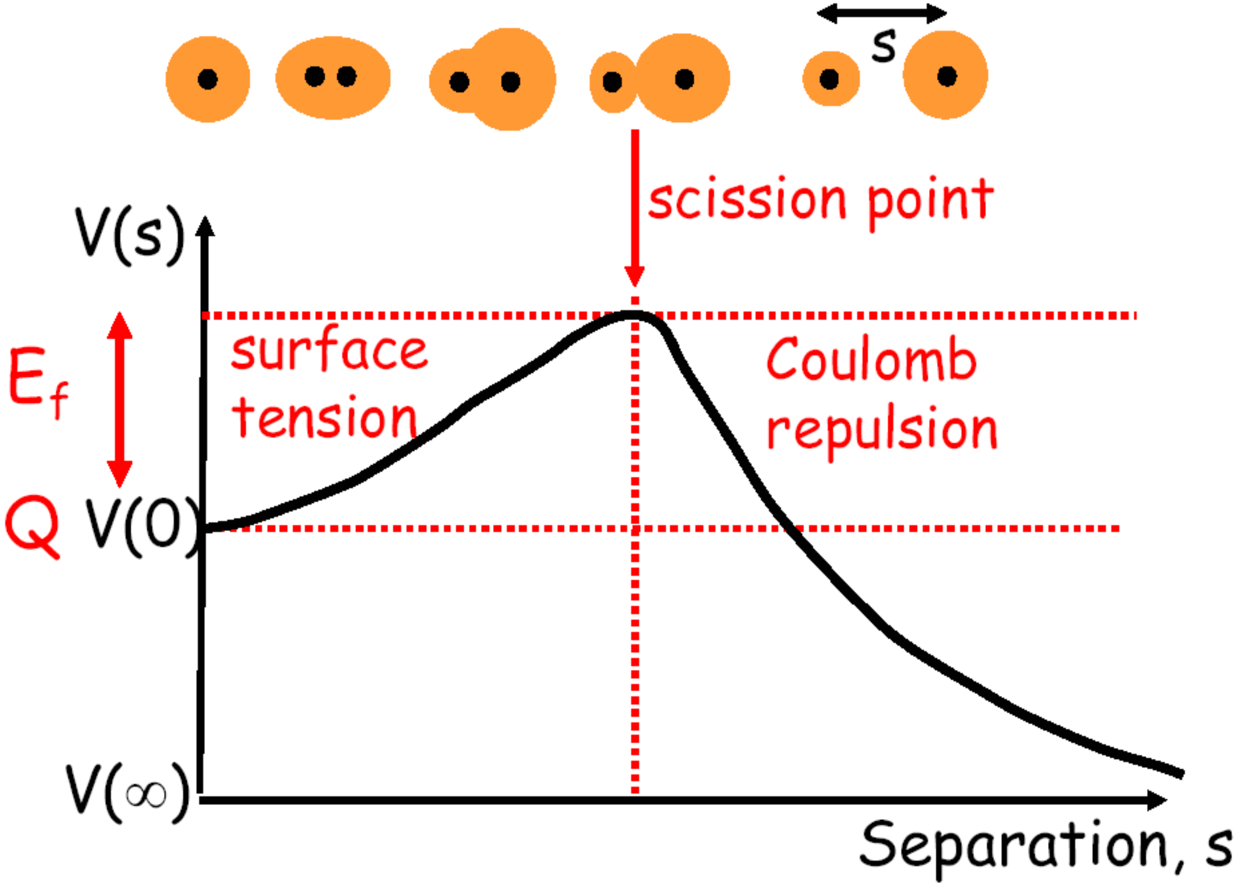
# Fission barrier

Deformation condition alone not sufficient. Fission process requires an intermediate state of the nucleus. Here the surface energy is increased, but the Coulomb energy is not reduced sufficiently to allow fission. At the scission point the Coulomb energy takes over and the two fragments separate.



# Fission barrier

Q = energy released → K.E. of fragments.

$E_f$ = fission activation energy

$$E_f = a_c A^{2/3}\left(\frac{Z^2}{A} - \frac{2a_s}{a_c}\right)\frac{\varepsilon^2}{5} \approx \underline{6\,\text{MeV}} \qquad {}^{236}_{92}\text{U}$$

Spontaneous fission is possible if tunnelling occurs (c.f. $\alpha$ decay).

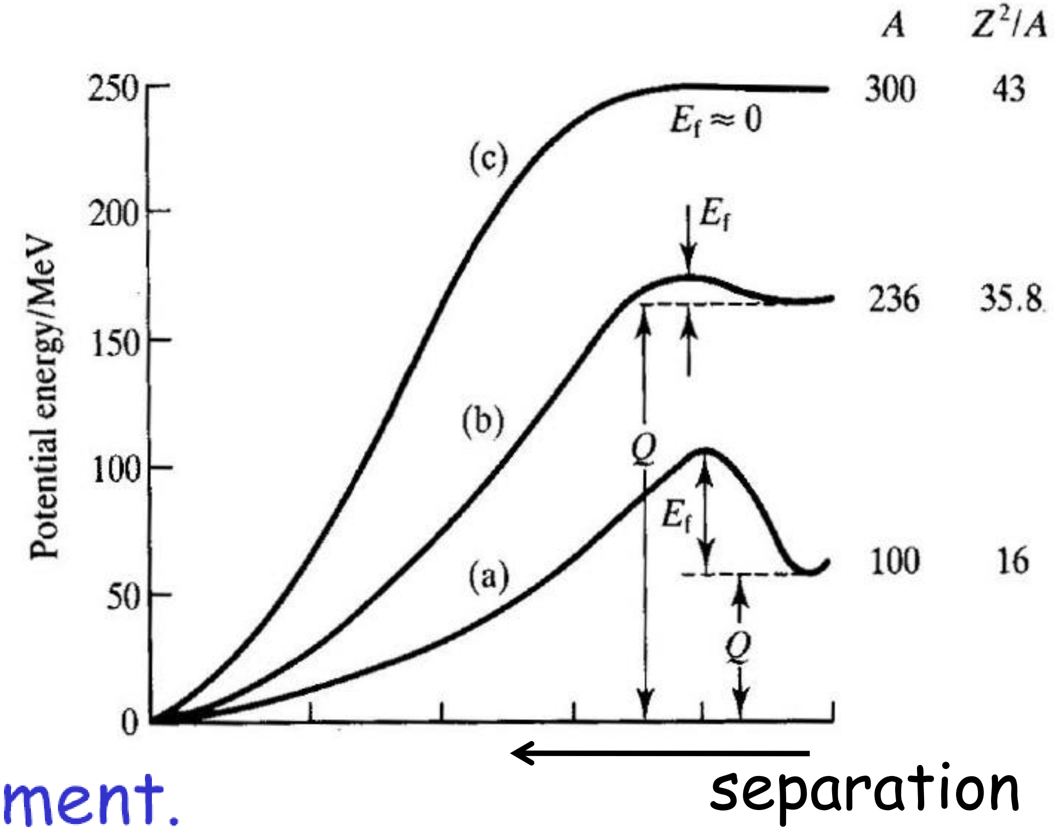
Tunnelling probability depends on

- $Z^2/A$.

# fission

- $Z^2/A$.

$$E_f \sim \frac{Z^2}{A}$$



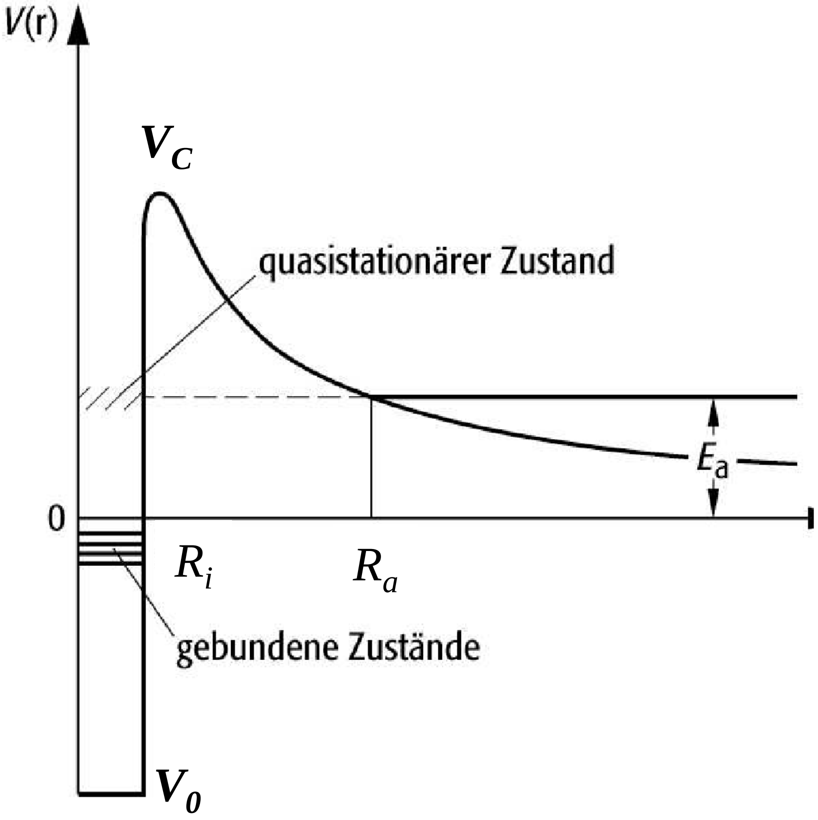
- Mass of fragment.

$$P = e^{-2G} \quad G \sim m^{1/2}$$

Large mass → low probability for tunnelling e.g. $10^6$ less probable than $\alpha$ decay for $^{238}_{92}U$

# α-decay reminder

Das *α*-Teilchen bewegt sich in einem mittleren Potenzial *V*(*r*) im Tochterkern. Im Inneren des Kern ist $V_0$ konstant, außerhalb des Kernradius *R* ist es ein reines Coulomb-Potenzial
*Beispiel* $^{238}U$: $V_0$ ~ -100 MeV *R* = ~ 10 fm



Ansatz für die Zerfallswahrscheinlichkeit pro Zeiteinheit *λ*:

$$\lambda = S \cdot \omega \cdot P$$

- *S* Wahrscheinlichkeit daß sich bereits im Kerninneren ein *α*-Teilchen gebildet hat

- *ω* Frequenz, mit der das Teilchen an die Barriere stößt:

$$\omega = \frac{1}{\Delta t} = \frac{v}{2R} = \frac{\sqrt{2V_0 / M}}{2R}$$

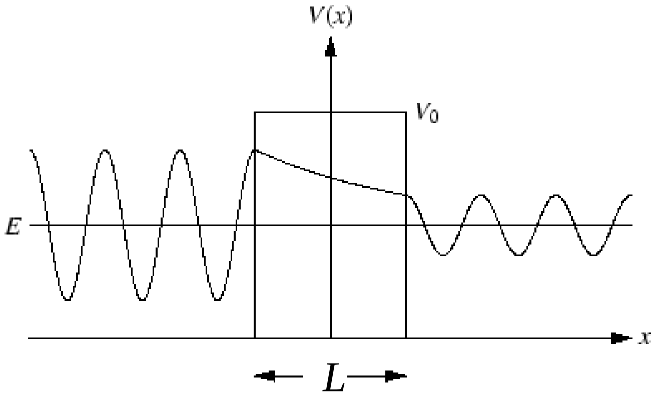
- *P* ist die Penetrabilität, die Wahrscheinlichkeit für einen Tunnelprozess.

Bemerkung:
Das *α*-Teilchen kann im Kern auch Bahndrehimpuls $l$ tragen. Diesen Vernachlässigen wir im folgenden, d.h. es gilt nur für Zerfälle zwischen Grundzuständen mit $l=0$.

# $\alpha$-decay and tunnel effect

Quantenmechanik: Tunnelwahrscheinlichkeit läßt sich für ein endliches Kastenpotential exakt berechnen:

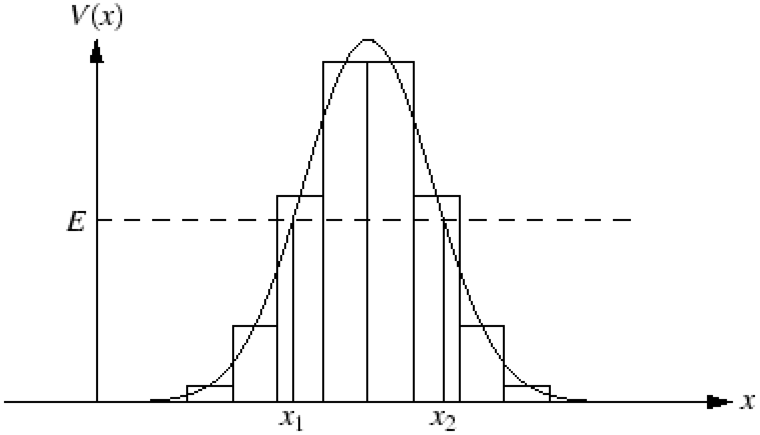
$$T(E) = \exp\left(-\frac{2L}{\hbar}\sqrt{2m(V_0 - E)}\right)$$



Für einen allgemeinen Potenzialberg ist dies nicht möglich.
Näherungsformel: Zwischen den klassischen Umkehrpunkten wird das Potential in $n$ kleine Schwellen der Breite $\Delta x$ zerlegt.

$$T(E) = \prod_{i=1}^{n} T_i(E) = \exp\left\{-\int_{x_1}^{x_2} dx \left(\frac{2}{\hbar}\sqrt{2m(V(x) - E)}\right)\right\}$$

Annahmen:
- Exponentialfaktor ist wesentlich größer als Eins
- *WKB (Wentzel, Kramers, Brillouin) Näherung* für kontinuierliche Potentialberge



# $\alpha$-decay

*P* ist die Penetrabilität, Wahrscheinlichkeit für Tunnelprozess:

$P = e^{-2G}$ mit Gamow Faktor G

$$G(E_\alpha) = \int_{R_i}^{R_a} dr \left( \frac{1}{\hbar} \sqrt{2m(V(r) - E)} \right)$$

$$\text{mit } V(R_a) = E_\alpha = \frac{2Ze^2}{R_a}$$

$$G(E_\alpha) = \frac{2}{\hbar} \sqrt{2m} \int_{R_i}^{R_a} dr \sqrt{\frac{2Ze^2}{r} - E}$$

$$= \frac{2}{\hbar} \sqrt{\frac{2m}{E}} 2Ze^2 \left\{ \arccos \sqrt{\frac{R_i}{R_a}} - \sqrt{\frac{R_i}{R_a} - \frac{R_i^{\ 2}}{R_a^{\ 2}}} \right\}$$

Grobe Abschätzung für die Unterschiede in Penetrabilität wg Massenunterschied zwischen alpha-Zerfall und Spaltung A~115-130:

$P_\alpha / P_{fis} \sim 10^6$

# Induced fission

Induced fission of nuclei occurs when a nucleus captures a low energy neutron receiving enough energy to climb the fission barrier.

e.g. $n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow X^* + Y^* \rightarrow X + Y + \kappa n$

$\kappa \sim 2.4$

prompt neutrons

If excitation energy > fission activation energy, fission will occur for zero energy neutrons

→ thermal neutrons.

# Induced fission

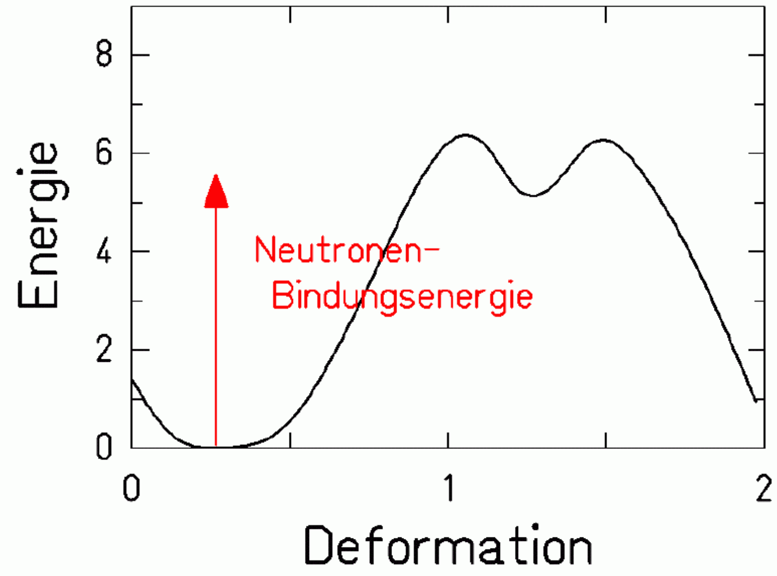
Otherwise need to supply energy using K.E. of n.

e.g. $n + {}^{238}U \rightarrow {}^{239}U^*$ $E_f \sim 6$ MeV

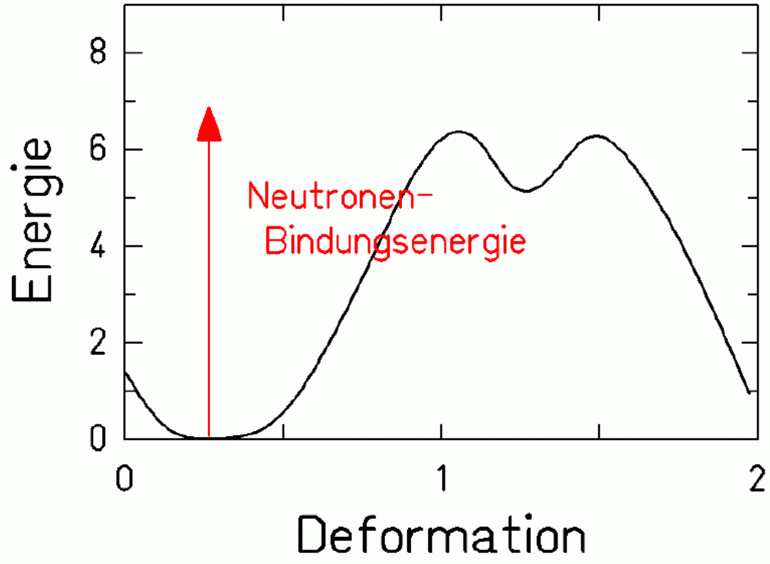
$E_n = 0$ $E \sim 5$ MeV no thermal fission

$E_n = 1.4$ MeV $E \sim 6.4$ MeV fission

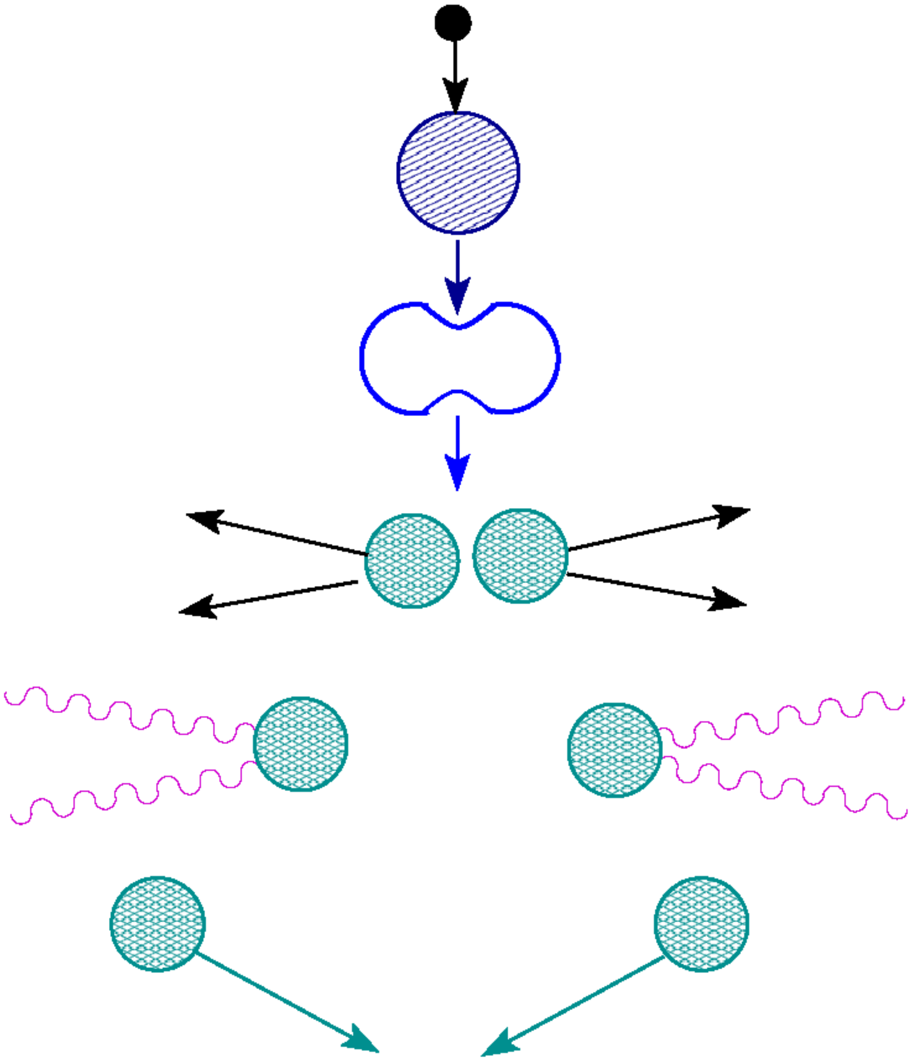
$^{238}$U even neutron number



$^{235}$U odd neutron number



# fission time sequence



t = 0

Neutron

$^{235}$U

$^{236}$U*

n emission prompt

t > $10^{-14}$ s

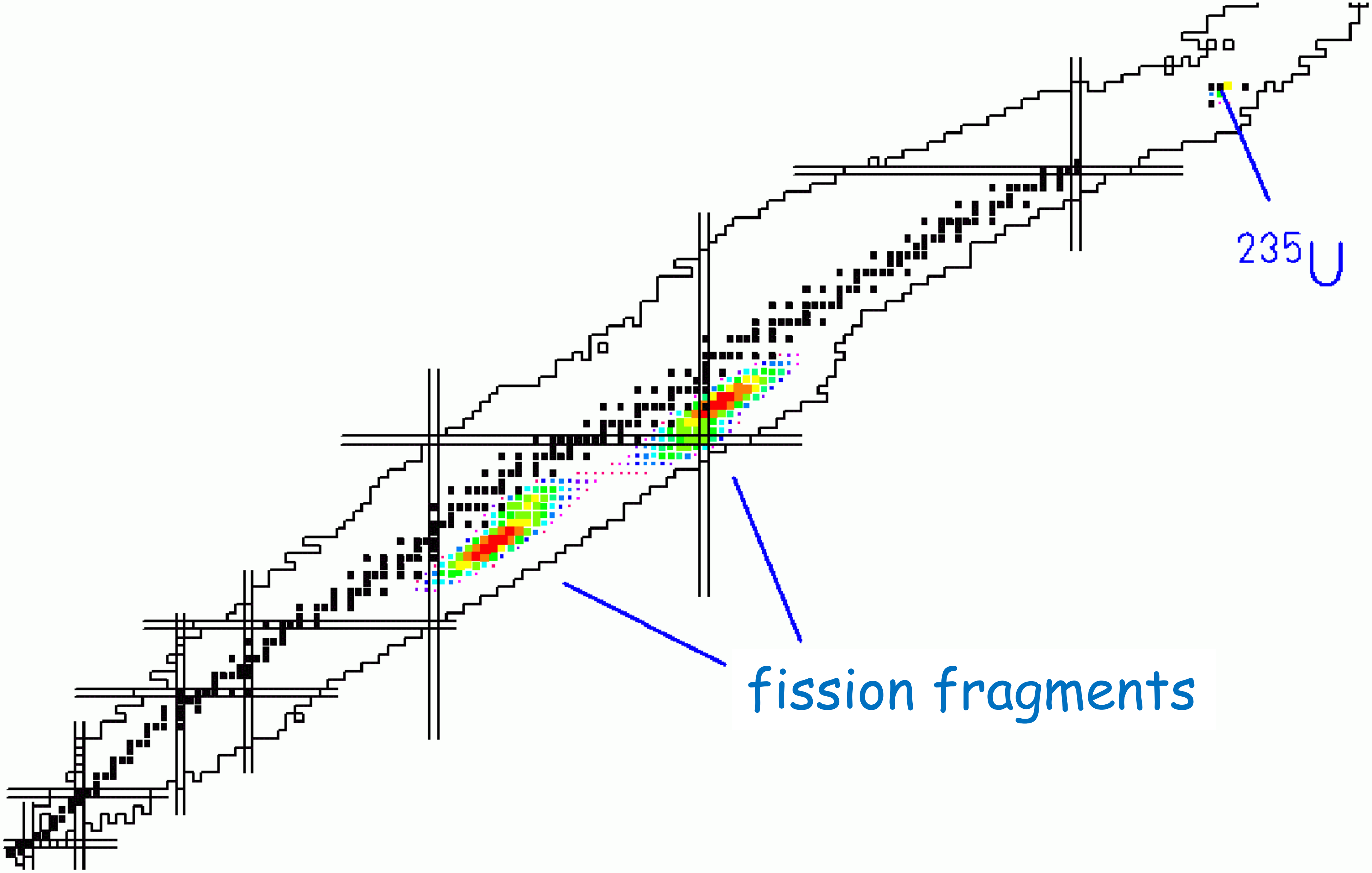
γ emission prompt

t > $10^{-10}$ s

excited instable
fission fragments

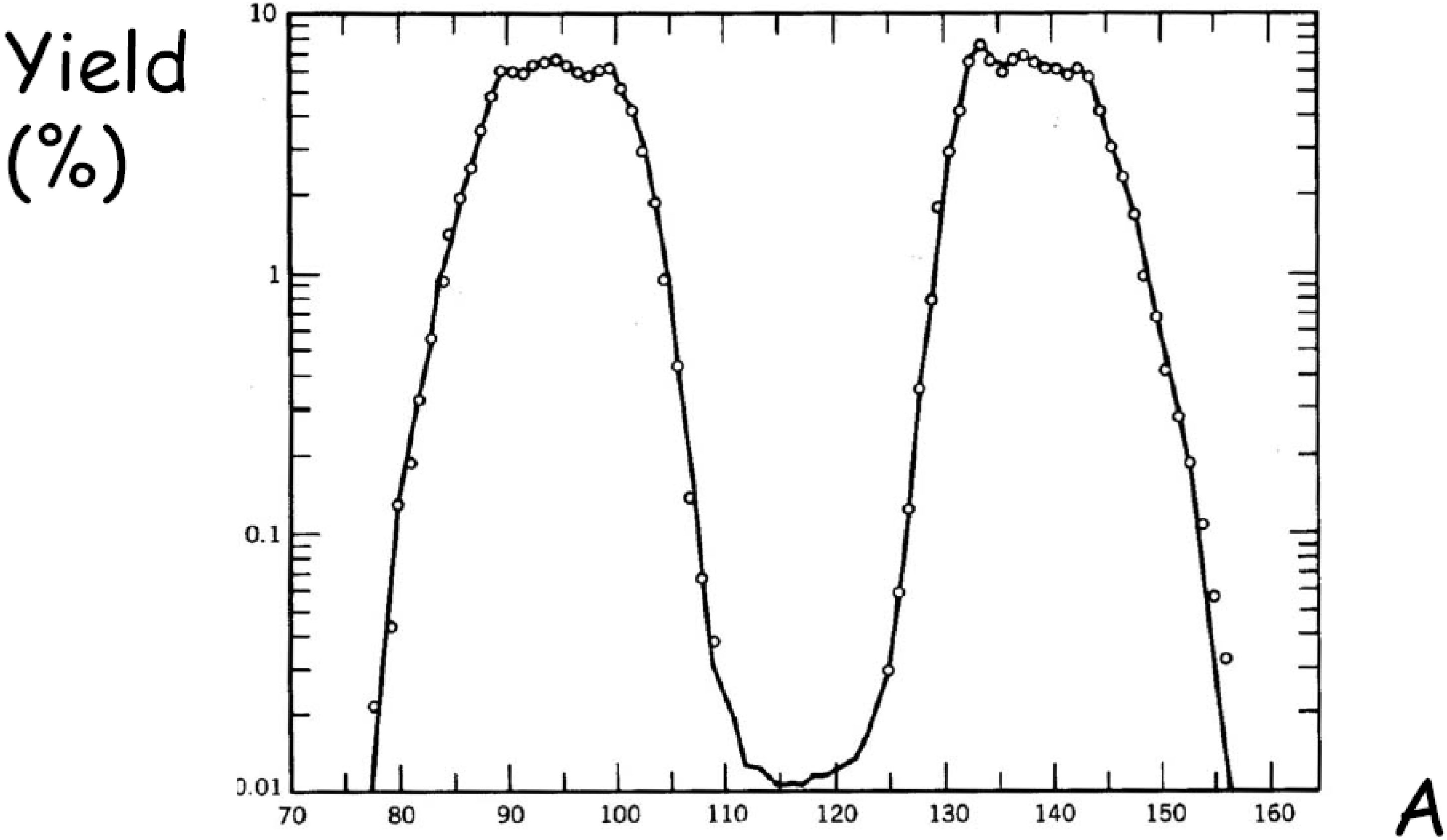
β, γ and delayed neutron emission from excited instable fission fragments

# Fission fragment distribution of $^{235}$U



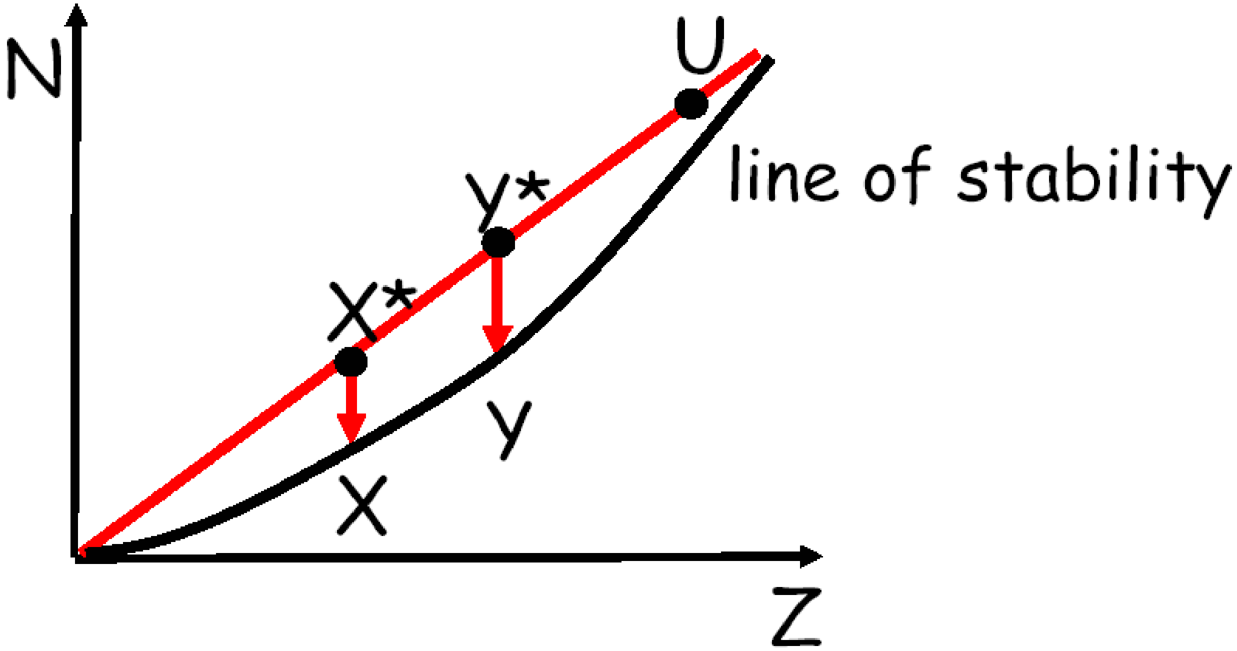
# Mass distribution of fission fragments

- Masses of fragments are unequal, in general. Tend to have Z, N near magic numbers.



# Prompt and delayed neutrons

- Fragments tend to have same Z/N ratio as parent → neutron rich nuclei which emit prompt neutrons ($10^{-16}$s)



X,Y β decay more slowly → delayed n emission
(~1 delayed n/100 fissions)

# fission chain reaction

- per fission 2–3 free neutrons are generated within ~ $10^{-14}$ s

  -> ***prompt neutrons***
- neutron emission after β–decay -> ***delayed neutrons***
- small fraction of approximately 0.65% of all neutron emission is delayed
- very important to control self sustained chain reaction in nuclear reactor

A chain reaction can be sustained is at least 1 n/fission induces another fission process.

Define k = number of neutrons from one fission which induce another

Reactors → $k = 1$ critical
$k < 1$ subcritical
$k > 1$ supercritical

# Heavy nuclei, breeding

Heavy fissile nuclei and natural abundance

$^{232}$Th: Z = 90, N = 142

$^{234}$U: Z = 92, N = 142, (0,0055%)

$^{235}$U: Z = 92, N = 143, (0,72%)

$^{238}$U: Z = 92, N = 146, (99.2745%)

- Nuclei with odd neutron number are fissile, cross sections of (n,f) reaction with thermal neutrons are very high.

Breeding of fissile material

- Nuclei with odd neutron number can be produced by n-capture of thermal neutron with even n-number in reactor -> ‚breeding'.

Via n-capture reaction an odd n-number nucleus is formed.

example: $^{238}U(n,\gamma)\,^{239}U$, $^{239}U \xrightarrow{-\beta} {}^{239}Np$, $^{239}Np \xrightarrow{-\beta} {}^{239}Pu$

- Natural fission fuel for conventional fission reactor is $^{235}$U.

# Chain reaction, critical mass

| Isotop | critical mass* in kg | |
|---|---|---|
| $^{235}$U | 22,8 kg | highly enriched, enrichment from 0,72 % to > 90% |
| $^{233}$U | 7,5 kg | obtained from n-capture or breeding of $^{232}$Th |
| $^{239}$Pu | 5,6 kg | obtained from n-capture or breeding of $^{238}$U |

*critical mass for a spherical configuration and a neutron reflector

Critical mass: mass value which is needed to sustain a chain reaction of fission.

The critical mass of an arbitrarily shaped fissile amount of material is mainly determined by ratio of surface area to volume.

Important is also surrounding material. Neutron reflecting and not absorbing material favours chain reaction.