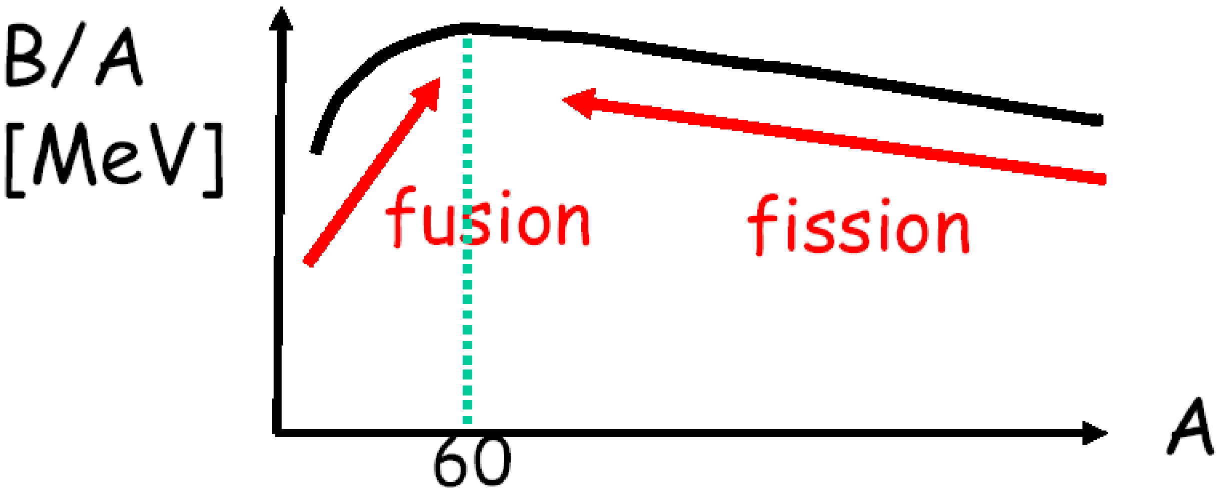
Fusion and fission



Most stable form of nuclear matter is at A~60. Expect a large amount of energy released in the fission of a heavy nucleus into two medium nuclei and in the fusion of two light nuclei into a single medium nucleus.

$$B(A,Z) = a_V A - a_S A^{2/3} - \frac{a_C Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + \delta(A)$$

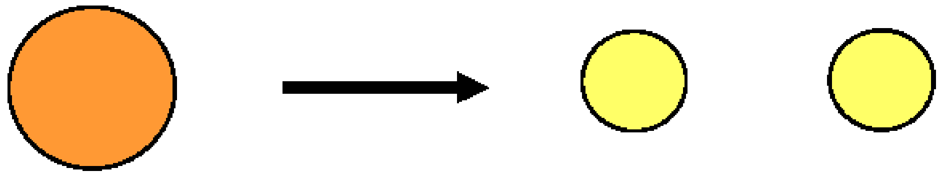
Fusion and fission

Fission occurs because the total Coulomb repulsion energy of p's in a nucleus is reduced if the nucleus splits into two smaller nuclei. The nuclear surface energy increases in the process, but its magnitude is much smaller.

Fusion occurs because the two low *A* nuclei have too large a surface area for their volume. The surface area decreases when they amalgamate. Coulomb energy increases, but its magnitude is too small.

Spontaneous fission

Spontaneous Fission



(A,Z) (A_1,Z_1) (A_2,Z_2)

Expect spontaneous fission to occur if

$$Q = B(A_1,Z_1) + B(A_2,Z_2) - B(A,Z) > 0$$

Assume $\frac{A_1}{A} = \frac{Z_1}{Z} = y_1$ and $\frac{A_2}{A} = \frac{Z_2}{Z} = y_2$; $y_1 + y_2 = 1$

$$Q = a_s A^{2/3} - a_s A_1^{2/3} - a_s A_2^{2/3} + a_c \frac{Z^2}{A^{1/3}} - a_c \frac{Z_1^2}{A_1^{1/3}} - a_c \frac{Z_2^2}{A_2^{1/3}}$$

Spontaneous fission

calculation

$$Q = a_s A^{2/3} - a_s A_1^{2/3} - a_s A_2^{2/3} + a_c \frac{Z^2}{A^{1/3}} - a_c \frac{Z_1^2}{A_1^{1/3}} - a_c \frac{Z_2^2}{A_2^{1/3}}$$

$$= a_s A^{2/3} - a_s \frac{A^{2/3}}{Z^{2/3}} Z_1^{2/3} - a_s \frac{A^{2/3}}{Z^{2/3}} Z_2^{2/3} + a_c \frac{Z^2}{A^{1/3}} - a_c \frac{A_1^2 Z^2}{A^2} \frac{1}{A_1^{1/3}} - a_c \frac{A_2^2 Z^2}{A^2} \frac{1}{A_2^{1/3}}$$

$$= a_s A^{2/3} \left(1 - \frac{Z_1^{2/3}}{Z^{2/3}} - \frac{Z_2^{2/3}}{Z^{2/3}} \right) + a_c \frac{Z^2}{A^{1/3}} - a_c \frac{Z^2}{A^{1/3}} \frac{A_1^{5/3}}{A^{5/3}} - a_c \frac{Z^2}{A^{1/3}} \frac{A_2^{5/3}}{A^{5/3}}$$

$$= a_s A^{2/3} \left(1 - y_1^{2/3} - y_2^{2/3} \right) + a_c \frac{Z^2}{A^{1/3}} \left(1 - y_1^{5/3} - y_2^{5/3} \right)$$

fission

Maximum energy release when

$$\frac{\partial Q}{\partial y_1} = 0 \quad (dy_2 = -dy_1)$$

$$\frac{\partial Q}{\partial y_1} = a_s A^{2/3}\left(-\frac{2}{3}y_1^{-1/3} + \frac{2}{3}y_2^{-1/3}\right) + a_c \frac{Z^2}{A^{1/3}}\left(-\frac{5}{3}y_1^{2/3} + \frac{5}{3}y_2^{2/3}\right)$$

$= 0$ when $y_1 = y_2 = 1/2$ symmetric fission

Maximum $Q = 0.37\, a_c \frac{Z^2}{A^{1/3}} - 0.26\, a_s A^{2/3}$

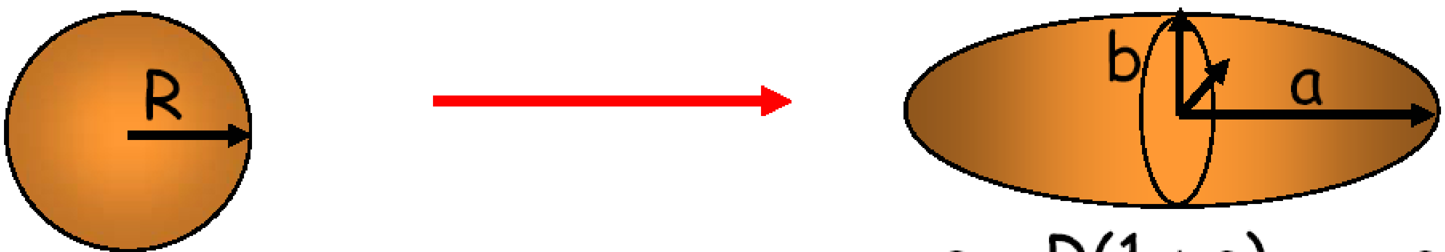
$a_s = 16.8$ MeV
$a_c = 0.72$ MeV

e.g. ${}^{238}_{92}U$ Maximum $Q \approx 200$ MeV

~10^6 > energy released in chemical reaction.

fission and deformation

Estimate mass at which nuclei become unstable to fission (i.e. point at which energy change due to deformation gives a change in B.E, $\Delta B > 0$)



$$a = R(1+\varepsilon) \qquad \varepsilon << 1$$

$$b = R(1+\varepsilon)^{-1/2}$$

$$\text{Volume} = \frac{4}{3}\pi a b^2 = \frac{4}{3}\pi R^3 = \text{CONSTANT}$$

volume term unchanged

Change in surface term

$$a_s A^{2/3} \longrightarrow a_s A^{2/3}\left(1+\frac{2}{5}\varepsilon^2\right)$$

fission and deformation

Change in Coulomb term

$$a_c \frac{Z^2}{A^{1/3}} \longrightarrow a_c \frac{Z^2}{A^{1/3}}\left(1-\frac{\varepsilon^2}{5}\right)$$

Change in Binding energy

$$\Delta B = B(\varepsilon) - B(0) = a_c A^{2/3}\left(\frac{Z^2}{A} - \frac{2a_s}{a_c}\right)\frac{\varepsilon^2}{5}$$

i.e. if $\frac{Z^2}{A} > \frac{2a_s}{a_c}$ $\Delta B > 0$ and nucleus unstable under deformation

$$\longrightarrow \frac{Z^2}{A} > 50$$

is only a rough indicator based on classical reasoning for spherical shapes of charged drops!!!

spontaneous fission