Energy production in sun

- Sequence of nuclear reactions: proton-proton chain

1. $p + p \rightarrow {}^{2}D + e^{+} + \nu_e$

2. ${}^{2}D + p \rightarrow {}^{3}He + \gamma$

3. ${}^{3}He + {}^{3}He \rightarrow {}^{4}He + p + p$

Total: $4\,p \rightarrow {}^{4}He + 2\,e^{+} + 2\nu_e$

Introduction

proton proton chain

$4\text{p} \rightarrow {}^4\text{He} + 2\text{e}^+ + 2\nu \quad \text{Q} = 26.73\text{MeV}$

possible first steps:

$\text{p} + \text{p} \rightarrow {}^2\text{He}$ Diproton unbound nucleus

$({}^2\text{He} + \text{p} \rightarrow {}^3\text{Li}$ also unbound)

- Weak interaction process

$\text{p} + \text{p} \rightarrow d + \text{e}^+ + \nu$

Q-value: $\text{Q} = 1.44$ MeV

kinetic energy: $\text{E}_{\text{kin}} = Q - 2m_e c^2 = 0.42 MeV$

$2m_p + 2m_e \rightarrow m_d + 3m_e \Rightarrow m_H + m_H \rightarrow m_{2H} + 2m_e$

kinetic energy is shared by positron and neutrino

- Theoretical cross section for the p + p reaction

total Hamiltonian: $\text{H} = \text{H}_\text{n} + \text{H}_\beta$

but H_β much smaller then $\text{H}_\text{n} \Rightarrow$ perturbation theory

Fermis Golden Rule:

$$\text{d}\sigma = \frac{2\pi}{\hbar} \frac{\rho(E)}{v_i} \left| \langle f | \text{H}_\beta | i \rangle \right|^2$$

$\rho(E)$ statistical factor, density of final states

v_i relative velocity in incident channel

$\left| \langle f | \text{H}_\beta | i \rangle \right|$ transition matrix element

$|i\rangle \equiv \text{p} + \text{p}$

$|f\rangle \equiv d + \text{e}^+ + \nu$

Phase space

$\rho(E) = \frac{dN}{dE}$ number of final states dN in energy interval dE (E and E + dE)

total phase space divided by unit volume $dn = V\frac{4\pi p^2 dp}{h^3}$

applying to pp reaction → states available for positrons and neutrinos

$$\mathrm{dN} = dn_e \cdot dn_\nu = \left(V\frac{4\pi p_e^2 dp_e}{h^3}\right)\left(V\frac{4\pi p_\nu^2 dp_\nu}{h^3}\right)$$

assume : - zero rest mass of neutrino $\mathrm{m}_\nu = 0$

- zero recoil energy of deuteron $\mathrm{E}_{kin,d} = 0$

$\Rightarrow$ Energy is shared by positron and neutrino

$$\mathrm{E} = \mathrm{E}_e + \mathrm{E}_\nu = \mathrm{E}_e + c\mathrm{p}_\nu$$

$p_\nu = \frac{E}{c} - \frac{E_e}{c}$ for constant $\mathrm{p_e}$

$$\frac{dp_\nu}{dE} = \frac{1}{c} \qquad dp_\nu = \frac{1}{c}dE \qquad \frac{dp_\nu}{dE_e} = -\frac{1}{c}$$

neutrino level density : $\frac{dn_\nu}{dE} = \left(V\frac{4\pi p_\nu^2}{h^3}\frac{dp_\nu}{dE}\right) = V\frac{4\pi}{h^3}p_\nu^2\frac{dp_\nu}{dE} = V\frac{4\pi}{h^3}\frac{1}{c^2}(E-E_e)^2\frac{1}{c}$

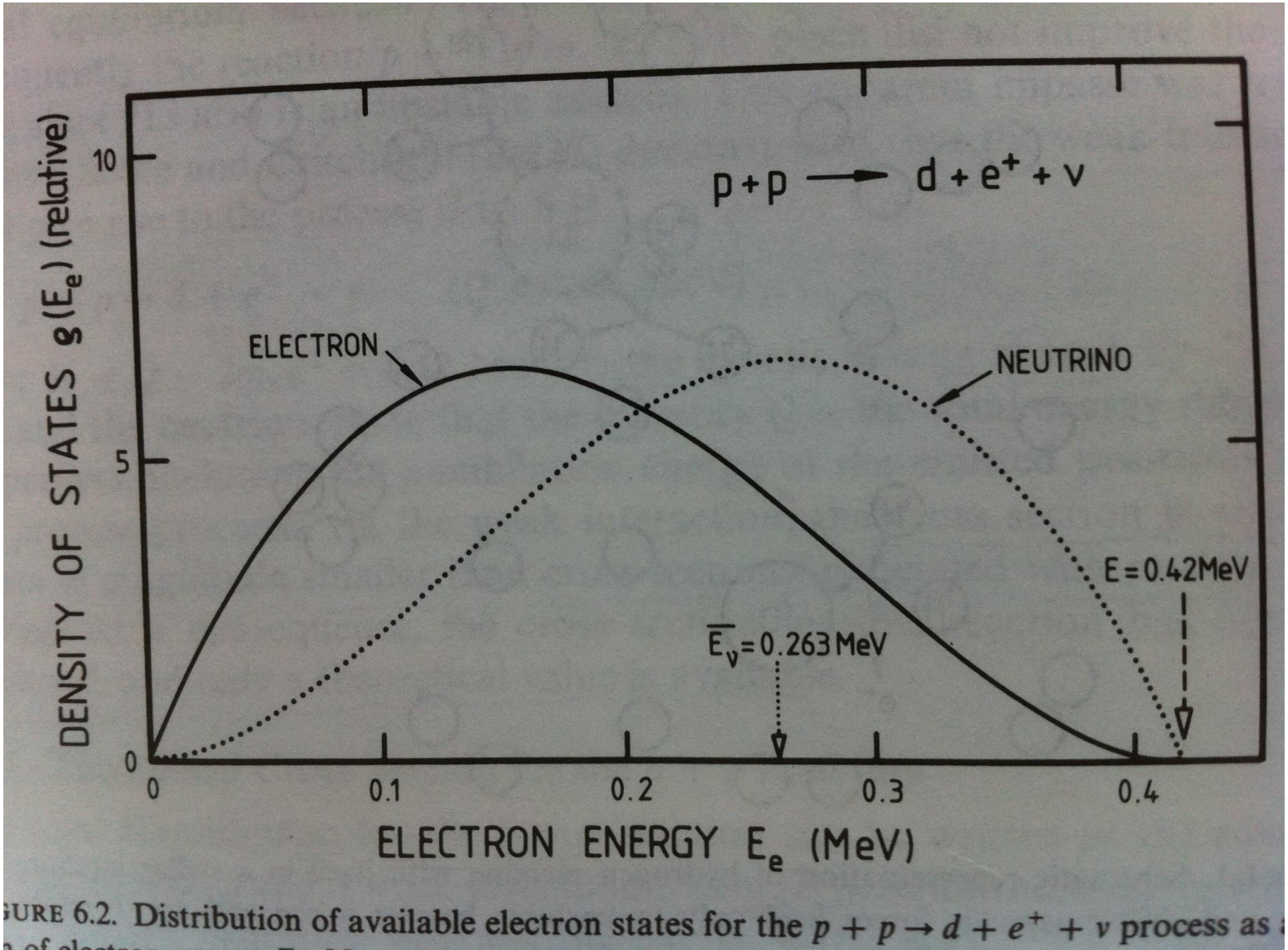
$$dn_e = \left(V\frac{4\pi p_e^2 dp_e}{h^3}\right)$$

$$\rho(E) = \frac{dN}{dE} = dn_e \cdot \frac{dn_\nu}{dE} = \left(V\frac{4\pi p_e^2 dp_e}{h^3}\right) \cdot V\frac{4\pi}{h^3}\frac{1}{c^2}(E-E_e)^2\frac{1}{c}$$

$$\rho(E) = V^2\frac{16\pi^2}{h^6}\frac{1}{c^3}p_e^2(E-E_e)^2 dp_e = \rho(E_e)dp_e$$

Phase space

$\rho(E) = \frac{dN}{dE}$ number of final states dN in energy interval dE (E and E + dE)



$$\rho(E) = V^2 \frac{16\pi^2}{h^6} \frac{1}{c^3} p_e^2 (E - E_e)^2 dp_e = \rho(E_e) dp_e$$

Matrix element

Back to the cross section for the p + p reaction, cross section depends only on positron variables

Fermis Golden Rule :

$$d\sigma = \frac{2\pi}{\hbar}\frac{\rho(E_e)}{v_i}\left|\langle f \left|\mathrm{H}_\beta\right| i\rangle\right|^2 dp_e$$

Matrix element : $H_{if} = \langle f \left|\mathrm{H}_\beta\right| i\rangle = \int_{Vol} \psi_f^* \mathrm{H}_\beta \psi_i \, d\tau = \int_{Vol} \left[\psi_d \psi_e \psi_\nu\right]^* \mathrm{H}_\beta \psi_i \, d\tau$

Wave function of positron and neutrino : plane waves

$$\psi_e = \frac{1}{V^{1/2}}\exp(i\vec{k}_e\vec{r}) \quad \text{and} \quad \psi_\nu = \frac{1}{V^{1/2}}\exp(i\vec{k}_\nu\vec{r})$$

Leptons are normalized to the volume.

For initial protons : $\int_{Vol} \psi_i^* \psi_i \, d\tau = 1$

Deuteron wave function vanishes rapidly outside nuclear radius $r_d = 1.7$ fm.

Mean neutrino energy from phase space : $\overline{E}_\nu = 0.26$ MeV.

$\Rightarrow \; k_\nu r_d = 2.2 \times 10^{-3}$

expansion of wave functions : $\psi_e = \frac{1}{V^{1/2}}\left[1 + i(\vec{k}_e\vec{r}) + \ldots\right]$

$$\psi_\nu = \frac{1}{V^{1/2}}\left[1 + i(\vec{k}_\nu\vec{r}) + \ldots\right]$$

Cross section

Matrix element after expansion :

$$H_{if} = \frac{1}{V}\int_{Vol} \Psi_d^* \mathrm{H}_\beta \Psi_i \, d\tau \quad \text{strength of weak interaction } \mathrm{H}_\beta \text{ is governed by coupling constant g}$$

(analog to electric charge as coupling constant α in EM interaction)

$$H_{if} = \frac{1}{V} g \int_{Vol} \Psi_d^* \Psi_i \, d\tau = \frac{1}{V} g M_{space} M_{spin}$$

$$d\sigma = \frac{2\pi}{\hbar}\frac{\rho(E_e)}{v_i}\left|\langle f|\mathrm{H}_\beta|i\rangle\right|^2 dp_e = \frac{2\pi}{\hbar}\frac{1}{v_i}V^2\frac{16\pi^2}{h^6}\frac{1}{c^3}p_e^2(E-E_e)^2\left(\frac{1}{V}gM_{space}M_{spin}\right)^2 dp_e$$

$$\sigma = \frac{2\pi}{\hbar}\frac{1}{v_i}\frac{16\pi^2}{h^6}\frac{1}{c^3}g^2 M_{space}^2 M_{spin}^2 \int_0^E p_e^2 (E-E_e)^2 dp_e$$

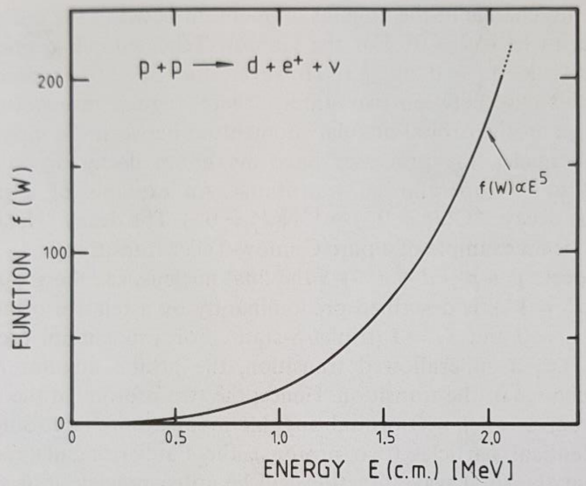
new variable : $w = \dfrac{E + m_e c^2}{m_e c^2}$

substitution : $\displaystyle\int_0^E p_e^2(E-E_e)^2 dp_e = \frac{(m_e c^2)^5}{c^3}\int_1^w (w_e^2-1)^{1/2}(w-w_e)^2 w_e dw_e = \frac{(m_e c^2)^5}{c^3} f(w)$

integral is proportional to E^5 $\; f(w) \propto w^5 \propto E^5$

$$\sigma = \frac{1}{v_i}\left[\frac{2\pi}{\hbar}\frac{16\pi^2}{h^6}\frac{1}{c^3}\frac{(m_e c^2)^5}{c^3}\right] f(w) g^2 M_{space}^2 M_{spin}^2 \qquad \text{constant values} \quad \alpha = \left[\frac{2\pi}{\hbar}\frac{16\pi^2}{h^6}\frac{1}{c^3}\frac{(m_e c^2)^5}{c^3}\right] = 1.45\cdot 10^{70}\frac{1}{eV^2\cdot s\cdot cm^6}$$

$$\sigma = \alpha \frac{1}{v_i} f(w) g^2 M_{space}^2 M_{spin}^2$$



The function $f(W)$, which is a measure of the total phase space available for t
$e^+ + \nu$ reaction, is shown as a function of energy E. As might be expected, the ava
pace increases rapidly with E. For large values of energy, the phase space and hen
ty for the process increase as the fifth power of the energy.

illustration of this function $f(W)$ is presented in Figure 6.3. F
es of W, i.e., large energies E, the function $f(W)$ approaches the for
$^5 \propto E^5$.

al cross section is given now by the equation

$$1\ \frac{2\pi}{\hbar}\frac{16\pi^2}{c^3h^6}\frac{(m_e c^2)^5}{c^3} f(W)g^2M_{\rm spin}^2 M_{\rm space}^2$$

$$\propto \frac{1}{v_i} f(W)g^2M_{\rm spin}^2 M_{\rm space}^2 , \qquad (6.$$

epresents the constants, which, when replaced by their numeric
ult in

$$\frac{m^5c^4}{2\pi^3\hbar^7} = 1.45 \times 10^{70}\ \mathrm{eV^{-2}\ s^{-1}\ cm^{-6}} .$$

ations of allowed weak-interaction phenomena indicate (Bla6
t among all possible types of couplin