Interaction of heavy charged particles in matter

Quantum mechanical results from Bethe-Bloch

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{2m_e\gamma^2 v^2 W_{\max}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$$

N_a : Avogadro constant 6.022×10^{23} mol^{-1}

r_e : class. electron radius 2.81×10^{-13} cm

m_e : electron mass

Z : charge number of abs. matter

A : atomic mass of abs. Materials

ρ : density of absorbing matter

z : charge of incoming particle

β, γ : $\beta = v/c$ $\gamma = 1/\sqrt{1-\beta^2}$

I : averaged ionisation potential

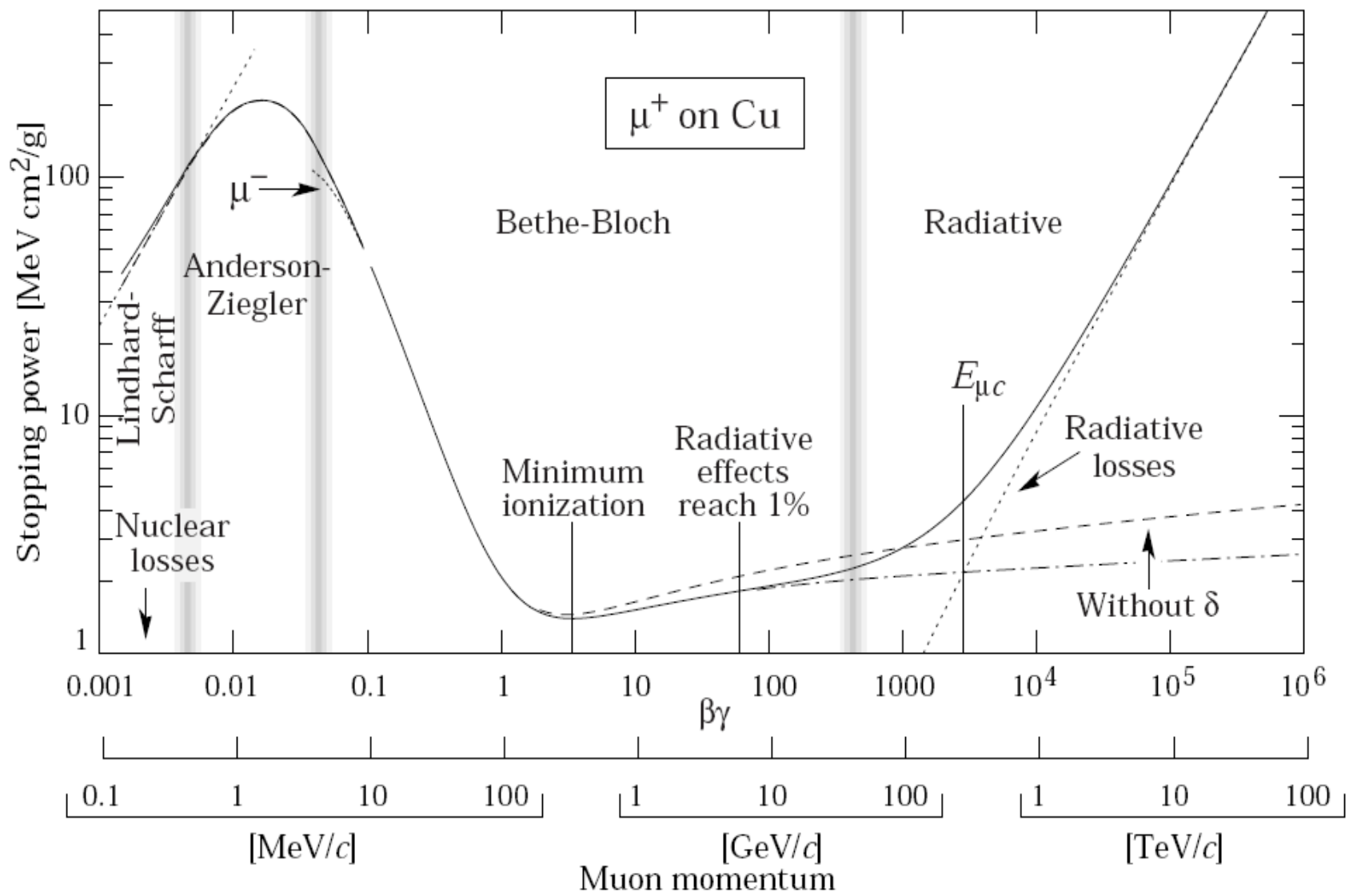
$W_{\max}$: max. energy transfer in single collision

two correction terms

δ : density correction (at relativistic energies)

C : shell correction (low energies)

Interaction of heavy charged particles in matter

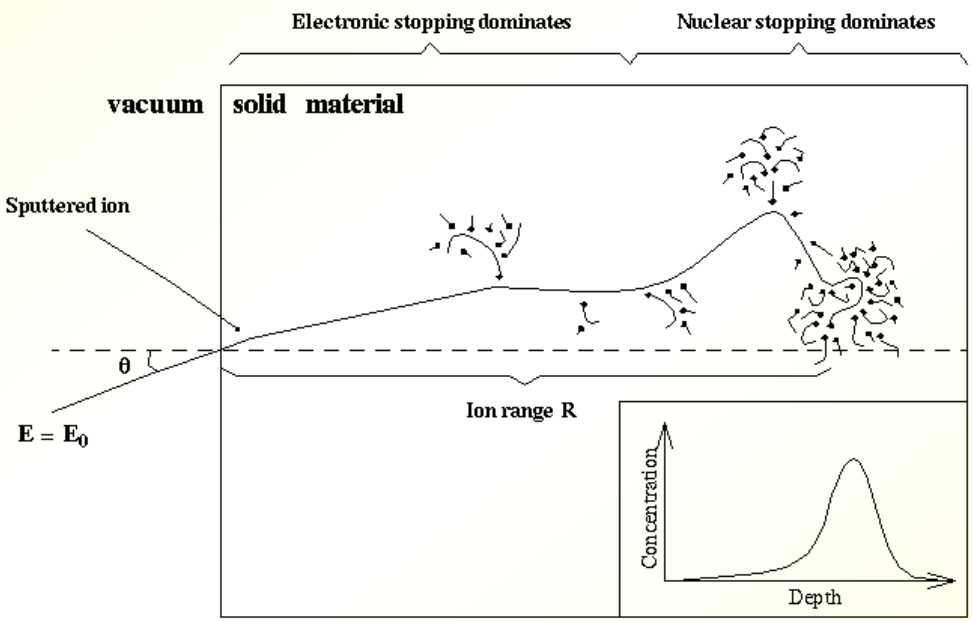


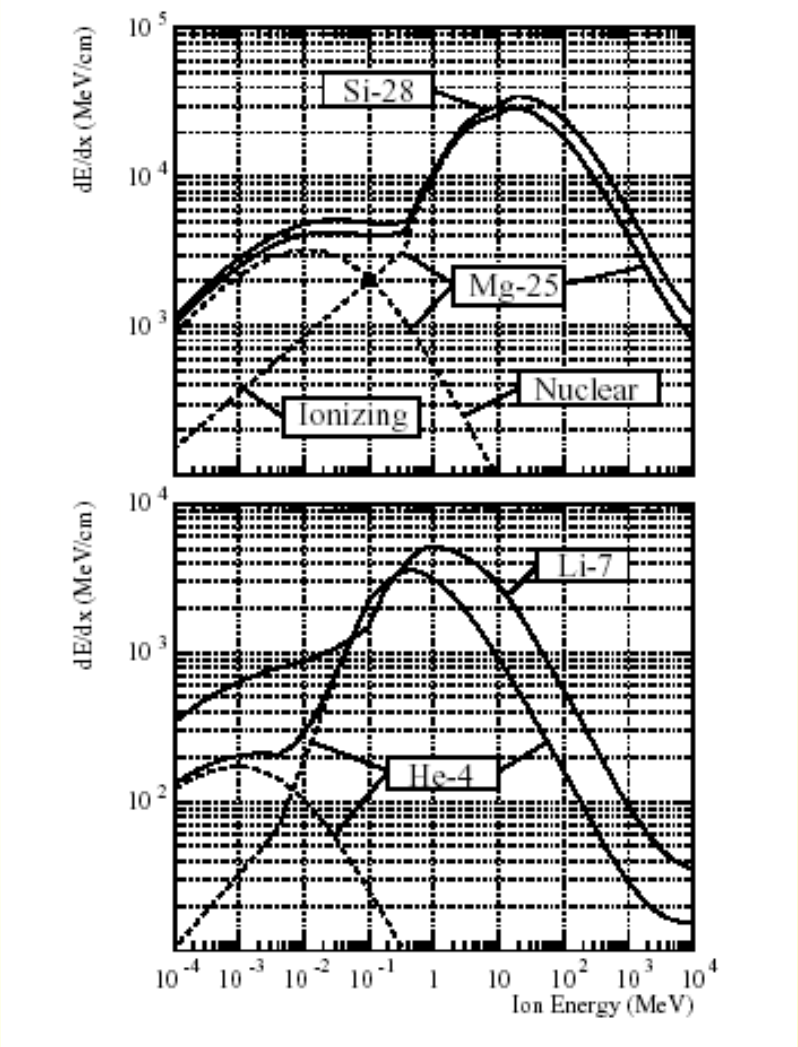
Interaction of heavy charged particles in matter

Electronic stopping: slowing down by inelastic collisions between electrons in matter and moving ion at E_{ion} >100 keV.

Nuclear stopping: elastic collisions between Ion and the full atoms. E_{ion}<100 keV

e.g.: Si-ion, E_{ion}=1 MeV, range in Si r=1-2μm





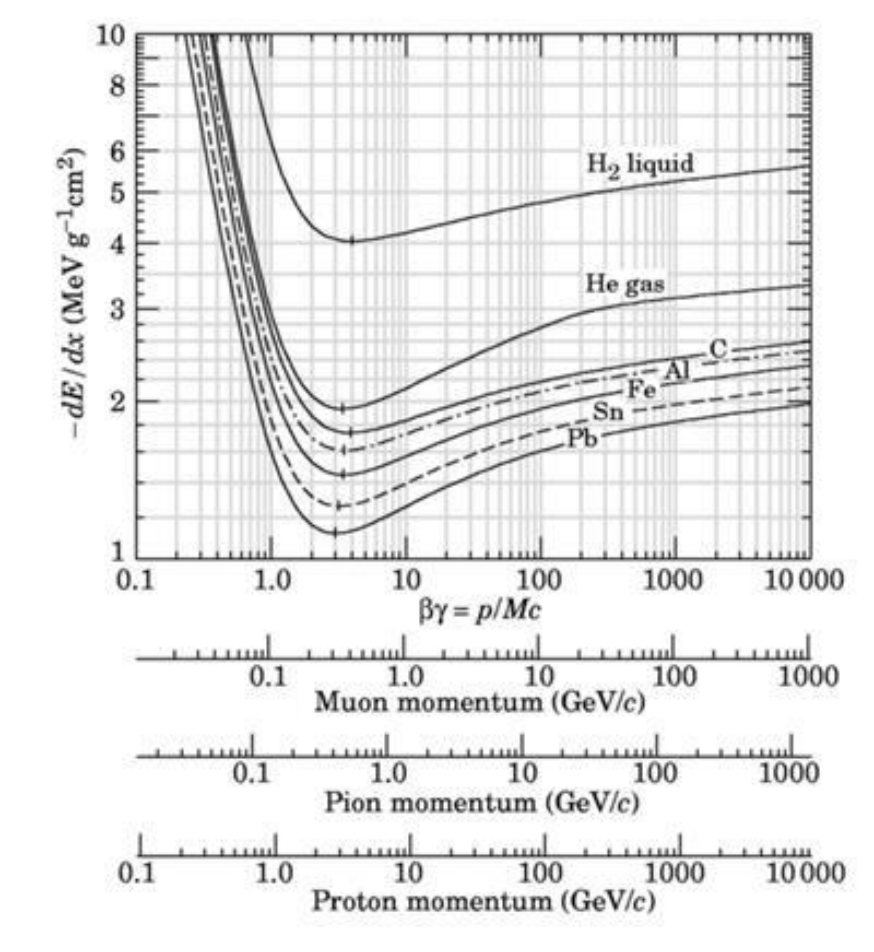
Interaction of heavy charged particles in matter

Energy loss, stopping power and range of charged particles

- Biggest energy deposition at end of track: Bragg peak

$$-\frac{dE}{d\varepsilon} = -\frac{1}{\rho}\frac{dE}{dx} = z^2 \frac{Z}{A} f(\beta, I)$$

- $dE/d\varepsilon$ is nearly independent of matter for equal particles.



Interaction of heavy charged particles in matter

Energy loss, stopping power and range of charged particles

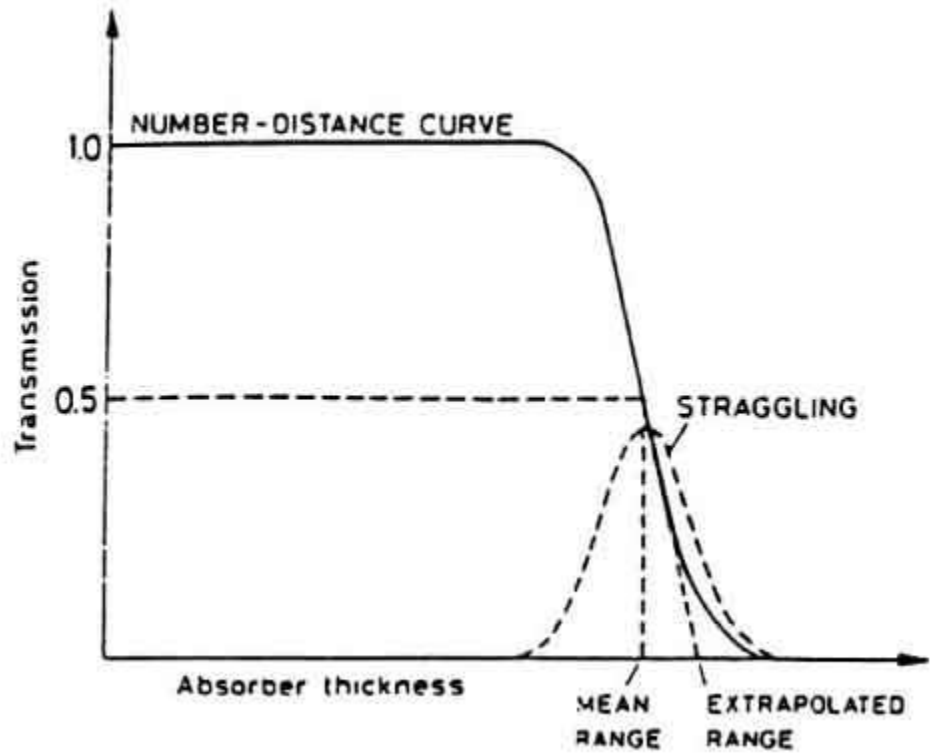
- average range for particle with kin. energy T :

$$S(T) = \int_0^T \left(\frac{dE}{dx} \right)^{-1} dE$$

- Range is not a precise quantity but smeared out, called range straggling.

- Number of interactions is statistically distributed.

- At low energies empirical constants are included in S(T).



Interaction of heavy charged particles in matter

Energy loss, stopping power and range of charged particles

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Number of interactions is statistically distributed.
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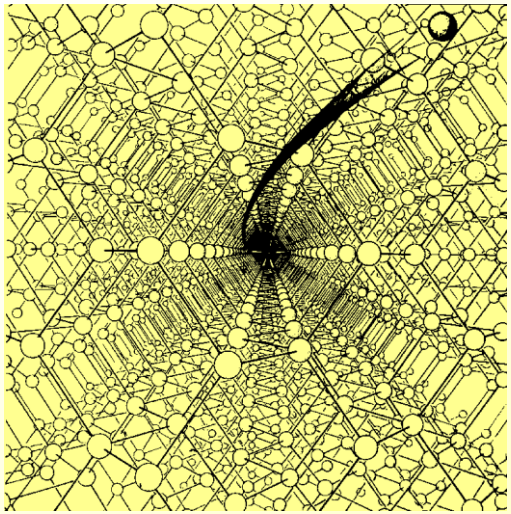
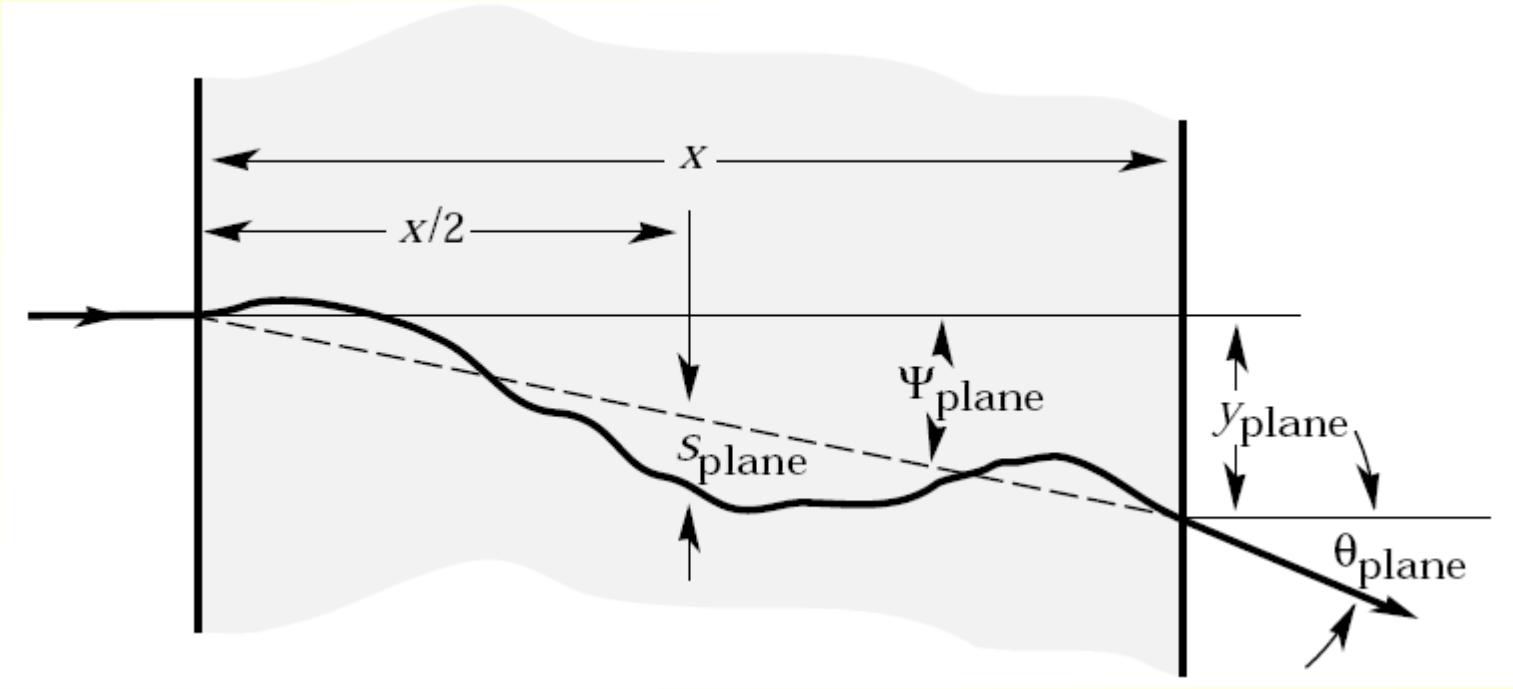
- Remark: In matter with spatial symmetry (e.g. crystals) Bethe-Bloch formula is not valid! Correlated scattering under certain geometrical conditions (critical angle with respect of crystal axis) reduces energy loss and enlarge range of particle.
Channeling effect



Interaction of heavy charged particles in matter

- Heavy particles do not lose only energy in matter



- … there is a chance for small changes in direction
- Small angle scattering

Interaction of heavy charged particles in matter

- Average scattering angle is nearly Gauss distributed for small deviations
- $$\theta_0 = \frac{13.6\ \text{MeV}}{\beta c p} z \sqrt{\frac{x}{X_0}} \left[1 + 0.038 \ln\left(\frac{x}{X_0} \right) \right]$$

  $X_0 \equiv$ radiation length
- Angular distribution:

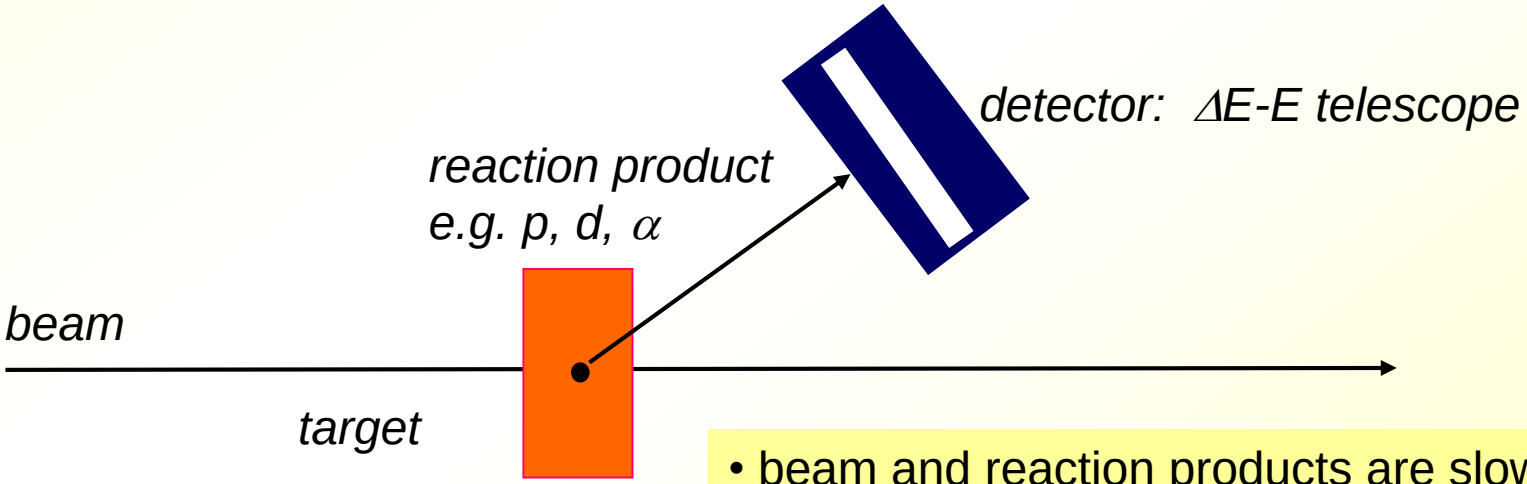
$$\frac{dN}{d\Omega} \propto \frac{1}{2\pi\theta_0^2} \exp\left(-\frac{\theta_{space}^2}{2\theta_0^2} \right)$$

$$\frac{dN}{d\theta_{plane}} \propto \frac{1}{\sqrt{2\pi}\theta_0} \exp\left(-\frac{\theta_{plane}^2}{2\theta_0^2} \right)$$

Interaction of charged particles in matter

Examples, applications for stopping power and range:
Energy loss of projectil and reaction products in target
e.g. $^{48}Ca + ^{208}Pb$ @ E_{beam}=200MeV *target thickness d=0.5 mg/cm*^{2} (s= 44 μm)
energy loss of ^{48}Ca *in target 7.2 MeV*
-> equals width of excitation function $^{208}Pb(^{48}Ca,2n)$ *- reaction*

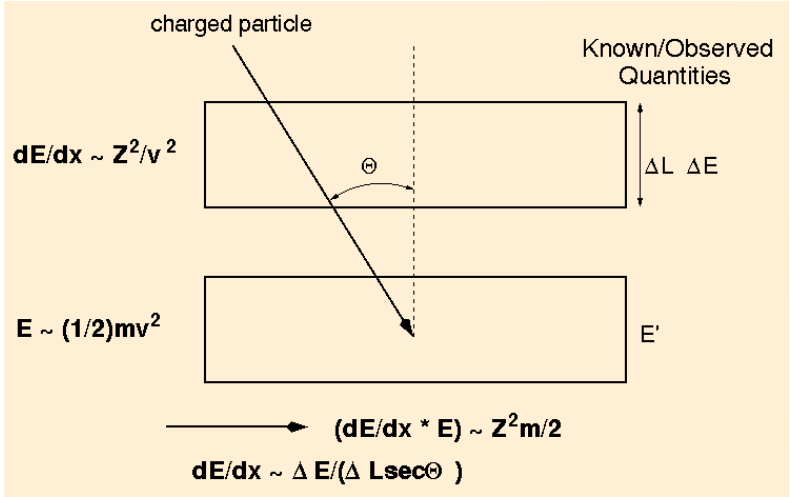
remark: thickness are given in d=mass/area
Units (e.g.): mg/cm^2 d = m/A = ρ s
ρ density, s thickness: s=d/ρ

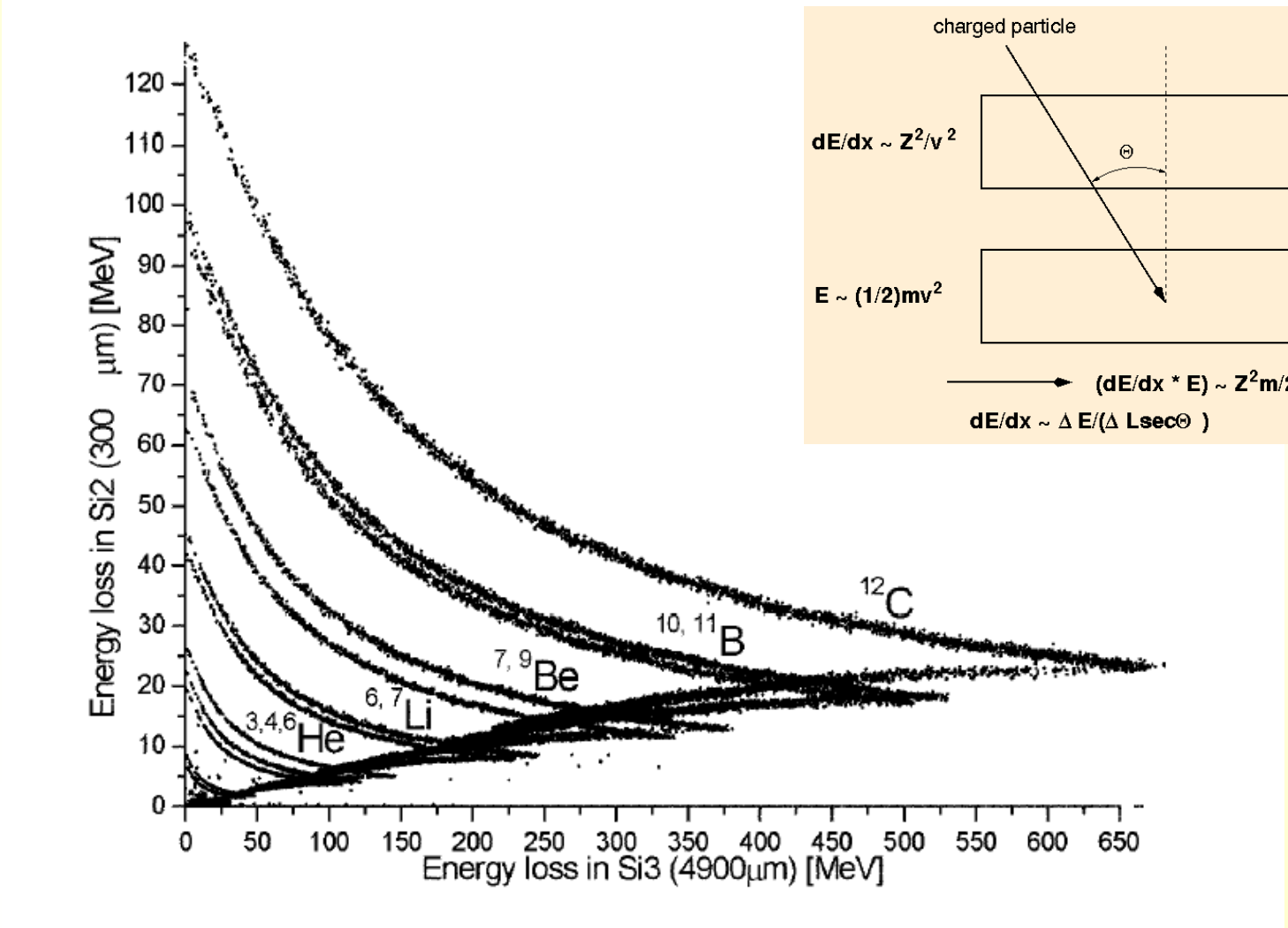


- beam and reaction products are slowed down.
- cross section and kinematics are changed.
- targets limit count rate and energy resolution.

Interaction of charged particles in matter

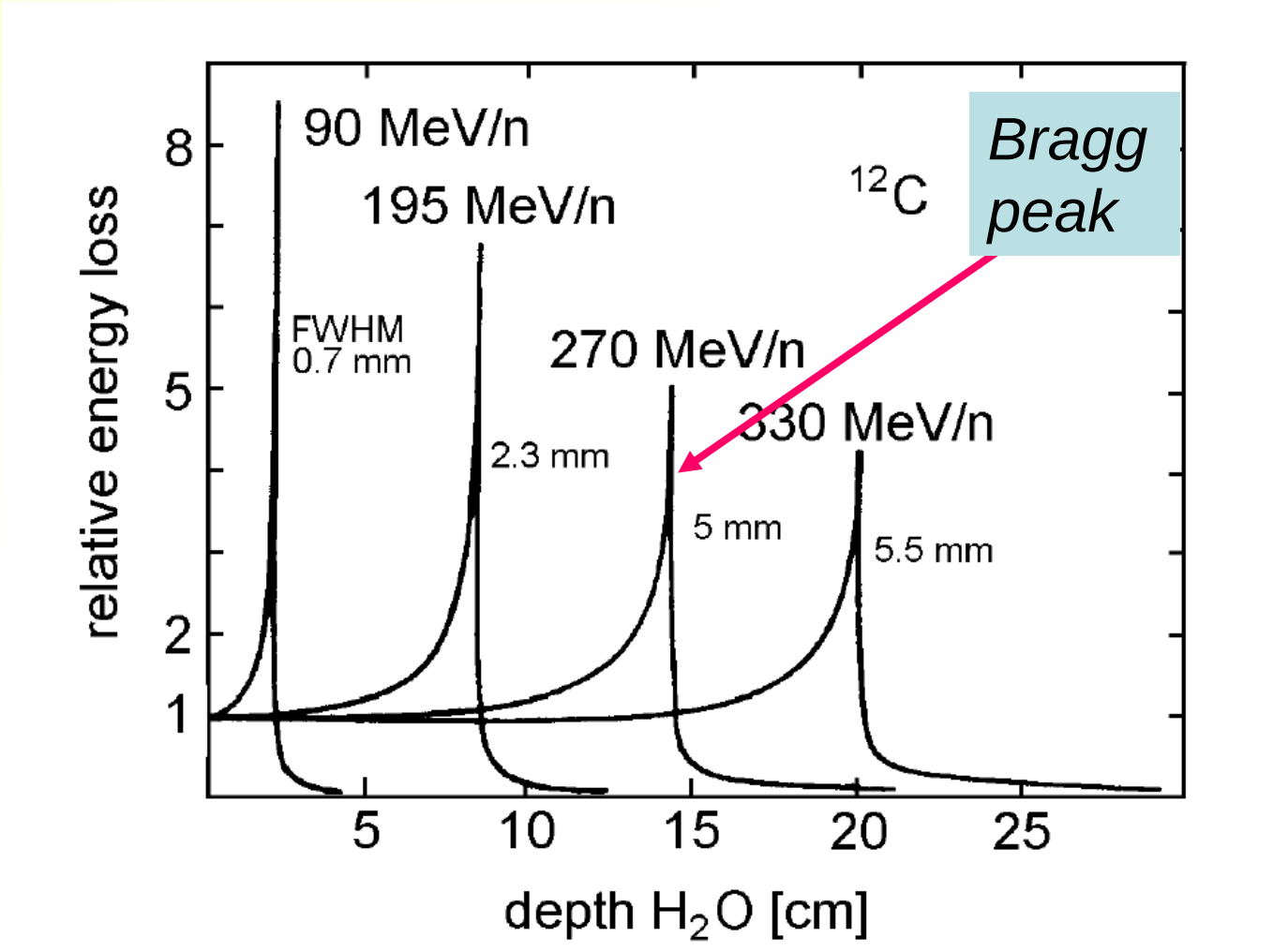
Example and application for stopping power and range:
Particle identification with ΔE – E telescope





Range of heavy ions in matter

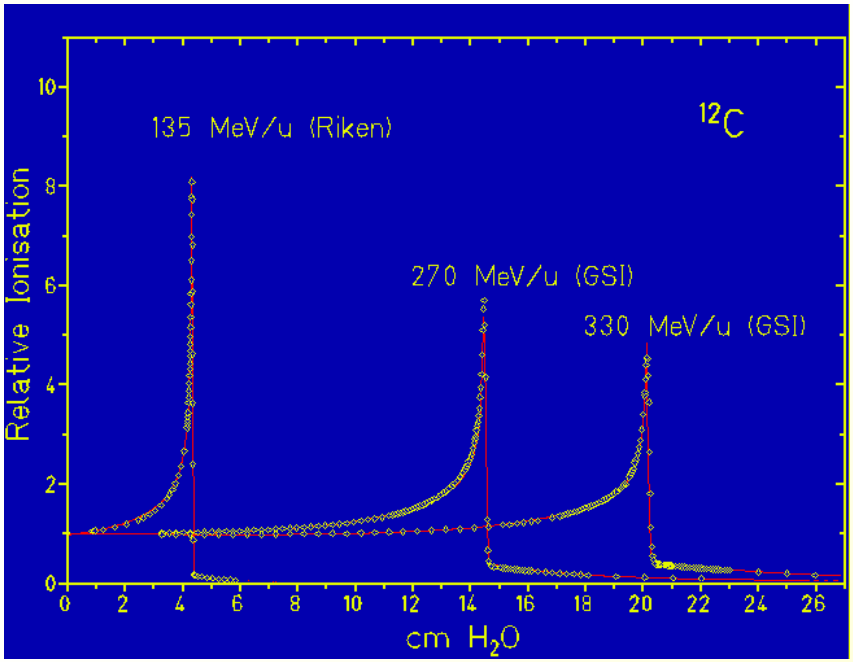
Range von ^{12}C ions in water *unit: MeV/n e.g. 90 MeV, n=12*
Elab=1,08 GeV



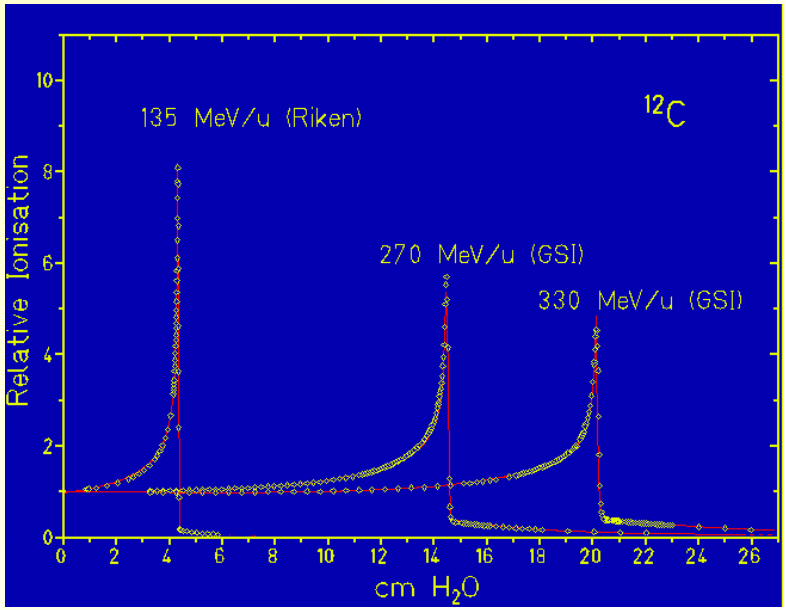
Tumor therapy with heavy ions

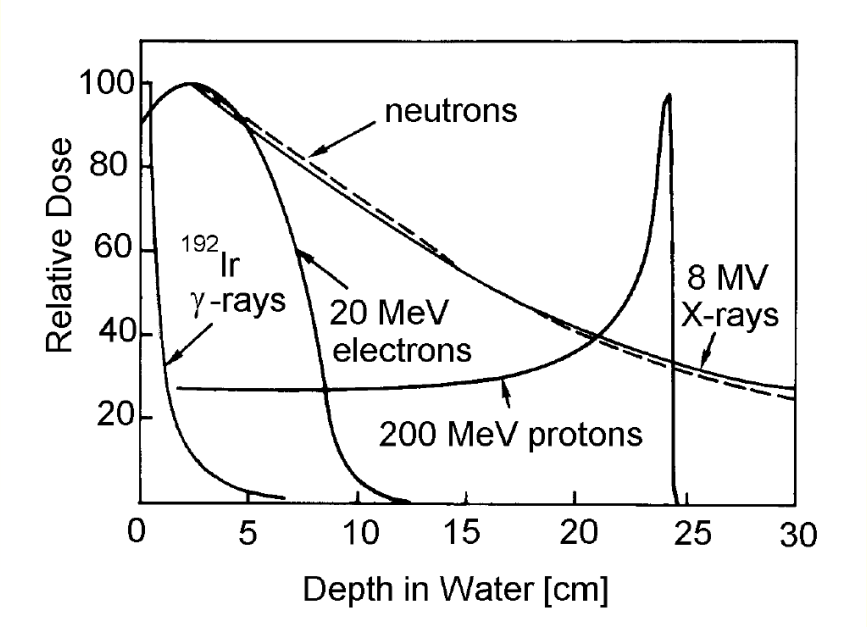
Heavy ion beams for radiation therapy of cancer tumors
Requirements:
- *profile of radiation dose – Bragg curve –*
- *increased relative biological efficiency of heavy ions*

Tumors :
- *brain tumors (chordoma, chondrosarcoma, mal. schwannoma, atyp. Meningeoma, adenoidcystic ca.)*
- *Tumors close to spine: (sacral chordoma, chondrosarcoma, soft tissue sarcoma)*



Compare with other treatments





Tumor therapy with ionising particle beams:

- *γ-rays and high energetic photons*
  strong absorption at surface

- *electrons*
  strong absorption at surface

- *neutrons*
  strongest absorption at surface
  local: $n + {}^{10}B \rightarrow {}^{7}Li + \alpha$

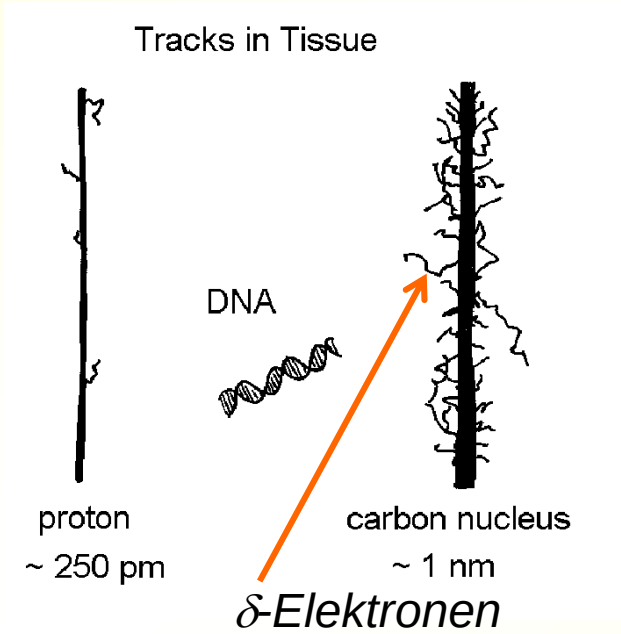
- *protons + heavy ions*
  highest biological efficacy
  at Bragg peak position

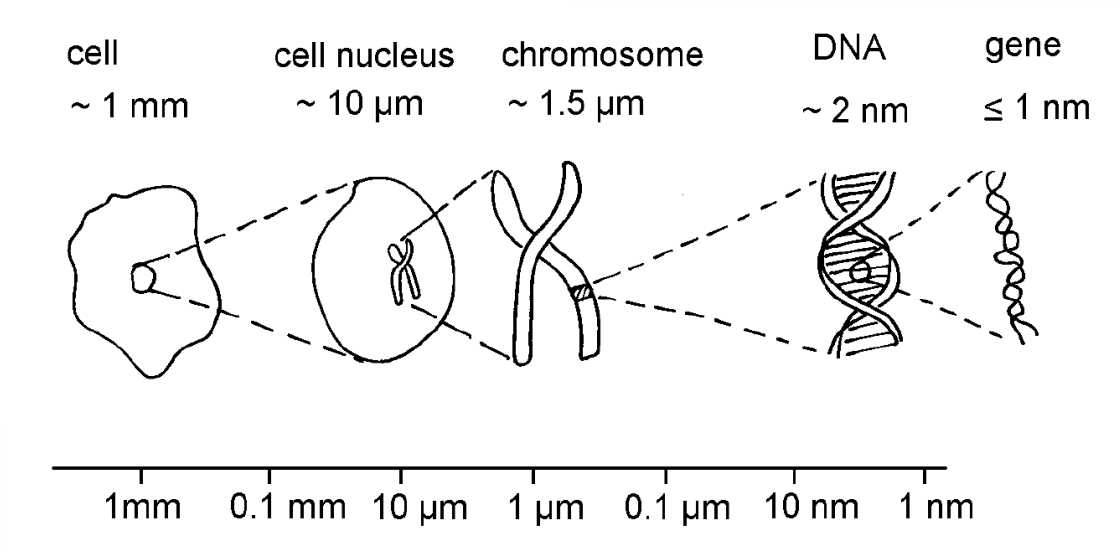
Biological Efficacy

Equivalent dose $$H = \frac{1}{m}\int \frac{dE}{dx}dx \cdot RBE$$ *Relativ. Biolog. Effectiveness*

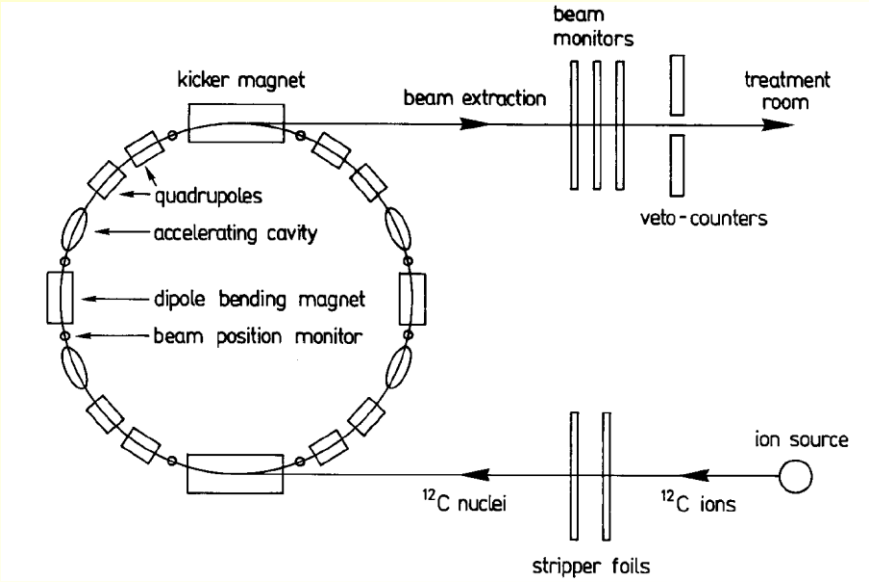
increased biological efficacy of ions due to DNA double

Increased transversal range of ionising radiation:





Tumor therapy: technical solutions

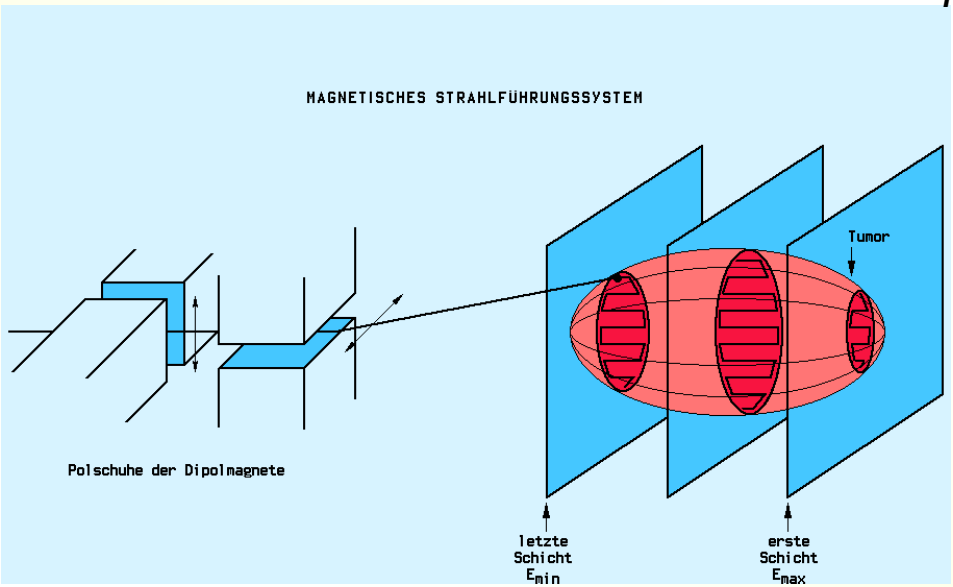


highest accelerator demands:

variation of beam energy: 80 – 450 MeVA,
2 – 30 cm H_2O range,
20 – 40 steps

Pencil beam: diameter d~1mm

Beam monitoring :- ~ 1 mm,
in front of patient e.g. ionisation chamber

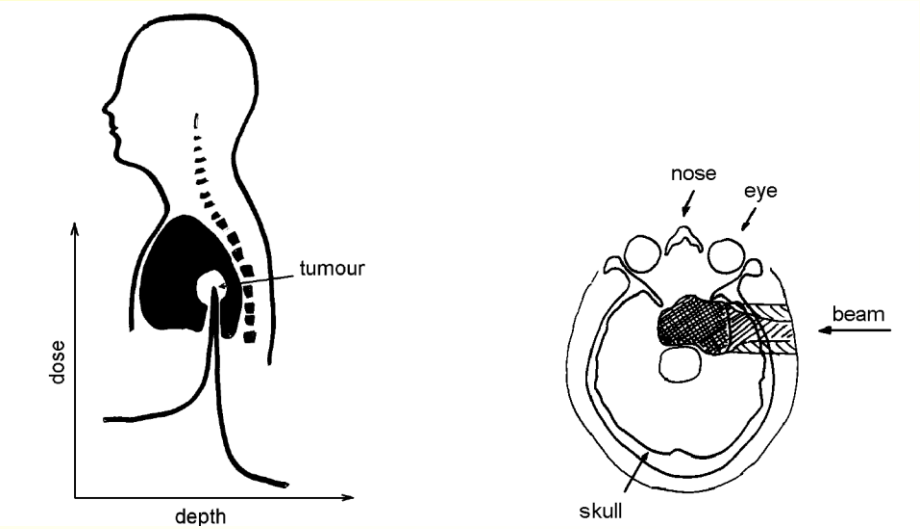


PET in situ (Tumor)

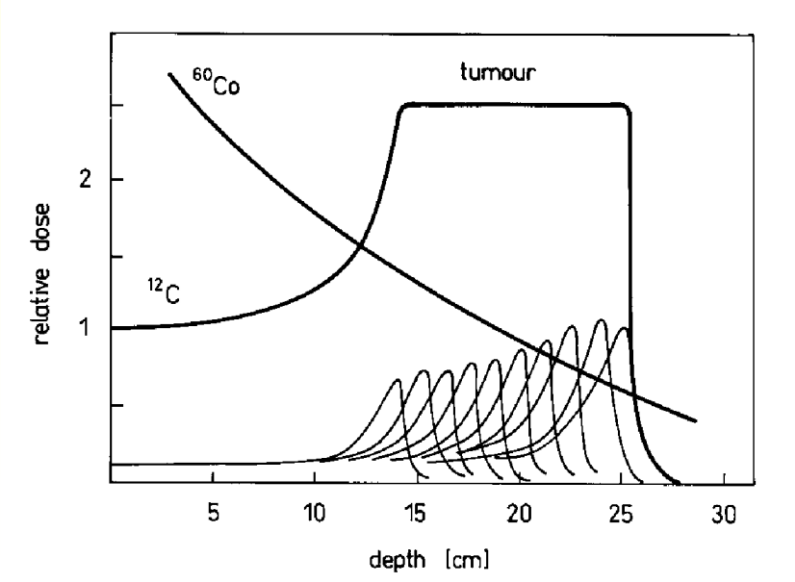
Raster scan of 3d tumor volume
in x-,y- direction by magnetic
bending (3d ´voxels`).

extrem high reliability of accelerator!

Tumor therapy strategy



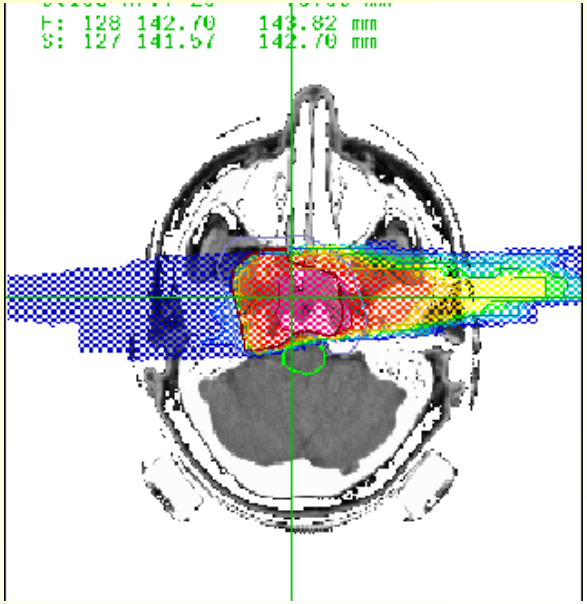
Treatment relies on exact position and size of tumor volume.
Radiation plan needs to know different tissues around tumor to determine the biological effectiveness.



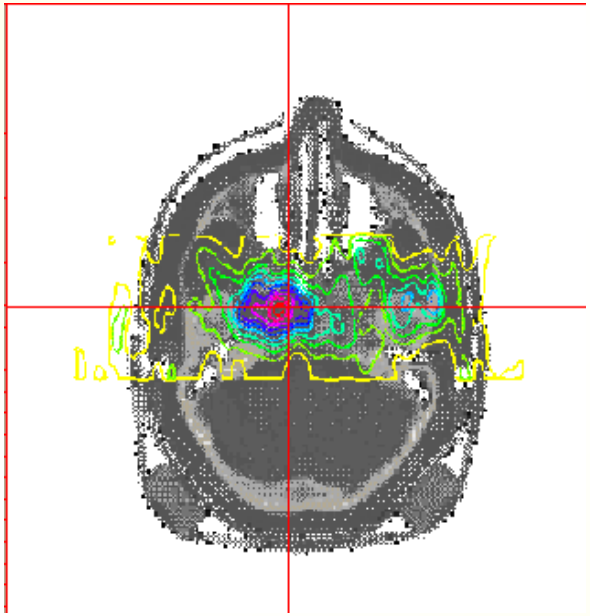
Monitoring via Positron-Emission-Tomography (PET)

- *Fragmentation of ^{12}C produces $^{10,11}C$*
- *β^+-decays:*
  $^{10}C \rightarrow {}^{10}B + \beta + \nu_e \quad T_{1/2}=19.3\ s$
  $^{11}C \rightarrow {}^{11}B + \beta^+ + \nu_e \quad T_{1/2}=20.38\ min$
- *Annihilation $e^+ + e^- \rightarrow \gamma + \gamma$*
- *correlated emission back-to-back of two 511 kev γ-rays*
- *pos. resolution ~ 2 - 3 mm*

Tumor therapy with heavy ions



Distribution of Verteilung radiation dose is overlaid with CT image of patient (red highest dose). Dose is optimized to achieve homogeneous biological dose profile in tumor volume.



Distribution of positron emitters detected with PET camera. PET is sensitive to stopped ^{11}C and ^{10}C reaction products.

Tumor therapy with heavy ions

Radiation therapy:

GSI, Darmstadt

Heidelberger Ionenstrahl-Therapiezentrum (HIT)

Interaction of charged particles in matter today electrons

Introductory remarks

- *Electrons ‚feel' electro-magnetic and weak interaction*
- *Electrons couple to photons and W- and Z-Boson of weak interaction*
- *Electrons ‚talk' easily to photons, remember pair creation and bremsstrahlung*

$$\sigma \propto r_e^{\ 2} \ = (e^2 / mc^2)^2$$

- *Electron and photon detection are entangled, e.g. electro-magnetic shower detection at high energies*

Relevance of electrons in nuclear and particle physics some examples:

- *beta decays*
- *conversion electrons (complementary to γ-spectroscopy)*
- *electron beams for electron-nucleon-scattering (structure functions)*
- *electron-positron collisions at high energies provide clean source for particle production and new discoveries*

- *electron is most abundant reaction product, often source of unwanted and tremendous back ground*

Interaction of electrons and positrons

Energy loss of electrons and positrons via collisions with electrons on atoms and electro-magnetic radiation (Bremsstrahlung)

$$\left(\frac{dE}{dx}\right)_{tot} = \left(\frac{dE}{dx}\right)_{rad} + \left(\frac{dE}{dx}\right)_{coll}$$

Modified calculation of energy loss

- large scattering angle of incoming particle

- scattering of identical particles -> Q.M. interference

Collisional Energy loss:

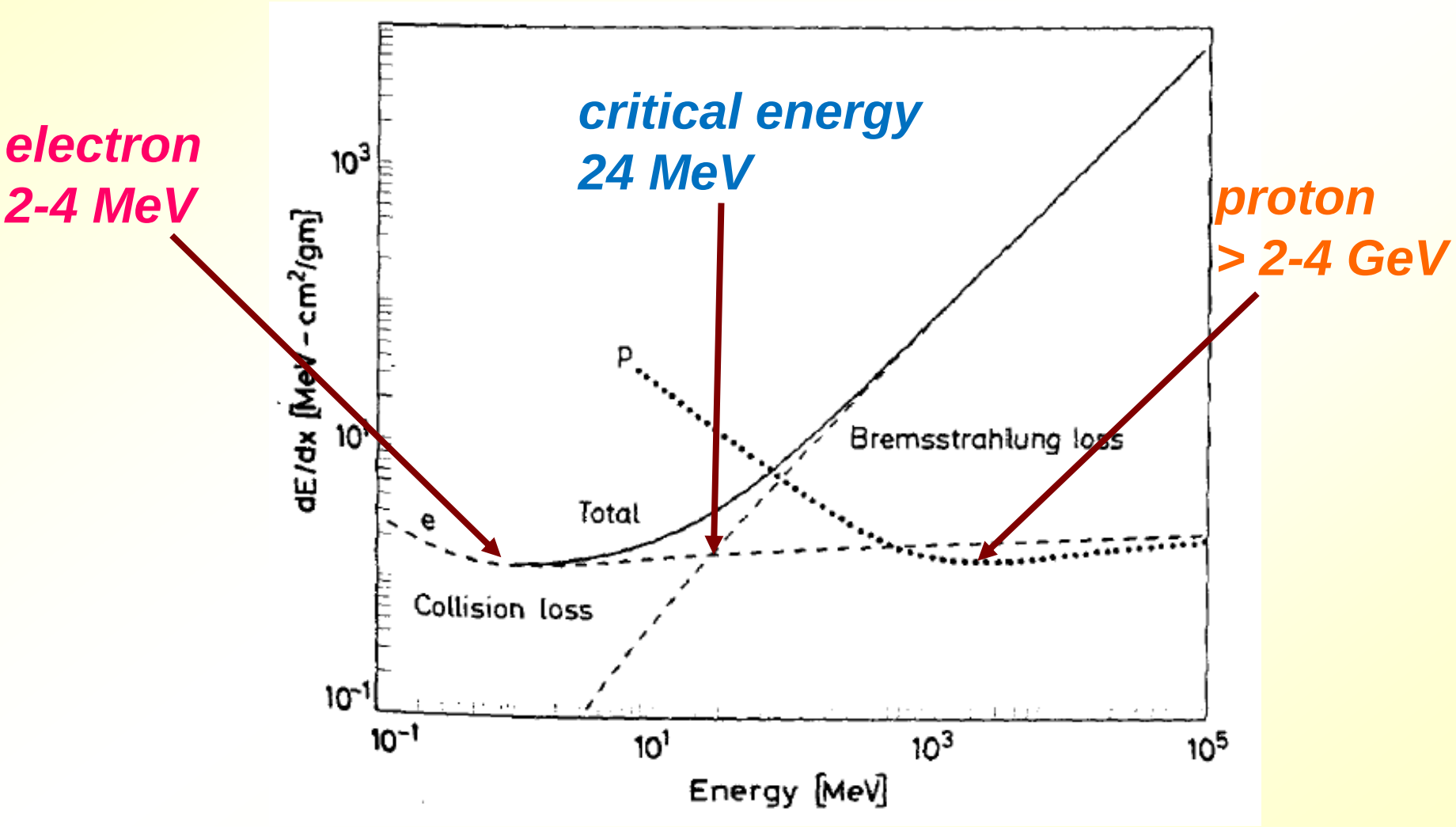
$$-\left(\frac{dE}{dx}\right)_{coll} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{1}{\beta^2}\left[\ln\left(\frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2}\right) + F(\tau) - \delta - 2\frac{C}{Z}\right]$$

τ: kinetic energy in units of $m_e c^2$

$F(\tau)$: $F(\tau) = 1 - \beta^2 + \ldots$ electrons $\quad F(\tau) = 2\ln 2 - \frac{\beta^2}{12}(23 + \ldots)$ positrons

Comparison of interactions

Bremsstrahlung loss and ionisation loss for electrons and protons in copper



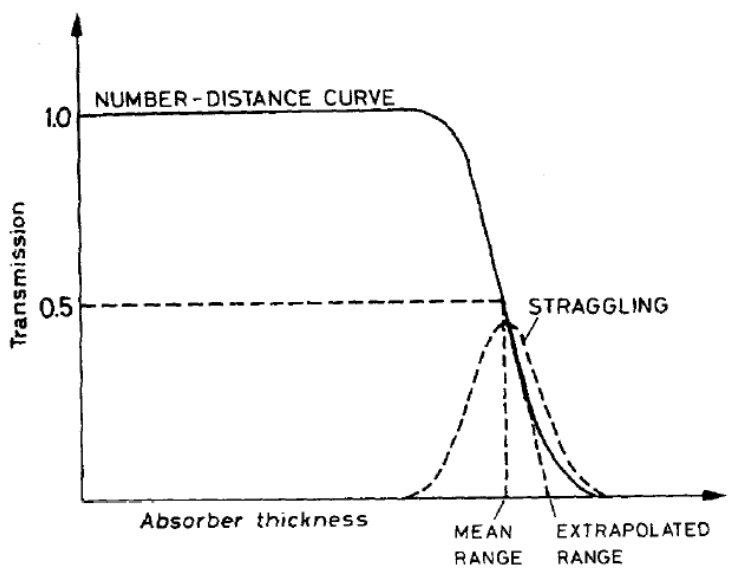
Range and energy loss

Variation in range and energy loss caused by statistics of absorbing collisions.

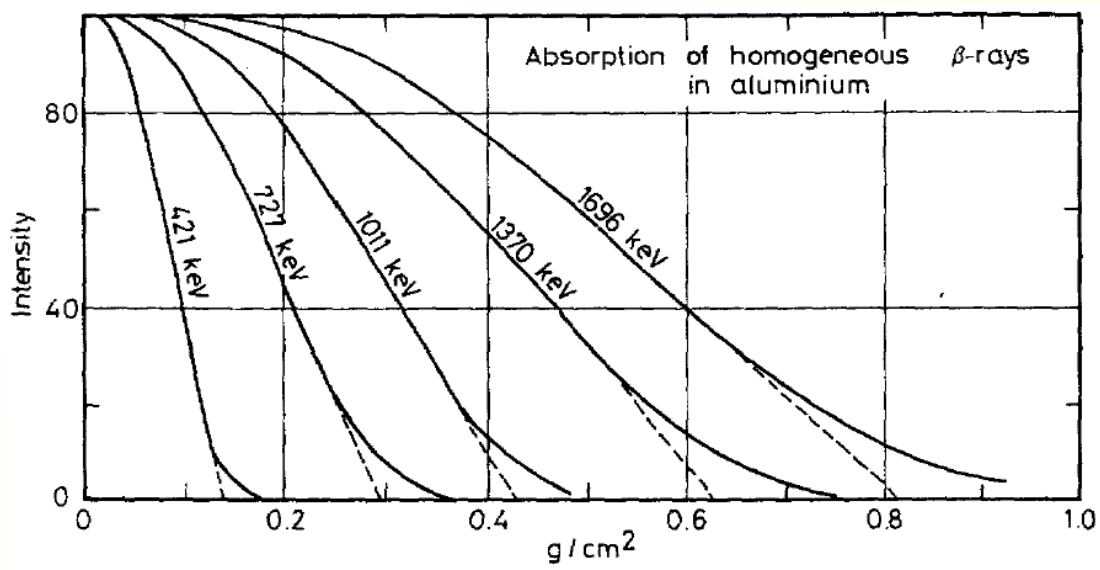
Energy loss distribution
- average energy loss –(dE/dx)
- energy loss straggling
- range straggling

Electron range fluctuates considerably due to multiple scattering
Large energy transfer via collisions is possible.
Range distribution is smeared out .

Heavy charged particle



electrons



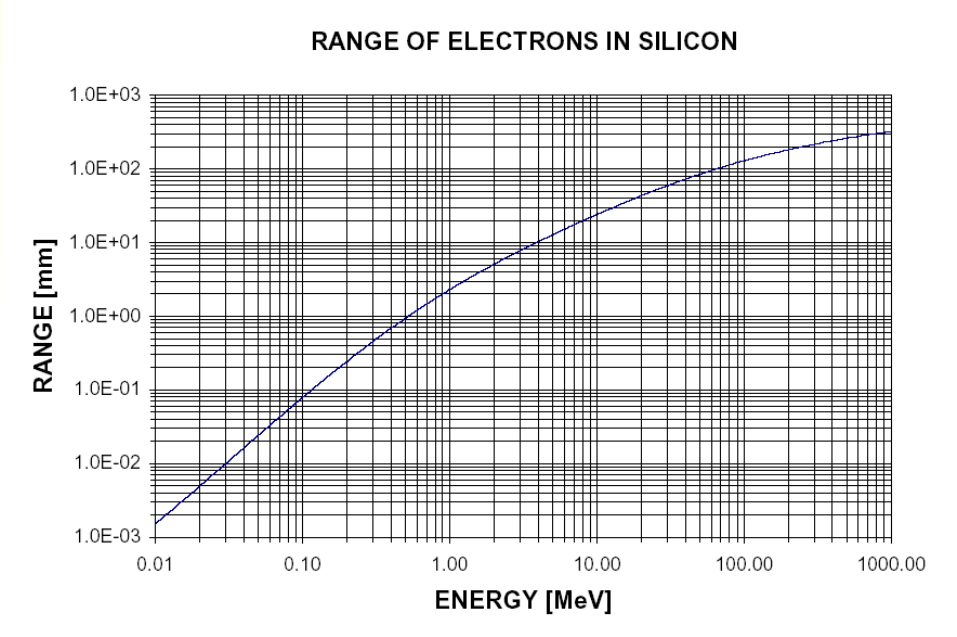
Transmission of electrons in aluminum [1 g/cm^2 = 3.7 mm]

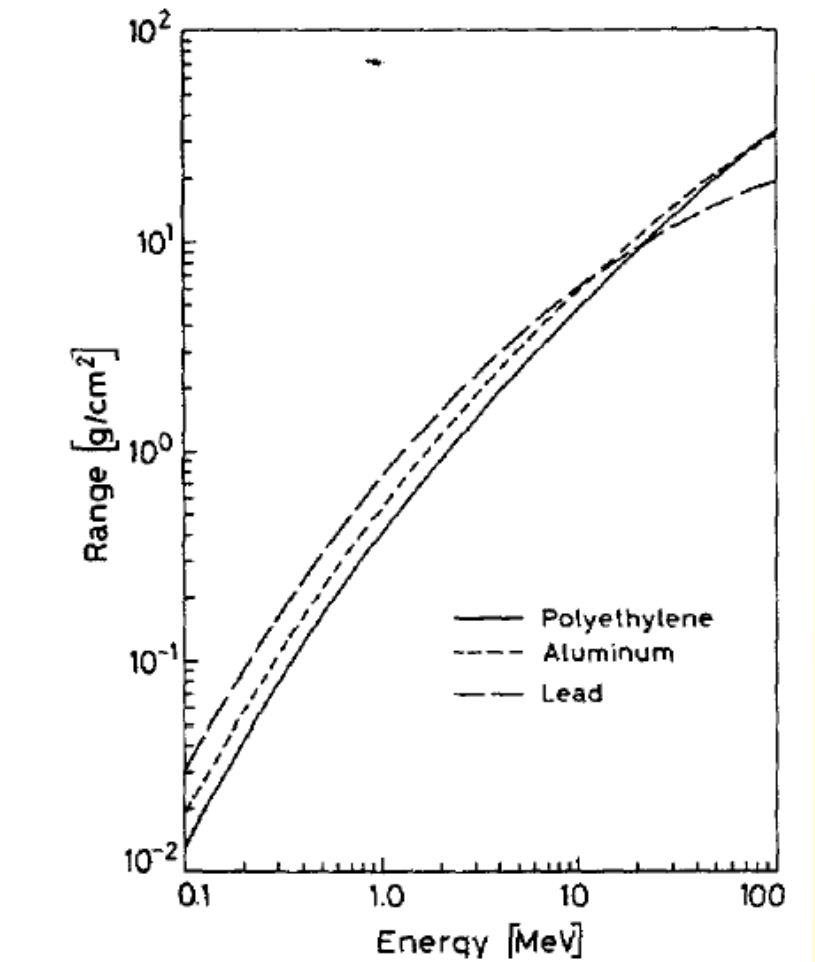
Electron range: low energies

Electron range

Polyethylän [1 g/cm^2 = 9.0 mm]
Aluminium [1 g/cm^2 = 3.7 mm]
Blei [1 g/cm^2 = 0.88 mm]

Range variation below critical energy is very energy dependent.





Absorption of β-electron

β – decay of ^{185}W

$^{185}W \rightarrow {}^{185}Re + \beta^- + \nu$

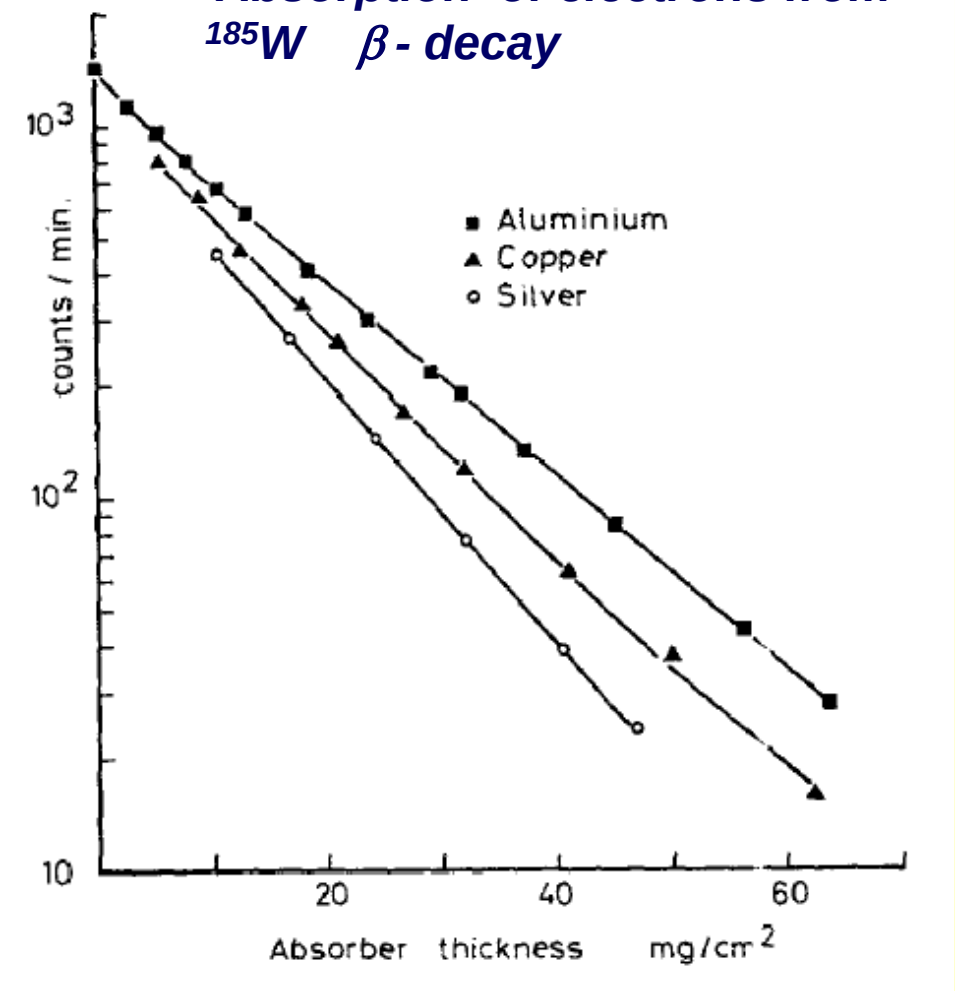
Q = 433 KeV, $T_{1/2}$ = 75.1 d

Neutrinos cause continuos electron spectrum

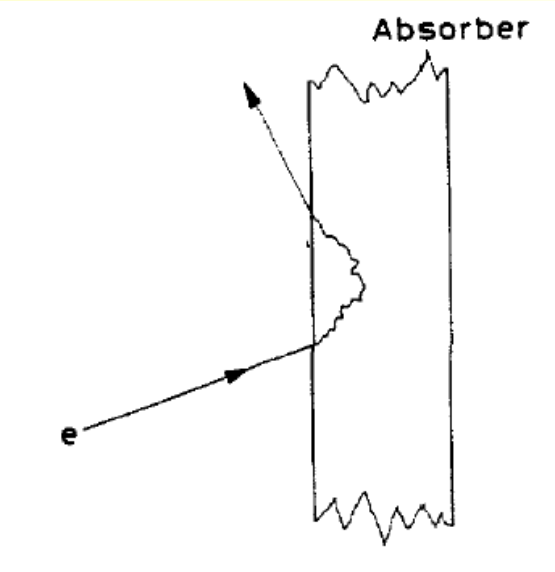
Folding of electron energy spectrum and range in matter shows exponentiel attenuation
$I = I_0 \exp(-\mu x)$
β – attenuation coeffizient μ

Exponentiel attenuation is not valid in general, only for simple β–decay schemse.

Absorption of electrons from ^{185}W β - decay

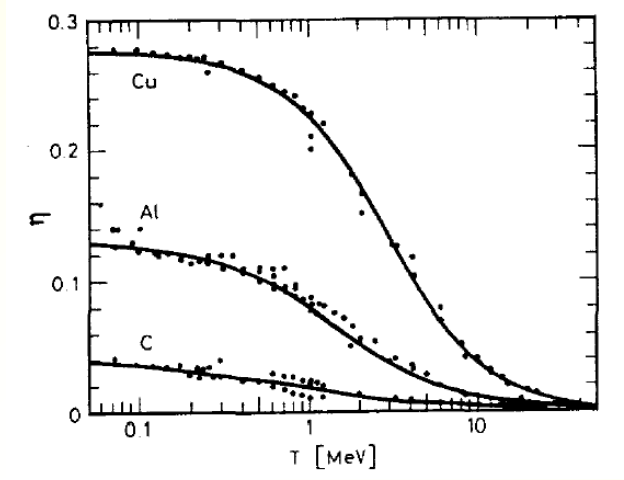


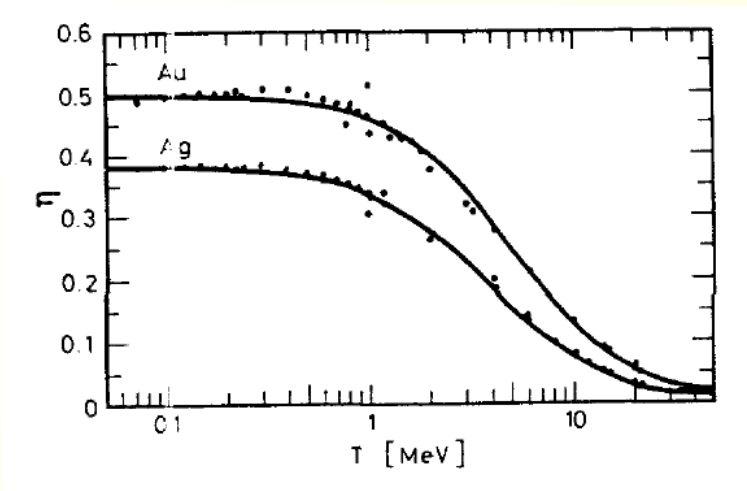
Electron back scattering



Scattering cause large deflection, even Backward angles and small energy losses. Depending in incident angle!
Electrons leave matter in most cases even below the detection threshold.
Reduced detection efficiency!

Strongly dependent on:
-Electron energy,
- At low energies strongest impact
- charge number Z of absorbing matter





η is backscattering coefficient, given for perpendicular incidence