Reminder: cross section for direct reaction

example: direct capture $A + x \rightarrow B + \gamma$

$$\sigma = \pi \lambda_x^2 \left| \langle B | H | x + A \rangle \right|^2 P_\ell(E)$$

"geometrical factor"
de Broglie wave length
of projectile

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

matrix element
contains nuclear
properties of
interaction

- Penetrability/ Transmission
probability of projectile to interact
with target nucleus
- Depends on angular momentum
of projectil ℓ and energy E

$$\sigma = \frac{1}{E} \cdot P_\ell(E) \cdot S(E)$$

σ = (strong energy dependence) x (weak energy dependence)

S(E) = astrophysical factor

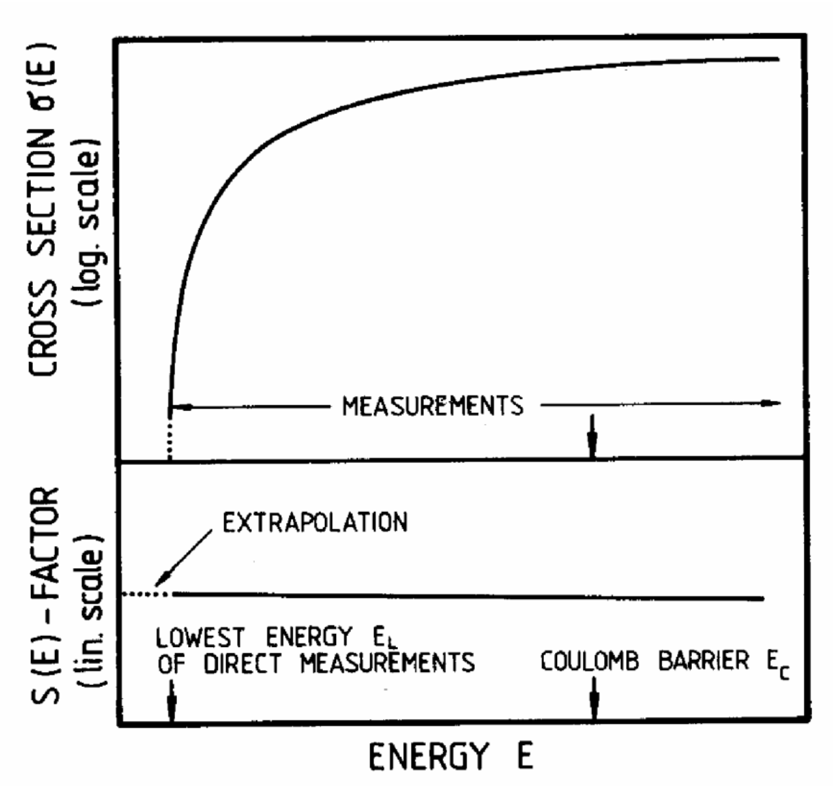
Contains nuclear physics of reaction, matrix element, wave functions, operator

Needed: penetrability $P_\ell(E)$

Transmission Probability depends on:

- Coulomb barrier (only charged particles)
- centrifugal barrier (for neutrons and charged particles)

Astrophysical S-factor



"astro physical S factor" contains detailed information on nuclear structure

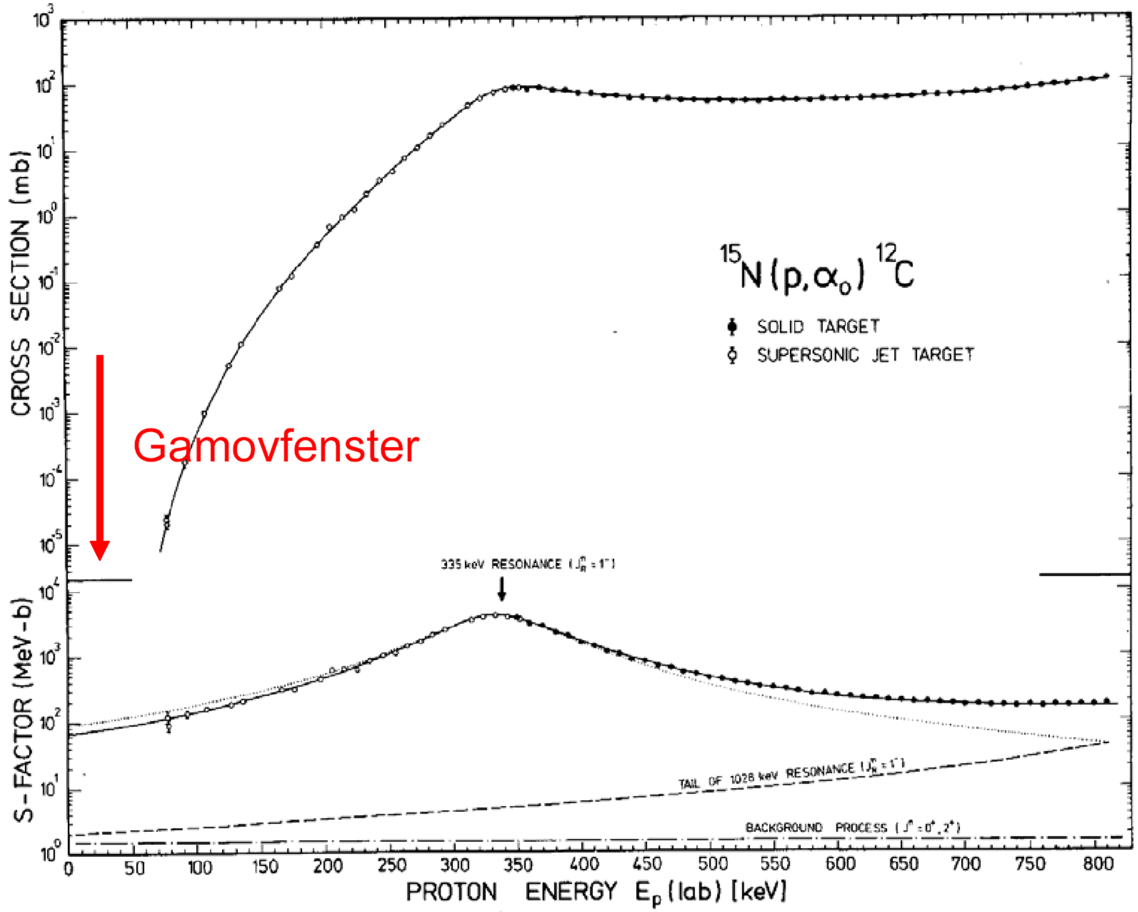
$$\sigma(E) = \frac{S(E)}{E} e^{-b/\sqrt{E}}$$

Relevant energy region for astro physics Is very low, typicalle at the limit and below measurable energy for nuclear reaction.

In some cases S(E) can be extrapolated.

Rolfs & Rodney p. 156

Astrophysical S-factor



Rolfs & Rodney p. 158

Typically data for astrophysical processes are given by S(E).
In some cases S(E) varies strongly with energy.

Gamow factor

Temperature to overcome the
Coulomb barrier in proton-proton system?

$$\frac{1}{2}\mu\langle v^2\rangle = \frac{3}{2}kT = \frac{Z_1 Z_2 e^2}{r}$$

$$T = \frac{2Z_1 Z_2 e^2}{3kr}$$

$$Z_1 = Z_2 = 1$$

$$r = 1\,\mathrm{fm}$$

$$T \approx 10^{10}\,\mathrm{K} \approx 1\,\mathrm{MeV}$$

compare:
Temp. in middle of sun: $T \sim 1.5 \times 10^7$ K (~1 keV)

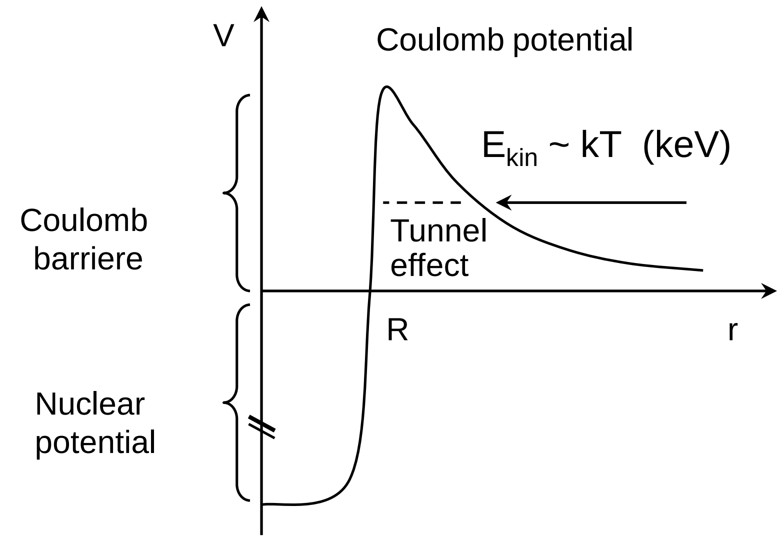
Also high energetic part of Maxwell- Boltzmann distribution is not sufficient to provide enough p-p reactions above barrier!

Tunnel effect is needed!

Tunneling Gamow factor

Projectil and target with charges Z_1 and Z_2

$$V_C = \frac{Z_1 Z_2 e^2}{R}$$



in units (MeV, amu, fm)

$$V_C[\mathrm{MeV}] = 1.44 \frac{Z_1 Z_2}{R[\mathrm{fm}]} \approx 1.2 \frac{Z_1 Z_2}{\left(A_1^{1/3} + A_2^{1/3}\right)}$$

example: $^{12}C(p,\gamma)$ $V_C = 3$ MeV

average kinetic energy in stellar plasma: kT ~ 1-100 keV!

⇒ Fusion reaction of two charged particles well below Coulomb barrier

⇒ Transmission probability is determined by tunnel effect

for $E \ll V_C$ and no angular momentum transfer tunnelling is given by:

$$P(E) \propto \exp(-2\pi\eta)$$

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v}$$

$$2\pi\eta = 31.29 Z_1 Z_2 \left(\frac{\mu}{E}\right)^{1/2}$$

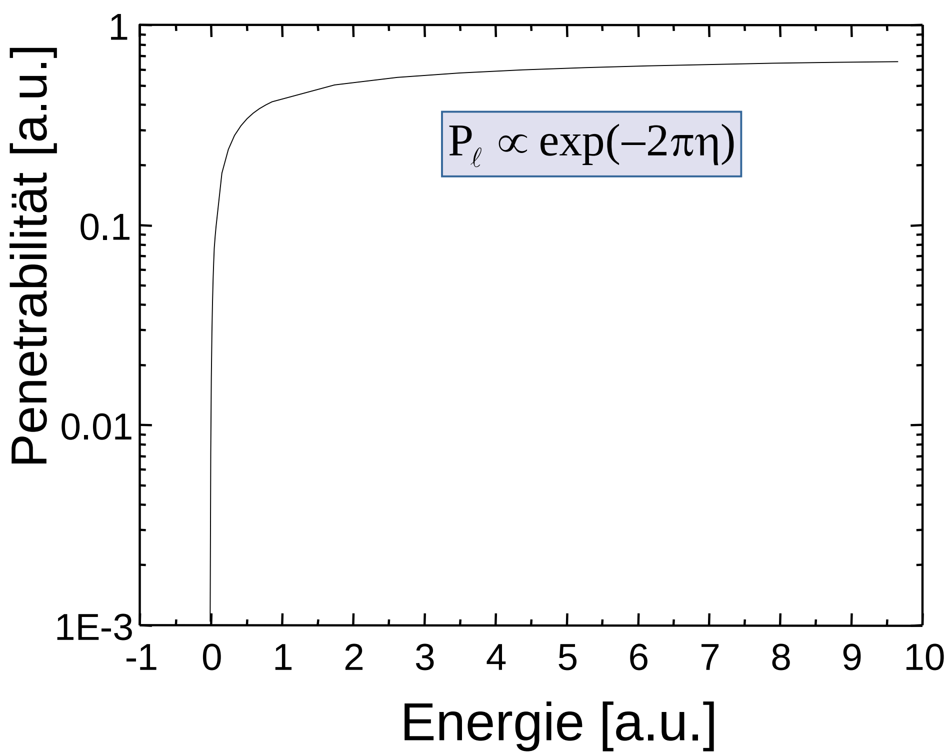
Gamow factor

η Sommerfeld parameter

with E- keV and μ- amu

Tunnel probability through Coulomb barrier for reactions with charged particles at energies $E \ll V_{coul}$

$$P_\ell \propto \exp(-2\pi\eta) = \exp\left(-\frac{b}{\sqrt{E}}\right)$$



$$\eta = \sqrt{\frac{\mu}{2E}}\frac{Z_1 Z_2 e^2}{\hbar}$$ Sommerfeld parameter

$$b = \sqrt{2\mu}\pi\frac{Z_1 Z_2 e^2}{\hbar}$$ b^2 Gamow energy

assume:

- complete ionisation
- no orbital angular momentum

$$\sigma = \frac{1}{E}\exp(-2\pi\eta)S(E)$$

Units of S(E): keV barn, MeV barn …

Additional orbital angular momentum causes in first order approximation a constant factor in addition to S-factor, decreases strongly with ℓ.

$$\langle\sigma v\rangle = \int \sigma(v)\phi(v)v\,dv = \int \sigma(E)\exp(-E/kT)E\,dE$$

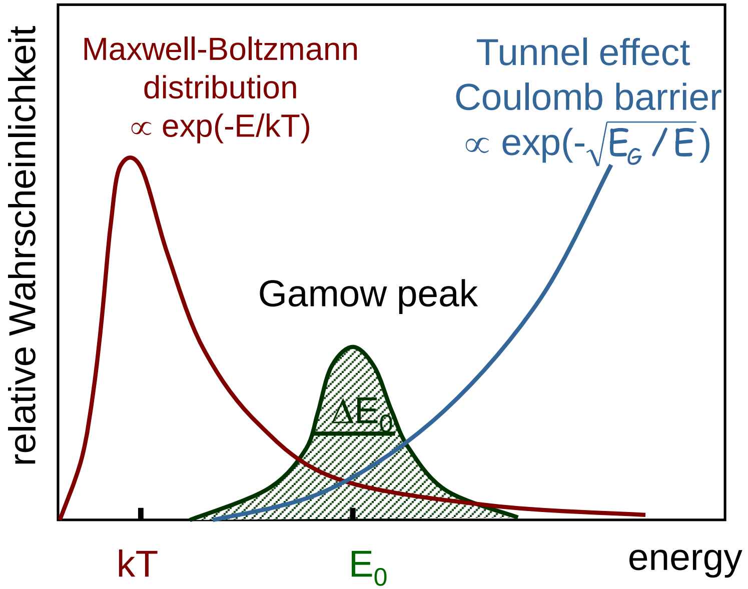
and substitution for σ: $$\langle\sigma v\rangle \propto \int S(E)\exp\left(-\frac{E}{kT}-\frac{b}{\sqrt{E}}\right)dE$$

maximum of reaction rate at E_0: $$\frac{d}{dE}\left[\exp\left(-\frac{E}{kT}-\frac{b}{\sqrt{E}}\right)\right]=0$$

Gamow peak

$$E_0 = \left(\frac{bkT}{2}\right)^{2/3} = 1.22\left(Z_1^2 Z_2^2 \mu\right)^{1/3} T_6^{2/3} \quad \text{keV}$$

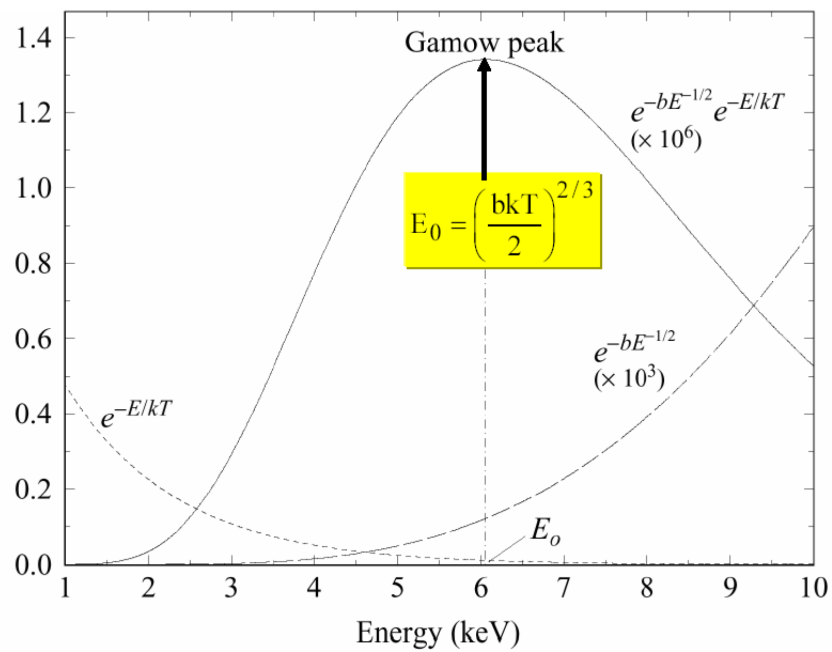
$$\Delta E = \frac{4}{\sqrt{3}}\sqrt{E_0 kT} = 0.749\left(Z_1^2 Z_2^2 \mu\right)^{1/6} T_6^{5/6} \quad \text{keV}$$



E_0 = relevant energy for astrophysics >> kT

Remark Gamow energy depends on **reaction** and **temperature**

$$\text{rate} \propto \exp\left(-\frac{E}{kT} - \frac{b}{\sqrt{E}}\right) \rightarrow E_0 = \left(\frac{bkT}{2}\right)^{2/3} = 1.22\left(Z_1^2 Z_2^2 \mu T_6^2\right)^{1/3} \text{keV}$$



Energy of Gamow window determines the Energy for different reactions at specific temperature T.

Example: sun T_6=15 (~1keV)

p+p: E_0 = 5.9 keV

p+^{14}N: E_0 = 26.5 keV

α+^{12}C: E_0 = 56 keV

^{16}O+^{16}O: E_0 = 237 keV

Reaction rate is proportional to heigth of Gamow peak at E_0:

p+p: I_{max} = 1.1 x 10^{-6}

p+^{14}N: I_{max} = 1.8 x 10^{-27}

α+^{12}C: I_{max} = 3.0 x 10^{-57}

^{16}O+^{16}O: I_{max} = 6.2 x 10^{-239}

Ligth elements burn first.
Then star shrinks due to gravity.
This causes increase of temperature until next heavier element is synthesized or burned