Interaction of heavy charged particles in matter

Quantum mechanical results from Bethe-Bloch

\[- \frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[ \ln \left( \frac{2m_e \gamma^2 v^2 W_{\text{max}}}{I^2} \right) - 2\beta^2 - \delta - 2\frac{C}{Z} \right]\]

\[N_a : \text{Avogadro constant} \quad 6.022 \times 10^{23} \ \text{mol}^{-1}\]
\[\rho : \text{density of absorbing matter}\]
\[r_e : \text{class. electron radius} \quad 2.81 \times 10^{-13} \ \text{cm}\]
\[z : \text{charge of incoming particle}\]
\[m_e : \text{electron mass}\]
\[\beta, \gamma : \beta = v/c \quad \gamma = 1/\sqrt{1 - \beta^2}\]
\[Z : \text{charge number of abs. matter}\]
\[A : \text{atomic mass of abs. Materials}\]
\[l : \text{averaged ionisation potential}\]
\[W_{\text{max}} : \text{max. energy transfer in single collision}\]

two correction terms
\[\delta : \text{density correction (at relativistic energies)}\]
\[C : \text{shell correction (low energies)}\]
Interaction of heavy charged particles in matter
Interaction of heavy charged particles in matter

Electronic stopping: slowing down by inelastic collisions between electrons in matter and moving ion at $E_{\text{ion}} > 100$ keV.

Nuclear stopping: elastic collisions between ion and the full atoms. $E_{\text{ion}} < 100$ keV

e.g.: Si-ion, $E_{\text{ion}} = 1$ MeV, range in Si $r = 1-2 \mu m$
Interaction of heavy charged particles in matter

Energy loss, stopping power and range of charged particles

- Biggest energy deposition at end of track: Bragg peak

\[-\frac{dE}{d\varepsilon} = -\frac{1}{\rho} \frac{dE}{dx} = z^2 \frac{Z}{A} f(\beta, I)\]

- \(dE/d\varepsilon\) is nearly independent of matter for equal particles.
Interaction of heavy charged particles in matter

**Energy loss, stopping power and range of charged particles**

- Average range for particle with kin. energy $T$:

$$S(T) = \int_0^T \left( \frac{dE}{dx} \right)^{-1} dE$$

- Range is not a precise quantity but smeared out, called range straggling.

- Number of interactions is statistically distributed.

- At low energies empirical constants are included in $S(T)$. 
Interaction of heavy charged particles in matter

**Energy loss, stopping power and range of charged particles**

- average range for particle with kin. energy $T$:

\[
S(T) = \int_0^T \left( \frac{dE}{dx} \right)^{-1} dE
\]

- Range is not a precise quantity but smeared out, called *range straggling*. Number of interactions is statistically distributed. At low energies empirical constant are included in $S(T)$.

- Remark: In matter with spatial symmetry (e.g. crystals) Bethe-Bloch formula is not valid! Correlated scattering under certain geometrical conditions (critical angle with respect of crystal axis) reduces energy loss and enlarge range of particle. **Channeling effect**
Interaction of heavy charged particles in matter

- Heavy particles do not lose only energy in matter

- ... there is a chance for small changes in direction

- Small angle scattering
Interaction of heavy charged particles in matter

- Average scattering angle is nearly Gauss distributed for small deviations

\[ \theta_0 = \frac{13.6 \text{ MeV}}{\beta cp} z \sqrt{\frac{x}{X_0}} \left[ 1 + 0.038 \ln \left( \frac{x}{X_0} \right) \right] \]

\[ X_0 \equiv \text{radiation length} \]

- Angular distribution:

\[ \frac{dN}{d\Omega} \propto \frac{1}{2\pi \theta_0^2} \exp \left( - \frac{\theta_{\text{space}}^2}{2\theta_0^2} \right) \]

\[ \frac{dN}{d\theta_{\text{plane}}} \propto \frac{1}{\sqrt{2\pi \theta_0}} \exp \left( - \frac{\theta_{\text{plane}}^2}{2\theta_0^2} \right) \]
Examples, applications for **stopping power and range**:

Energy loss of projectile and reaction products in target:
- e.g. $^{48}\text{Ca} + ^{208}\text{Pb} @ E_{\text{beam}}=200\text{MeV}$
- target thickness $d=0.5 \text{mg/cm}^2$ ($s=44 \text{\mu m}$)
- energy loss of $^{48}\text{Ca}$ in target $7.2 \text{MeV}$

-> equals width of excitation function $^{208}\text{Pb}(^{48}\text{Ca},2n)$ - reaction

remark: thickness are given in $d=\text{mass}/\text{area}$
Units (e.g.): $\text{mg/cm}^2$ $d = m/A = \rho \, s$
$\rho$ density, $s$ thickness: $s=d/\rho$

- beam and reaction products are slowed down.
- cross section and kinematics are changed.
- targets limit count rate and energy resolution.
Interaction of charged particles in matter

Example and application for stopping power and range:
Particle identification with $\Delta E - E$ telescope
Range of heavy ions in matter

Range von $^{12}\text{C}$ ions in water  unit: MeV/n  e.g. 90 MeV, n=12
Elab=1.08 GeV

![Graph showing the range of heavy ions in water with a Bragg peak at 12C.](image)

- 90 MeV/n
- 195 MeV/n
- 270 MeV/n
- 330 MeV/n

Bragg peak
Tumor therapy with heavy ions

Heavy ion beams for radiation therapy of cancer tumors

Requirements:
• profile of radiation dose – Bragg curve –
• increased relative biological efficiency of heavy ions

Tumors:
• brain tumors (chordoma, chondrosarcoma, mal. schwannoma, atyp. Meningeoma, adenoidcystic ca.)
• Tumors close to spine: (sacral chordoma, chondrosarcoma, soft tissue sarcoma)
Tumor therapy with ionising particle beams:

- $\gamma$-rays and high energetic photons
  strong absorption at surface

- electrons
  strong absorption at surface

- neutrons
  strongest absorption at surface
  local: $n + ^{10}\text{B} \rightarrow ^{7}\text{Li} + \alpha$

- protons + heavy ions
  highest biological efficacy
  at Bragg peak position

Compare with other treatments
Biological Efficacy

Equivalent dose  \[ H = \frac{1}{m} \int \frac{dE}{dx} \cdot RBE \]  

Relativ. Biolog. Effectiveness

increased biological efficacy of ions due to DNA double helix

Increased transversal range of ionising radiation:

- proton: \( \sim 250 \text{ pm} \)
- carbon nucleus: \( \sim 1 \text{ nm} \)

\( \delta \text{-Elektronen} \)

lengths:
- cell: \( \sim 1 \text{ mm} \)
- cell nucleus: \( \sim 10 \mu\text{m} \)
- chromosome: \( \sim 1.5 \mu\text{m} \)
- DNA: \( \sim 2 \text{ nm} \)
- gene: \( \leq 1 \text{ nm} \)
highest accelerator demands:

variation of beam energy: 80 – 450 MeVA, 2 – 30 cm H$_2$O range, 20 – 40 steps

Pencil beam: diameter d~1mm

Beam monitoring :- ~ 1 mm, in front of patient e.g. ionisation chamber

PET in situ (Tumor)

Raster scan of 3d tumor volume in x-, y-direction by magnetic bending (3d ´voxels´).

extrem high reliability of accelerator!
Treatment relies on exact position and size of tumor volume. Radiation plan needs to know different tissues around tumor to determine the biological effectiveness.

Monitoring via Positron-Emission-Tomography (PET)
- Fragmentation of $^{12}\text{C}$ produces $^{10,11}\text{C}$
- $\beta^+$-decays:
  - $^{10}\text{C}\rightarrow^{10}\text{B} + \beta^- + \nu_e$ $T_{1/2}=19.3\text{ s}$
  - $^{11}\text{C}\rightarrow^{11}\text{B} + \beta^+ + \nu_e$ $T_{1/2}=20.38\text{ min}$
- Annihilation $e^+ + e^- \rightarrow \gamma + \gamma$
- correlated emission back-to-back of two 511 kev $\gamma$-rays
- pos. resolution $\sim 2 - 3\text{ mm}$
Tumor therapy with heavy ions

Distribution of Verteilung radiation dose is overlaid with CT image of patient (red highest dose). Dose is optimized to achieve homogeneous biological dose profile in tumor volume.

Distribution of positron emitters detected with PET camera. PET is sensitive to stopped $^{11}$C and $^{10}$C reaction products.
Tumor therapy with heavy ions

Radiation therapy:

GSI, Darmstadt

Heidelberger Ionenstrahl-Therapiezentrum (HIT)
Interaction of charged particles in matter today electrons

Introductory remarks
• Electrons ‘feel’ electro-magnetic and weak interaction
• Electrons couple to photons and W- and Z-Boson of weak interaction
• Electrons ‘talk’ easily to photons, remember pair creation and bremsstrahlung

\[ \sigma \propto r_e^2 = \left( \frac{e^2}{mc^2} \right)^2 \]

• Electron and photon detection are entangled, e.g. electro-magnetic shower detection at high energies

Relevance of electrons in nuclear and particle physics some examples:
- beta decays
- conversion electrons (complementary to \( \gamma \)-spectroscopy)
- electron beams for electron-nucleon-scattering (structure functions)
- electron-positron collisions at high energies provide clean source for particle production and new discoveries

- electron is most abundant reaction product, often source of unwanted and tremendous back ground
Interaction of electrons and positrons

Energy loss of electrons and positrons via collisions with electrons on atoms and electro-magnetic radiation (Bremsstrahlung)

\[
\left( \frac{dE}{dx} \right)_{\text{tot}} = \left( \frac{dE}{dx} \right)_{\text{rad}} + \left( \frac{dE}{dx} \right)_{\text{coll}}
\]

Modified calculation of energy loss
- large scattering angle of incoming particle
- scattering of identical particles -> Q.M. interference

Collisional Energy loss:

\[
-\left( \frac{dE}{dx} \right)_{\text{coll}} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{1}{\beta^2} \left[ \ln \left( \frac{\tau^2 (\tau + 2)}{2(I / m_e c^2)^2} \right) + F(\tau) - \delta - 2 \frac{C}{Z} \right]
\]

\( \tau \): kinetic energy in units of \( m_e c^2 \)

\( F(\tau) \):
- \( F(\tau) = 1 - \beta^2 + ... \) electrons
- \( F(\tau) = 2 \ln 2 - \frac{\beta^2}{12} (23 + ...) \) positrons
Comparison of interactions

Bremsstrahlung loss and ionisation loss for electrons and protons in copper

- Electron 2-4 MeV
- Proton > 2-4 GeV
- Critical energy 24 MeV
Variation in range and energy loss caused by statistics of absorbing collisions.

Energy loss distribution
- average energy loss \(-\frac{dE}{dx}\)
- energy loss straggling
- range straggling

Electron range fluctuates considerably due to multiple scattering
Large energy transfer via collisions is possible.
Range distribution is smeared out.

Heavy charged particle

Transmission of electrons in aluminum \(1 \text{ g/cm}^2 = 3.7 \text{ mm}\)
Electron range: low energies

Electron range

Polyethylän \[1 \text{ g/cm}^2 = 9.0 \text{ mm}\]
Aluminium \[1 \text{ g/cm}^2 = 3.7 \text{ mm}\]
Blei \[1 \text{ g/cm}^2 = 0.88 \text{ mm}\]

Range variation below critical energy is very energy dependent.
Absorption of $\beta$-electron

$\beta$ – decay of $^{185}$W

$^{185}$W $\rightarrow$ $^{185}$Re + $\beta^-$ + $\nu$

$Q = 433$ KeV, $T_{1/2} = 75.1$ d

Neutrinos cause continuous electron spectrum

Folding of electron energy spectrum and range in matter shows exponential attenuation

$I = I_0 \exp(-\mu x)$

$\beta$ – attenuation coefficient $\mu$

Exponential attenuation is not valid in general, only for simple $\beta$–decay schemes.
Scattering cause large deflection, even Backward angles and small energy losses. Depending in incident angle! Electrons leave matter in most cases even below the detection threshold. Reduced detection efficiency!

Strongly dependent on:
- Electron energy,
- At low energies strongest impact
- charge number $Z$ of absorbing matter

$\eta$ is backscattering coefficient, given for perpendicular incidence