2. ${}^{2}D + p \rightarrow {}^{3}He + \gamma$
3. ${}^{3}He + {}^{3}He \rightarrow {}^{4}He + p + p$

Total: $4\ p \rightarrow {}^{4}He + 2\ e^{+} + 2\nu_e$

Introduction

proton proton chain

$4p \rightarrow {}^4He + 2e^+ + 2\nu \quad Q = 26.73 MeV$

possible first steps:

$p + p \rightarrow {}^2He$ Diproton unbound nucleus

$({}^2He + p \rightarrow {}^3Li$ also unbound)

- Weak interaction process

$p + p \rightarrow d + e^+ + \nu$

Q-value: $Q = 1.44$ MeV

kinetic energy: $E_{kin} = Q - 2m_e c^2 = 0.42 MeV$

$2m_p + 2m_e \rightarrow m_d + 3m_e \Rightarrow m_H + m_H \rightarrow m_{2H} + 2m_e$

kinetic energy is shared by positron and neutrino

- Theoretical cross section for the p + p reaction

total Hamiltonian: $H = H_n + H_\beta$

but H_β much smaller then $H_n \Rightarrow$ perturbation theory

Fermis Golden Rule:

$$d\sigma = \frac{2\pi}{\hbar} \frac{\rho(E)}{v_i} \left| \langle f | H_\beta | i \rangle \right|^2$$

$\rho(E)$ statistical factor, density of final states

v_i relative velocity in incident channel

$\left| \langle f | H_\beta | i \rangle \right|$ transition matrix element

$| i \rangle \equiv p + p$

$| f \rangle \equiv d + e^+ + \nu$

Phase space

$$\rho(E) = \frac{dN}{dE} \text{ number of final states dN in energy interval dE (E and E + dE)}$$

total phase space divided by unit volume $\quad dn = V\,\frac{4\pi p^2 dp}{h^3}$

applying to pp reaction $\rightarrow$ states available for positrons and neutrinos

$$\mathrm{dN} = dn_e \cdot dn_\nu = \left(V\,\frac{4\pi p_e^2 dp_e}{h^3}\right)\left(V\,\frac{4\pi p_\nu^2 dp_\nu}{h^3}\right)$$

assume :- zero rest mass of neutrino $\mathrm{m}_\nu = 0$

- zero recoil energy of deuteron $\mathrm{E}_{kin,d} = 0$

$\Rightarrow$ Energy is shared by positron and neutrino

$$\mathrm{E} = \mathrm{E}_e + \mathrm{E}_\nu = \mathrm{E}_e + c\mathrm{p}_\nu$$

$$p_\nu = \frac{E}{c} - \frac{E_e}{c} \quad \text{for constant } \mathrm{p_e}$$

$$\frac{dp_\nu}{dE} = \frac{1}{c} \qquad dp_\nu = \frac{1}{c}dE \qquad \frac{dp_\nu}{dE_e} = -\frac{1}{c}$$

$$\text{neutrino level density}: \frac{dn_\nu}{dE} = \left(V\,\frac{4\pi p_\nu^2}{h^3}\,\frac{dp_\nu}{dE}\right) = V\,\frac{4\pi}{h^3}\,p_\nu^2\,\frac{dp_\nu}{dE} = V\,\frac{4\pi}{h^3}\,\frac{1}{c^2}\left(E - E_e\right)^2\frac{1}{c}$$

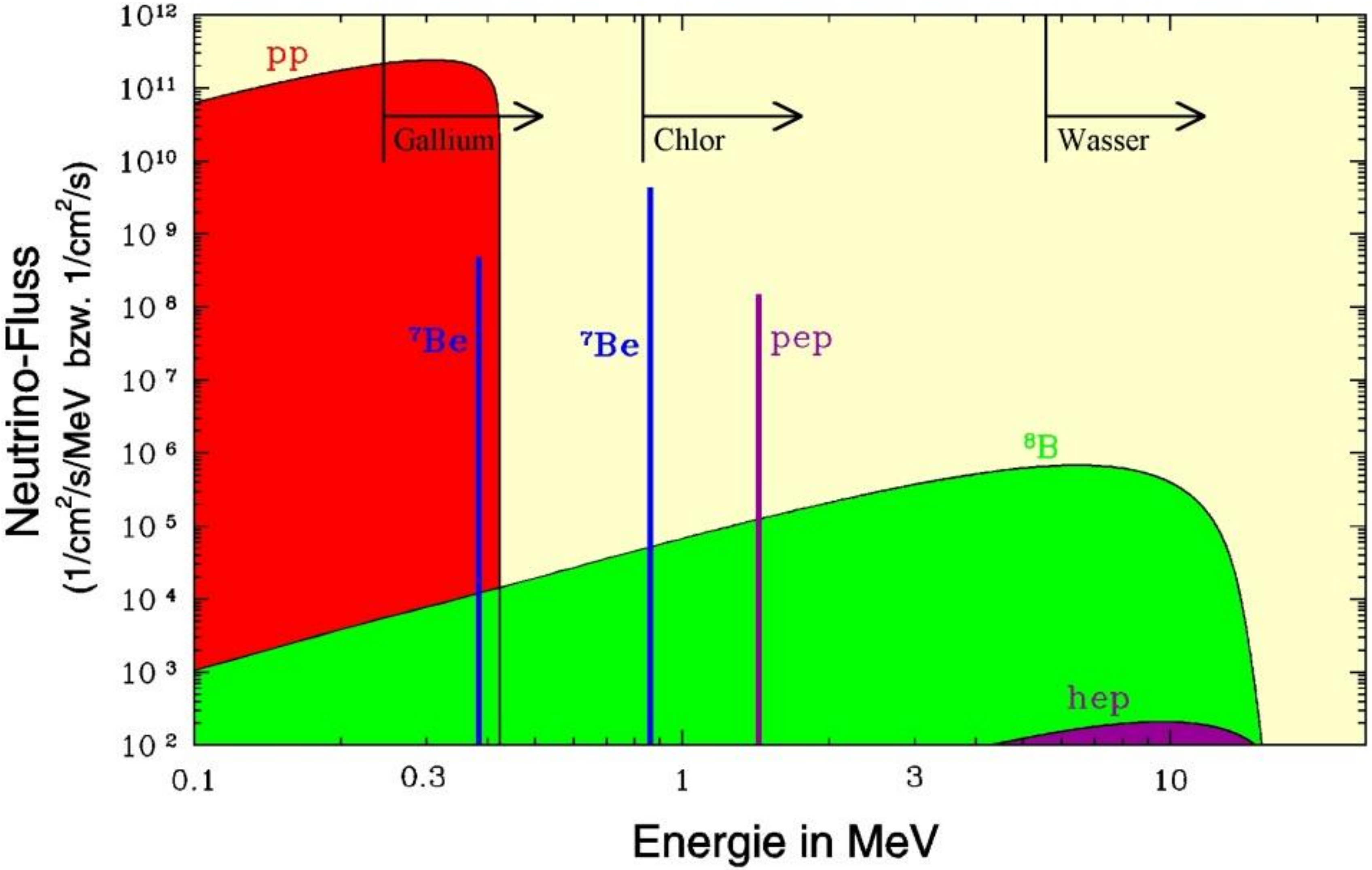
$$dn_e = \left(V\,\frac{4\pi p_e^2 dp_e}{h^3}\right)$$

$$\rho(E) = \frac{dN}{dE} = dn_e \cdot \frac{dn_\nu}{dE} = \left(V\,\frac{4\pi p_e^2 dp_e}{h^3}\right)\cdot V\,\frac{4\pi}{h^3}\,\frac{1}{c^2}\left(E - E_e\right)^2\frac{1}{c}$$

$$\rho(E) = V^2\,\frac{16\pi^2}{h^6}\,\frac{1}{c^3}\,p_e^2\left(E - E_e\right)^2 dp_e = \rho(E_e)dp_e$$

Phase space

$\rho(E) = \frac{dN}{dE}$ number of final states dN in energy interval dE (E and E + dE)



$$\rho(E) = V^2 \frac{16\pi^2}{h^6} \frac{1}{c^3} p_e^2 (E - E_e)^2 dp_e = \rho(E_e) dp_e$$

Detectors: main features

$R_{lab} = \sigma \cdot \varepsilon \cdot I_p \cdot \rho \cdot N_{av} / A$ → **High efficiency**

$R_{lab} > B_{nat} + B_{beam\ induced}$

$= B_{cosm} + B_{env}$

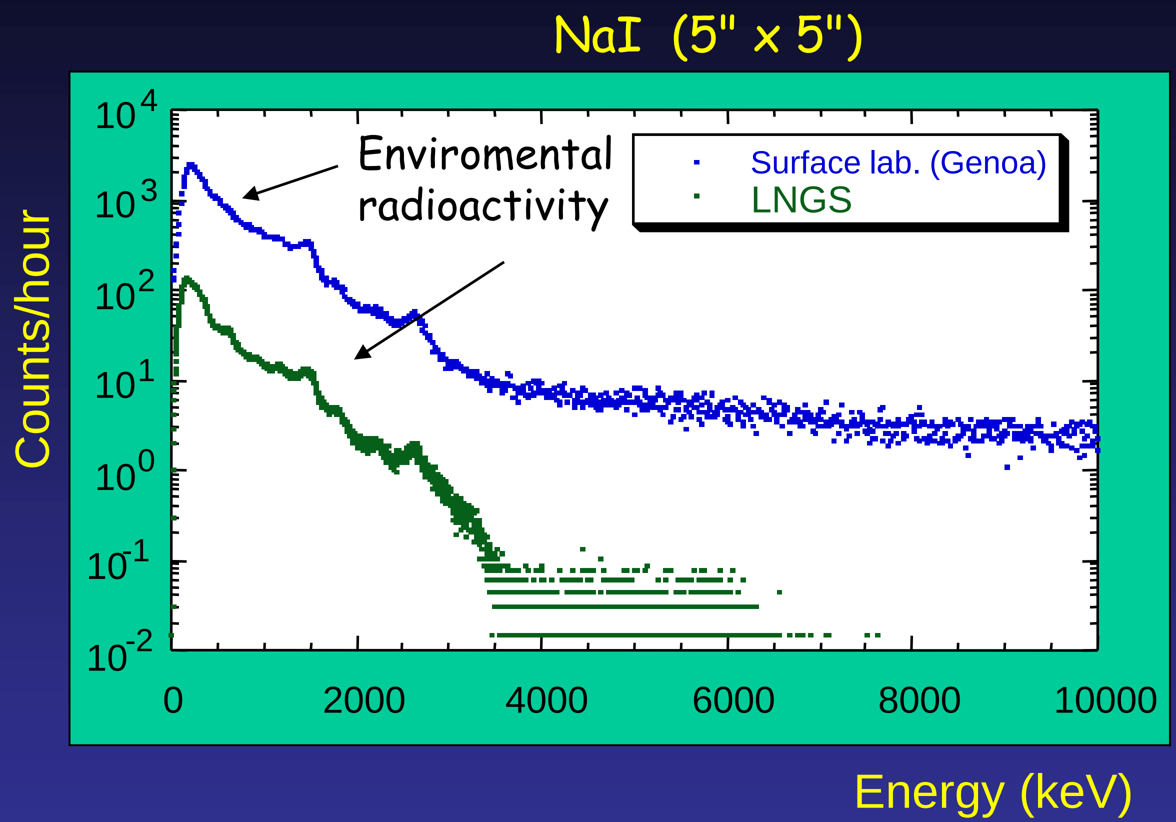
→ Enviromental radioactivity has to be considered underground

→ Low beam induced bck (pure beam & targets)

→ High resolution

Reduction of cosmic background @ LNGS

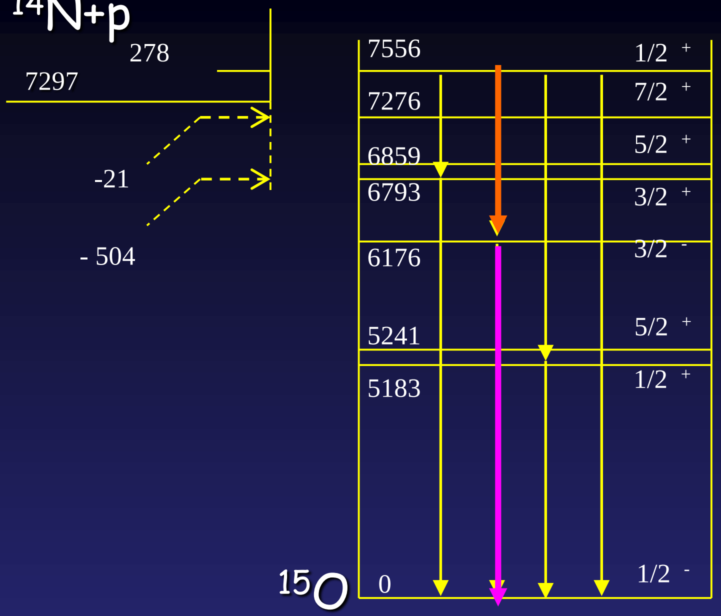
Example: γ-detectors



At E_γ > 3 MeV the background is drastically reduced

Radiative reaction measurements are favoured underground especially for ***high Q-value reactions***

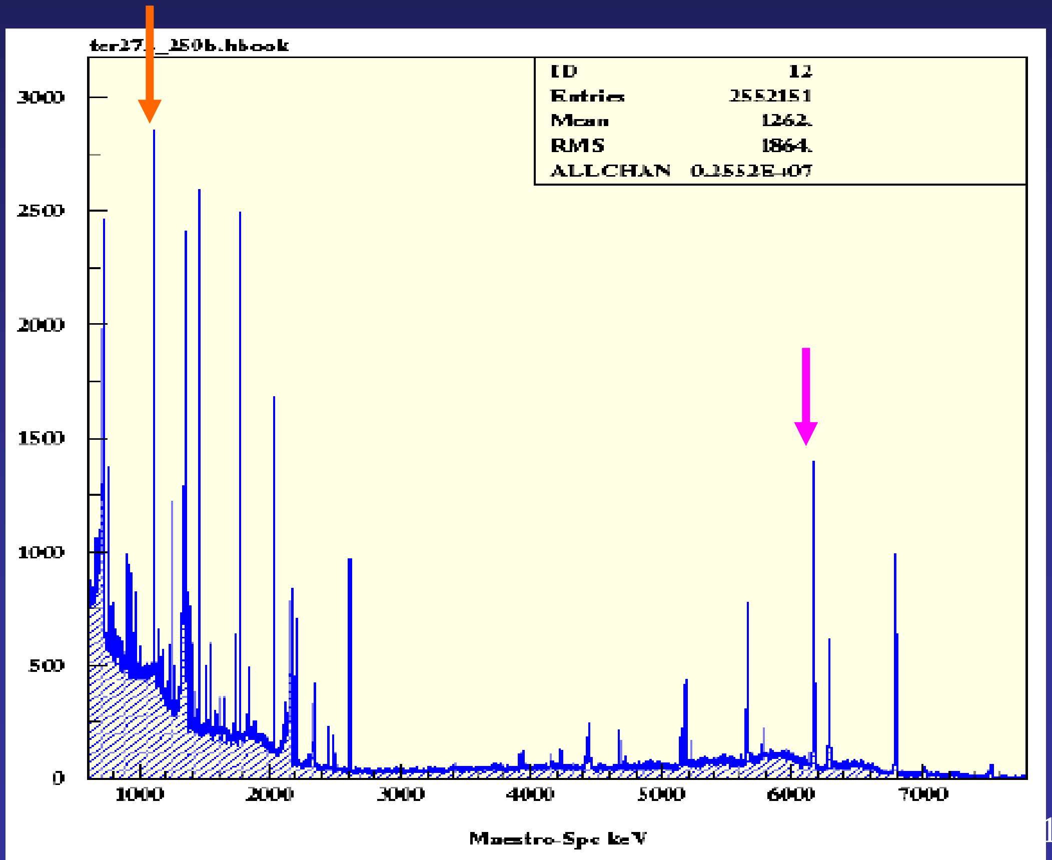
Which kind of detector? (an example)



$^{14}N(p,\gamma)^{15}O$ Q = 7.3 MeV

several γ cascades

If information on single γ transition is required $\rightarrow$ high resolution detector



HpGe detector

High resolution $\Delta E/E \sim 0.2\%$

Lower efficiency

For σ_{tot} measurements → 4π detector

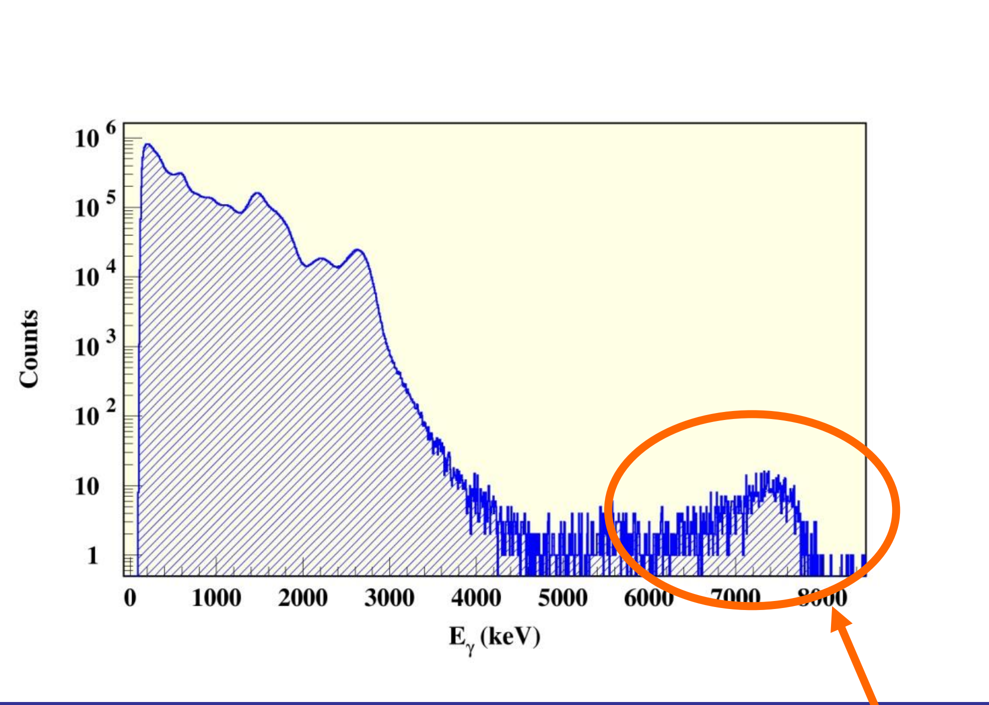
BGO summing crystal ε = 65%

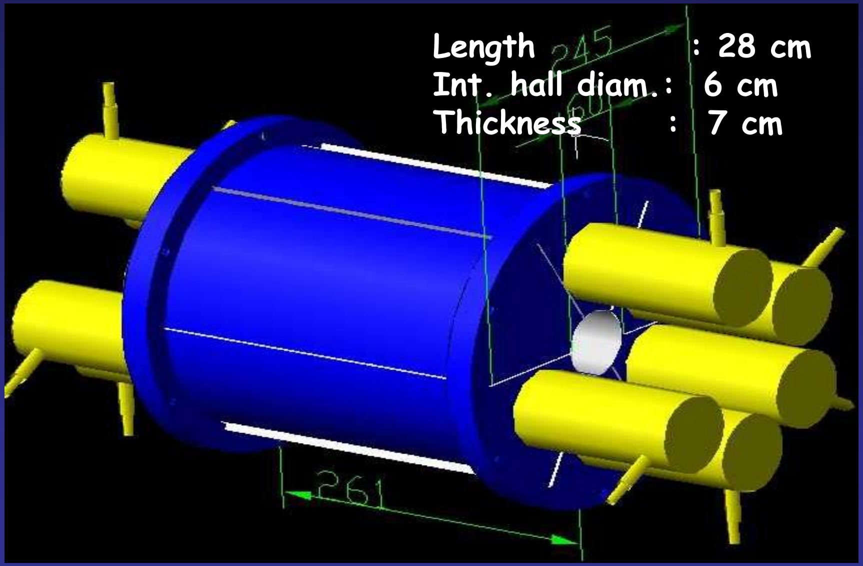
$^{14}N(p,\gamma)^{15}O$

High efficiency

Low resolution

BGO reaction spectrum E_p=120 keV

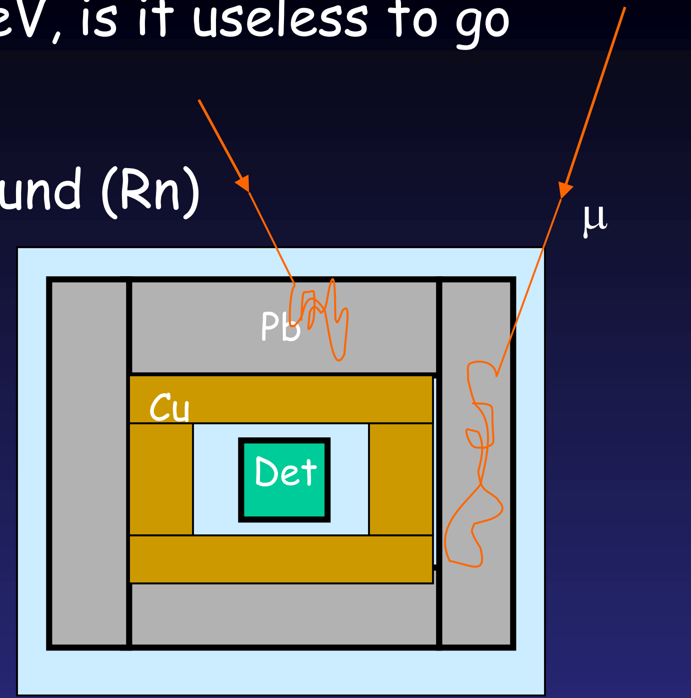




All the γ cascades are summed together to a summing peak at E_γ=Q+E_{cm}

If the Q-value of the nuclear reaction is < 3MeV, is it useless to go underground ? Cu

Enviromental radioactivity is present underground (Rn)



Detectors can be shielded passively with proper Pb-Cu shield as on surface

BUT underground passive shielding is more effective since μ flux, that create secondary γs in the shield, is suppressed.

Example: $^{4}He(^{3}He,\gamma)^{7}Be$

Q=1.6 MeV

3 γs emitted

Detector: HpGe 135%

Expected attenuation for 1.6 MeV γs: 10^{-5}-10^{-6} (GEANT4 simulations)