Interaction of charged particles in matter

Charged particles in matter: (1) energy loss (2) scattering

Caused by: - inelastic collisions with electrons
- elastic scattering with nuclei and atoms

Other process: - emission of Cherenkov radiation
- nuclear reactions
- Bremsstrahlung

Divide the description (i) heavy charged particles (μ, π, p, α, nuclei)
(ii) electrons and positrons

Electrons: - larger energy transfer and bending due to low mass
- scattering of identical particles
- Bremsstrahlung

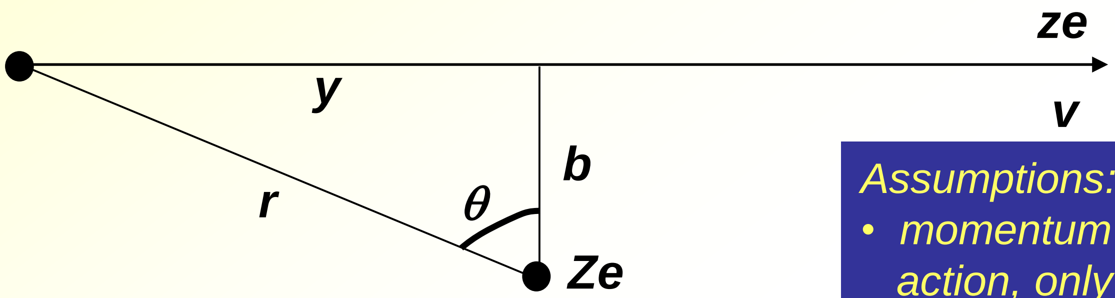
$$\sigma \propto r_e{}^2 = (e^2 / mc^2)^2$$

→ *requires separate description of e^- and e^+*

Interaction of heavy charged particles in matter

Bohrs calculation – classical assumption and Ansatz

Consider moving particle of charge ze passing by with v the stationary charge Ze



Assumptions:

- *momentum approximation: short interaction, only transversal moment. transfer*
- *target remains non-relativistic*

$$\text{force trans.}: F_x = \frac{Zze^2}{r^2}\cos\theta \qquad r = b/\cos\theta$$

$$= \frac{Zze^2}{b^2}\cos^3\theta$$

$$\text{momentum onto target}: \quad \Delta p = \int_{-\infty}^{\infty} F_x dt \qquad dt = dy/v \qquad y = b\tan\theta \qquad dy = b\frac{1}{\cos^2\theta}\mathrm{d}\theta$$

$$= \frac{Zze^2}{b^2}\int_{-\pi/2}^{\pi/2}\cos^3\theta\,\frac{b}{v\cos^2\theta}d\theta = \frac{2Zze^2}{b\ v}$$

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Bohrs calculation

$$\text{Energy increase of target atom}: \quad E_T = \frac{(\Delta p)^2}{2m_T} = \frac{2Z^2 z^2 e^4}{m_T b^2 v^2}$$

$$\text{Energy transfer to nuclei is negligible}: \quad \frac{Z^2 / Am_p}{Z^2 / m_e} \approx \frac{m_e}{2m_p}$$

$$\text{Energy increase of one electron}: \quad \Delta E = \frac{2z^2 e^4}{m_e v^2 b^2}$$

Energy loss to all electrons in distance b and $\mathrm{b} + db$, and thickness dx

Electronen density N_e

$$-dE(b) = \Delta E(b) N_e dV = \frac{4\pi\ z^2 e^4}{m_e v^2} N_e \frac{db}{b} dx \qquad \text{and} \qquad dV = 2\pi b\, db\, dx$$

Interaction of heavy charged particles in matter

Integration from $b = 0$ to ∞ does not work:

(i) ∞ in contrary to momentum approximation.

(ii) small distance $\rightarrow$ infinite E transfer - nonsense!

$$-\frac{dE}{dx} = \frac{4\pi\, z^2 e^4}{m_e v^2} N_e \ln \frac{b_{\max}}{b_{\min}}$$

Bohr's arguments for $b_{\min}$ and $b_{\max}$

1. classical limit: maximum E-transfer in central collision: $2\gamma^2 m_e v^2 = \dfrac{2z^2e^4}{m_e v^2 b_{\min}^2}$

$$\gamma = \frac{1}{\sqrt{(1-\beta^2)}} \quad \text{and} \quad \beta = \mathrm{v/c}$$

Q.M.: angular momentum and uncertainty: $L = p_{cms} b_{\min}$ and $\Delta L \Delta \Phi \approx h$

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2. no E - transfer in very peripheral collisions,

$\vec{E}$ - field is only an adiabatic distortion.

'Adiabatic invariance': collision time is long with respect to revolution frequency of electrons.

typical collision time: $t \cong b/v \Rightarrow t \cong b/(\gamma v)$ (relativistic)

condition: $b/(\gamma v) \leq \tau = 1/\bar{\upsilon}$ with averaged frequency of all e^- in atom.

average ionisation potential of atoms: $I_o = h\upsilon, \quad b_{\max} \approx h\gamma v / I_o$

Bohrs formula

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \frac{\gamma^2 m_e v^3}{z e^2 \bar{\upsilon}}$$

Q.M. approximation

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \frac{\gamma^2 v^2 m_e}{I_o}$$

Interaction of heavy charged particles in matter

Quantum mechanical results from Bethe-Bloch

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{2m_e\gamma^2 v^2 W_{\max}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$$

N_a : Avogadro constant 6.022 x 10^{23} mol^{-1}

r_e : class. electron radius 2.81 x 10^{-13} cm

m_e : electron mass

Z : charge number of abs. matter

A : atomic mass of abs. Materials

ρ : density of absorbing matter

z : charge of incoming particle

β, γ : $\beta = \text{v/c}$ $\gamma = 1/\sqrt{1-\beta^2}$

I : averaged ionisation potential

$W_{\max}$: max. energy transfer in single collision

two correction terms

δ : density correction (at relativistic energies)

C : shell correction (low energies)

Interaction of heavy charged particles in matter

average ionisation potential: - difficult to calculate exactly for all shells
- semi-empirical approximation $I \sim Z$

Density correction: - Electric field of moving charge is polarizing atoms and shields charges at larger distances.
- Polarisation depends on density of matter.
- reduced dE/dx at higher energies
- empirical approx.

Shell correction: -correction at small energies of incoming particles, velocity is comparable to Bohr- or e^- *-velocity.*
- at low energies strong decrease of dE/dx.

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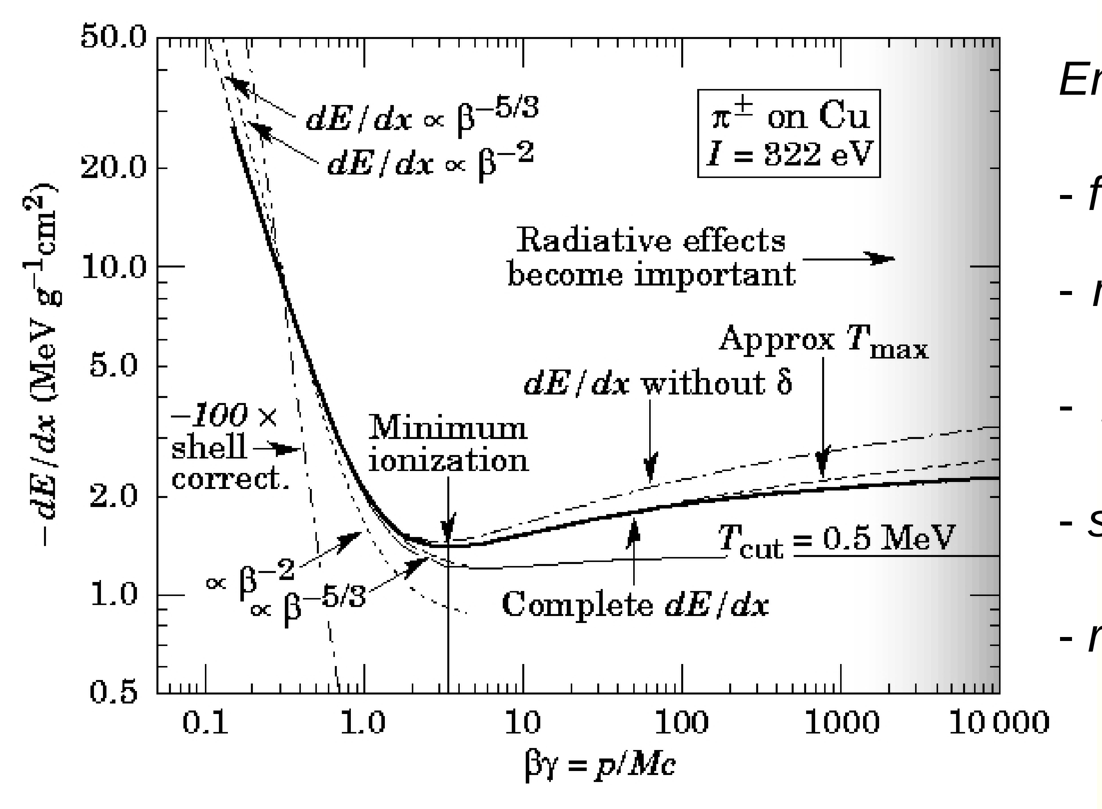

Energy dependence of energy loss

- *for non-relativistic energy: ~1/β^2*
- *minimum at v~0.96c*
- *independent of mass*
- *slow relativistic increase*
- *reduced by density effect*

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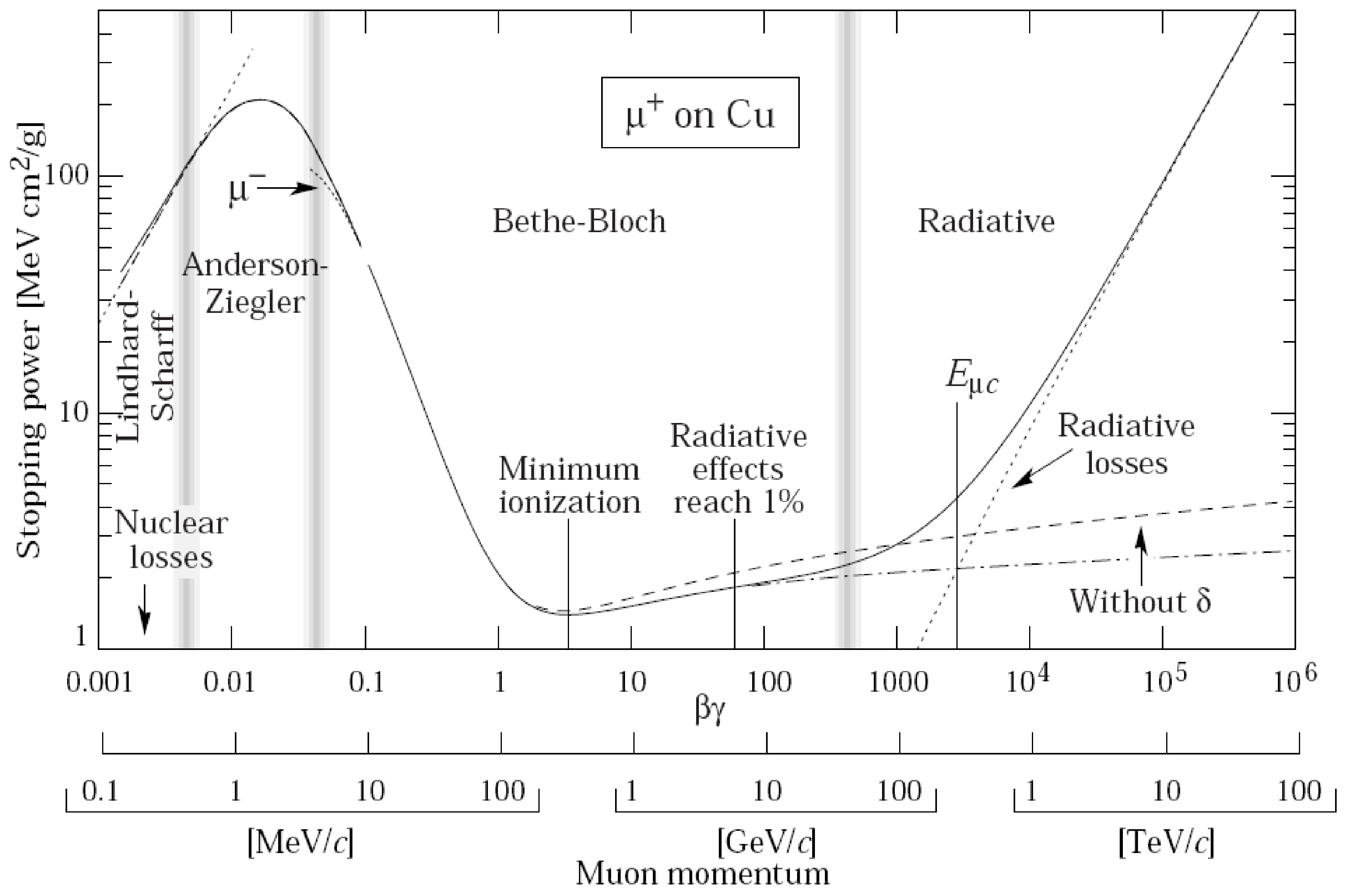

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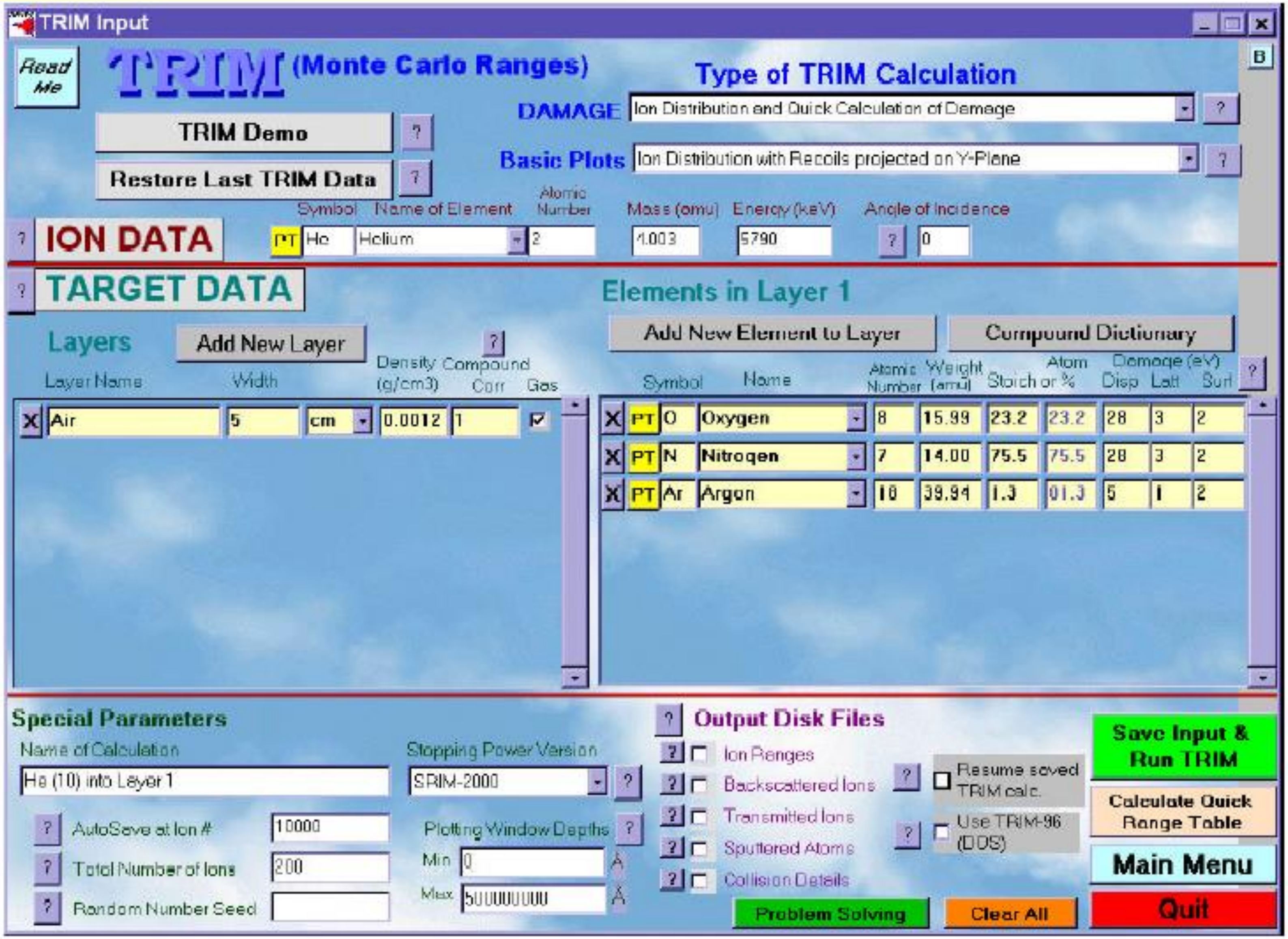


DAMAGE: Ion Distribution and Quick Calculation of Damage

Basic Plots: Ion Distribution with Recoils projected on Y-Plane

ION DATA

Symbol	Name of Element	Atomic Number	Mass (amu)	Energy (keV)	Angle of Incidence
PT He	Helium	2	4.003	5790	0

TARGET DATA

Layers

	Layer Name	Width		Density (g/cm3)	Compound Corr	Gas
X	Air	5	cm	0.0012	1	☑

Elements in Layer 1

		Symbol	Name	Atomic Number	Weight (amu)	Atom Stoich	or %	Damage (eV) Disp	Latt	Surf
X	PT	O	Oxygen	8	15.99	23.2	23.2	28	3	2
X	PT	N	Nitrogen	7	14.00	75.5	75.5	28	3	2
X	PT	Ar	Argon	18	39.94	1.3	01.3	5	1	2

Special Parameters

Name of Calculation: He (10) into Layer 1

Stopping Power Version: SRIM-2000

AutoSave at Ion #: 10000

Total Number of Ions: 200

Random Number Seed:

Plotting Window Depths: Min 0 Å, Max 50000000 Å

Output Disk Files

- ☐ Ion Ranges
- ☐ Backscattered Ions
- ☐ Transmitted Ions
- ☐ Sputtered Atoms
- ☐ Collision Details

☐ Resume saved TRIM calc.

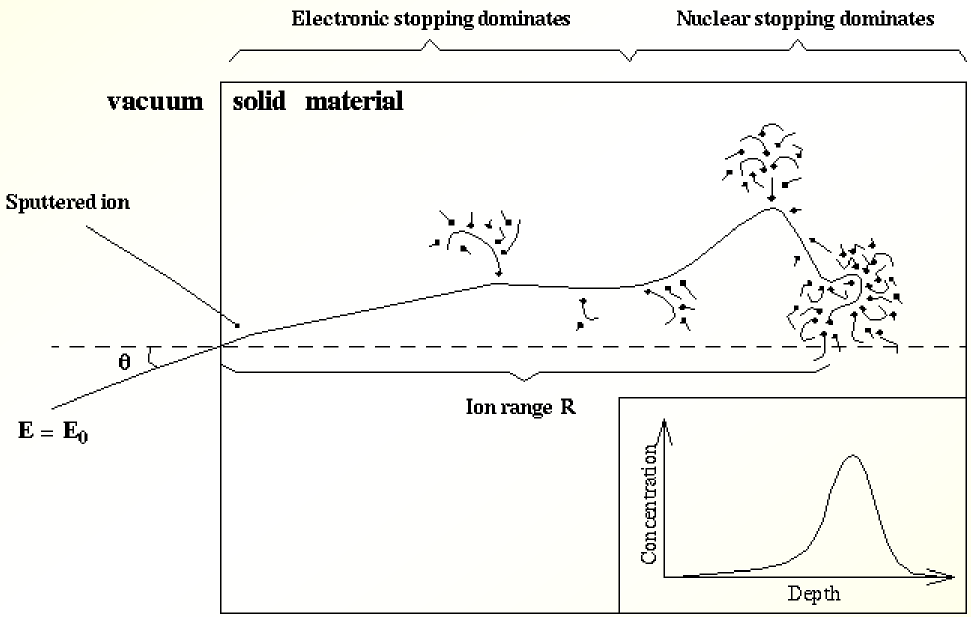
☐ Use TRIM-96 (DOS)

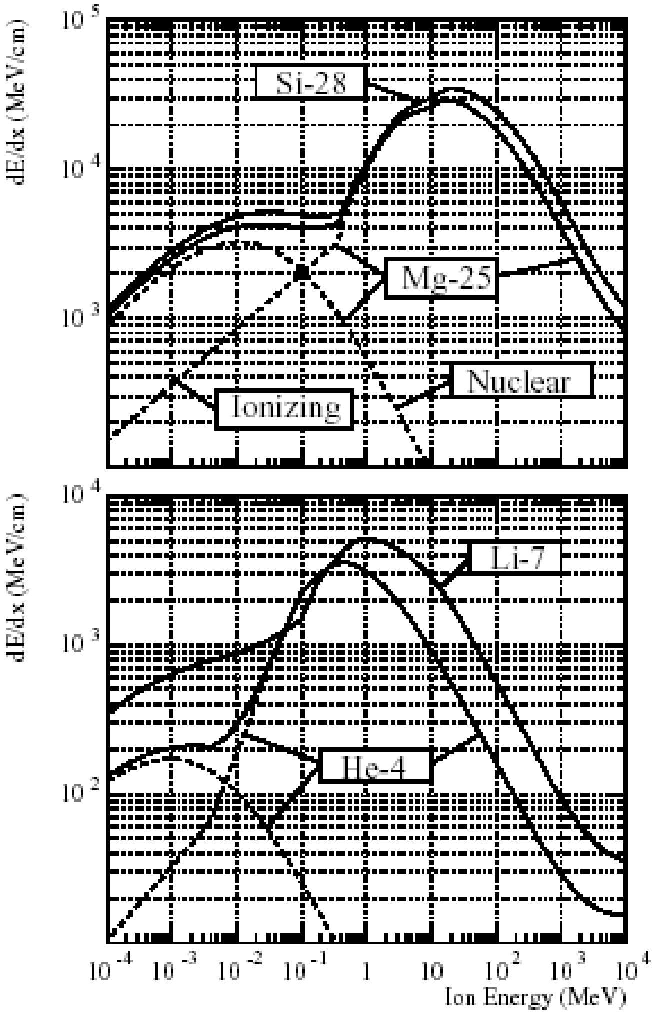
Interaction of heavy charged particles in matter

Electronic stopping: slowing down by inelastic collisions between electrons in matter and moving ion at E_{ion} >100 keV.

Nuclear stopping: elastic collisions between Ion and the full atoms. E_{ion}<100 keV

e.g.: Si-ion, E_{ion}=1 MeV, range in Si r=1-2μm





Interaction of heavy charged particles in matter

Energy loss, stopping power and range of charged particles

- Largest energy deposition at end of track: Bragg peak

$$-\frac{dE}{d\varepsilon}=-\frac{1}{\rho}\frac{dE}{dx}=z^2\frac{Z}{A}f(\beta,I)$$

- dE/dε is nearly independent of matter for equal particles.
- average range for particle with kin. energy T :

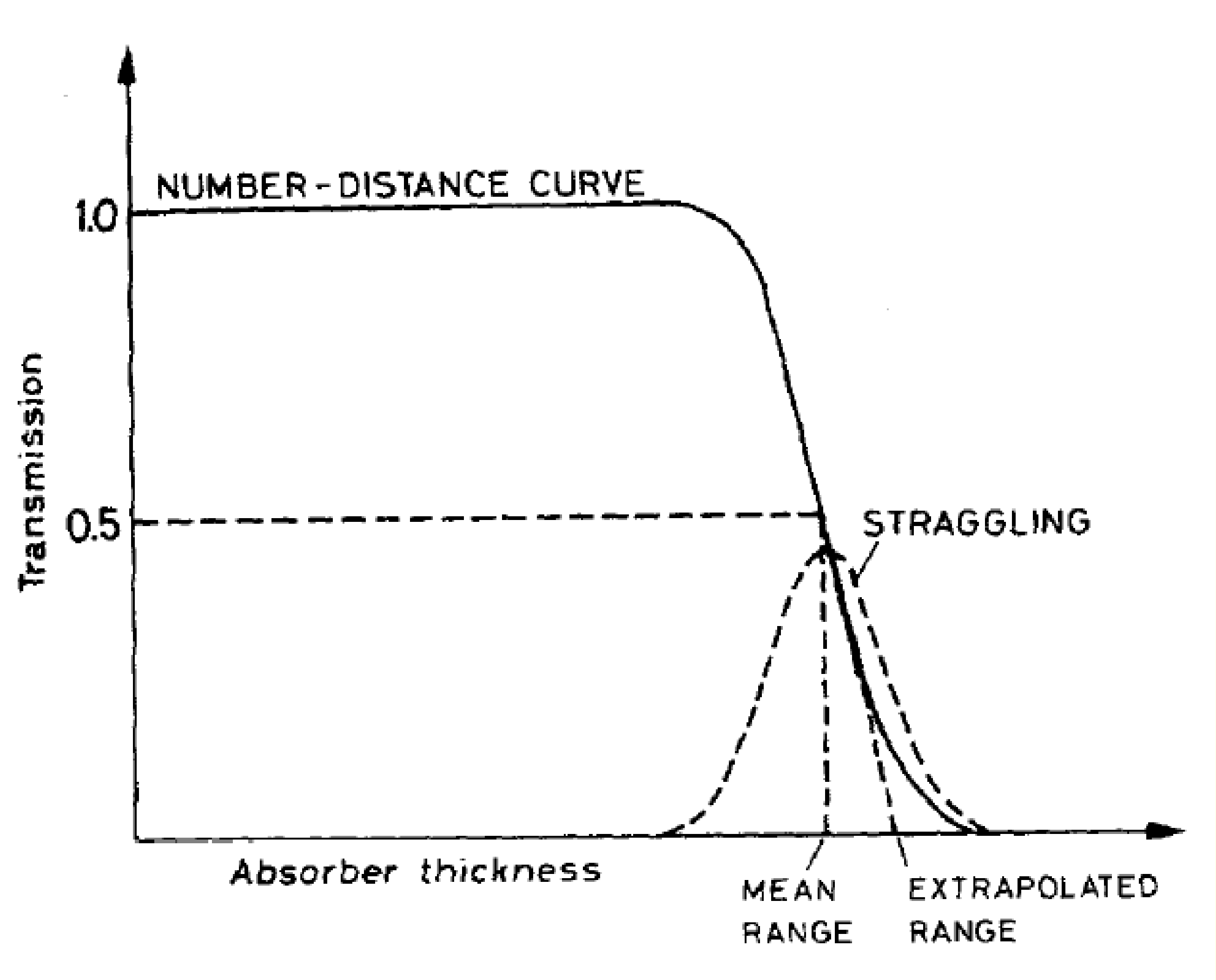
$$S(T)=\int_0^T\left(\frac{dE}{dx}\right)^{-1}dE$$

- Range is not a precise quantity but smeared, called range straggling. Number of interactions is statistically distributed. At low energies empirical constants are included in S(T).

- Remark: In matter with spatial symmetry (e.g. crystals) Bethe-Bloch formula is not valid! Correlated scattering at certain geometrical conditions (critical angle with respect of crystal axis) reduces energy loss and enlarge range of particle. Channeling effect

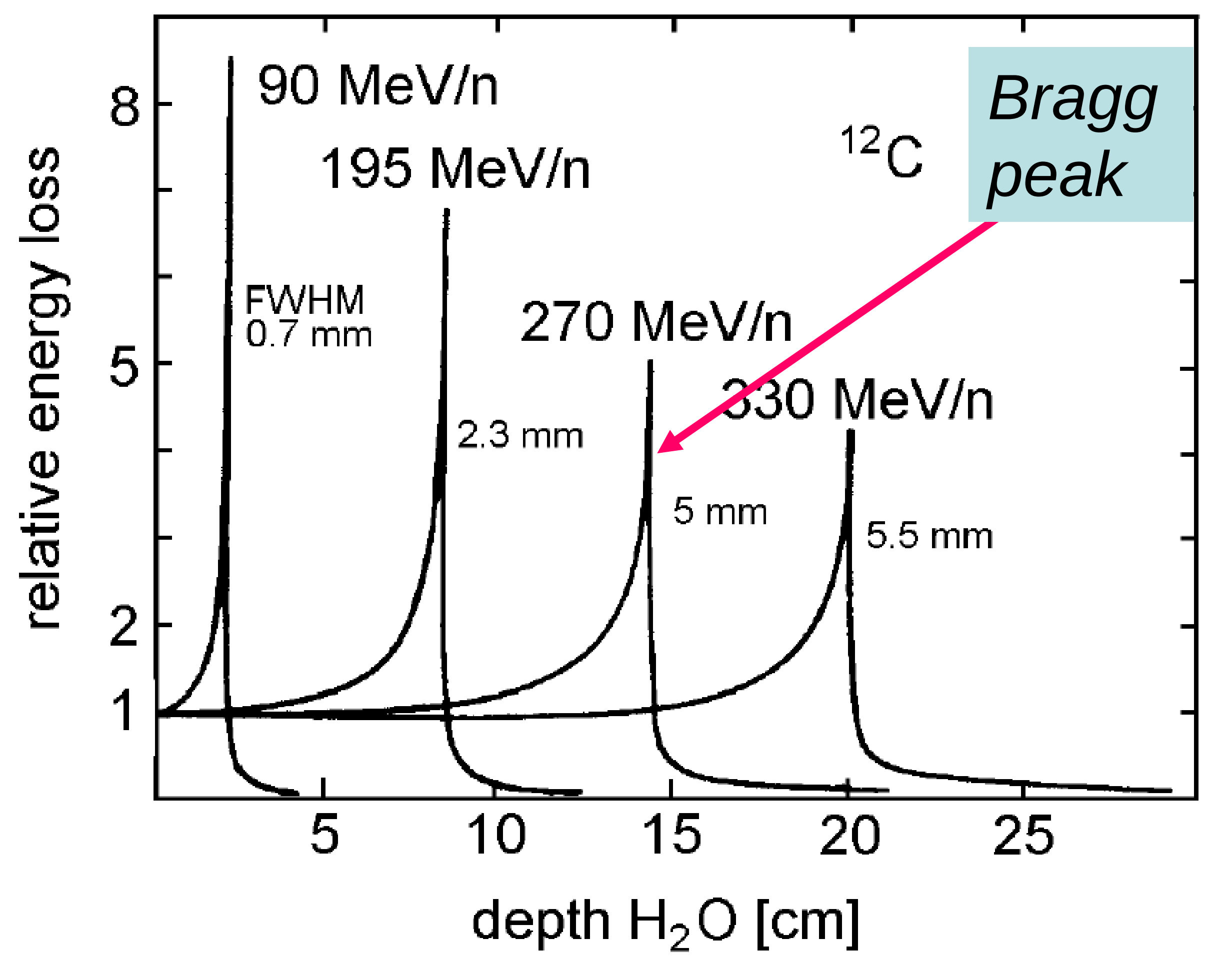
Interaction of heavy charged particles in matter

Range and straggling



Range of heavy ions in matter

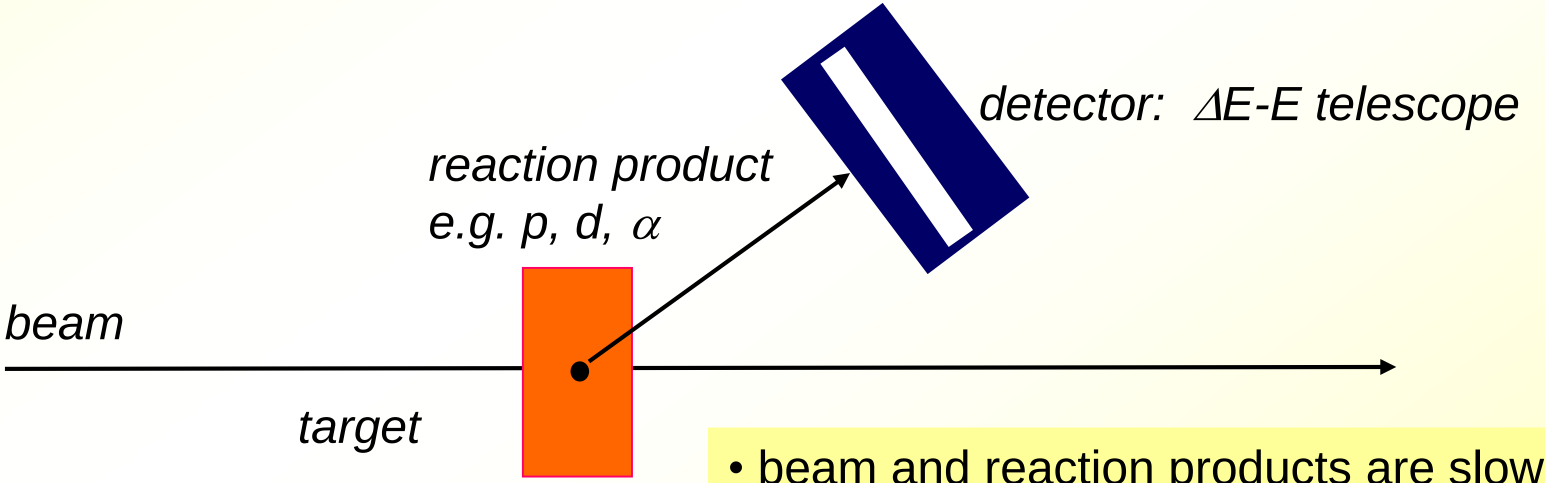
Range von ^{12}C ions in water *unit: MeV/n e.g. 90 MeV, n=12*
Elab=1,08 GeV



Interaction of charged particles in matter

Examples, applications for stopping power and range:
Energy loss of projectil and reaction products in target
e.g. ^{48}Ca+ ^{208}Pb @ E_{beam}=200MeV *target thickness* d=0.5 mg/cm^2 (s= 44 μm)
energy loss of ^{48}Ca *in target 7.2 MeV*
-> equals width of excitation function ^{208}Pb(^{48}Ca,2n) *- reaction*

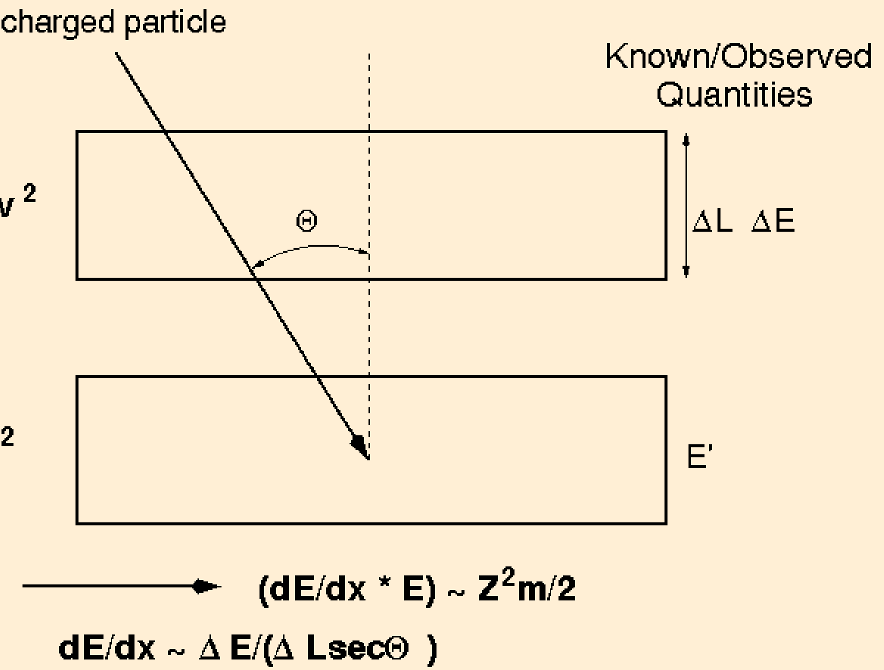
remark: thickness are given in d=mass/area
Units (e.g.): mg/cm^2 $d = m/A = \rho\, s$
ρ density, s thickness: $s=d/\rho$



- beam and reaction products are slowed down.
- cross section and kinematics are changed.
- targets limit count rate and energy resolution.

Interaction of charged particles in matter

Example and application for stopping power and range:
Particle identification with ΔE – E telescope



$dE/dx \sim Z^2/v^2$

$E \sim (1/2)mv^2$

$\longrightarrow (dE/dx * E) \sim Z^2m/2$

$dE/dx \sim \Delta E/(\Delta L \sec\Theta)$