Interaction of charged particles in matter today electrons

Introductory remarks

- *Electrons ‚feel' electro-magnetic and weak interaction*
- *Electrons couple to photons and W- and Z-Boson of weak interaction*
- *Electrons ‚talk' easily to photons, remember pair creation and Bremsstrahlung*

$$\sigma \propto r_e^{\,2} = (e^2 / mc^2)^2$$

- *Electron and photon detection are entangled, e.g. electro-magnetic shower detection at high energies*

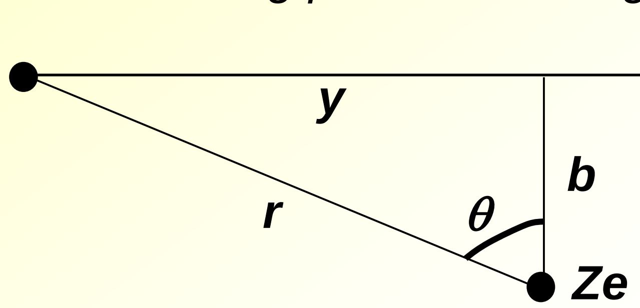
Relevance of electrons in nuclear and particle physics some examples:

- beta decays
- conversion electrons (complementary to γ-spectroscopy)
- electron beams for electron-nucleon-scattering (structure functions)
- electron-positron collisions at high energies provide clean source for particle production and new discoveries

- electron is most abundant reaction product, often source of unwanted and tremendous back ground

Interaction of heavy charged particles in matter

Consider moving particle of charge ze passing by with v the stationary charge Ze



Assumptions:

- *momentum approximation: short inter-action, only transversal moment. transfer*
- *target remains non-relativistic*

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{2m_e\gamma^2 v^2 W_{max}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$$

Bethe-Bloch

N_a : Avogadro constant 6.022 x 10^{23} mol^{-1}

r_e : class. electron radius 2.81 x 10^{-13} cm

m_e : electron mass

Z : charge number of abs. matter

A : atomic mass of abs. matter

two correction terms

ρ : density of absorbing matter

z : charge of incoming particle

β, γ : $\beta = v/c$ $\gamma = 1/\sqrt{1-\beta^2}$

I : averaged ionisation potential

W_{max} : max. energy transfer in single collision

δ : density correction (at relativistic energies)

C : shell correction (low energies)

Interaction of electrons and positrons

Energy loss of electrons and positrons via collisions with electrons on atoms and electro-magnetic radiation (Bremsstrahlung)

$$\left(\frac{dE}{dx}\right)_{tot} = \left(\frac{dE}{dx}\right)_{rad} + \left(\frac{dE}{dx}\right)_{coll}$$

Modified calculation of energy loss

- large scattering angle of incoming particle

- scattering of identical particles -> Q.M. interference

Collisional energy loss:

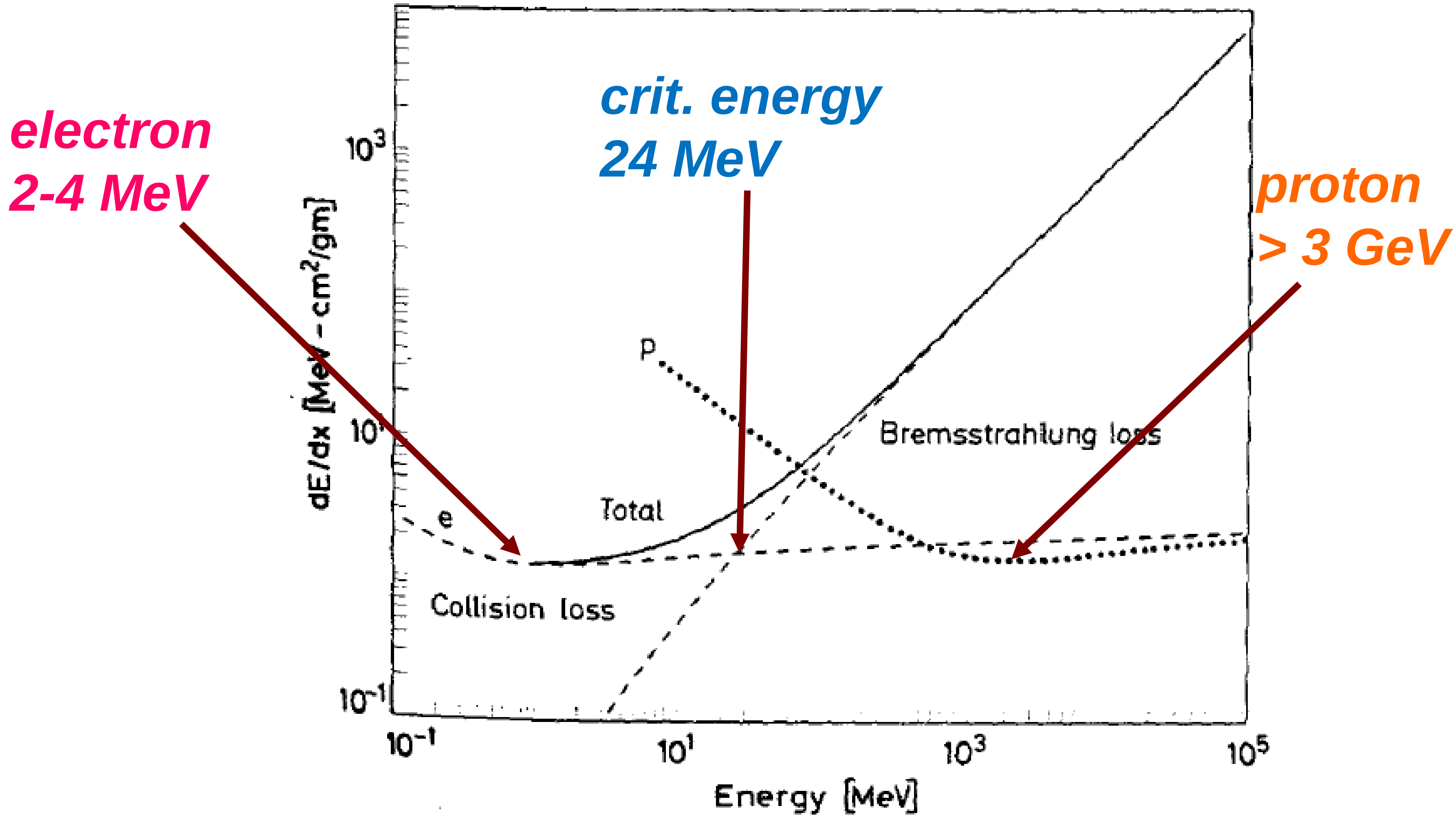
$$-\left(\frac{dE}{dx}\right)_{coll} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{1}{\beta^2} \left[\ln\left(\frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2} \right) + F(\tau) - \delta - 2\frac{C}{Z} \right]$$

τ : kinetic energy in units of $m_e c^2$

$F(\tau)$: $F(\tau) = 1 - \beta^2 + \ldots$ electrons $\qquad$ $F(\tau) = 2\ln 2 - \frac{\beta^2}{12}(23 + \ldots)$ positrons

Comparison of interactions

Bremsstrahlung loss and ionisation loss for electrons and protons in copper



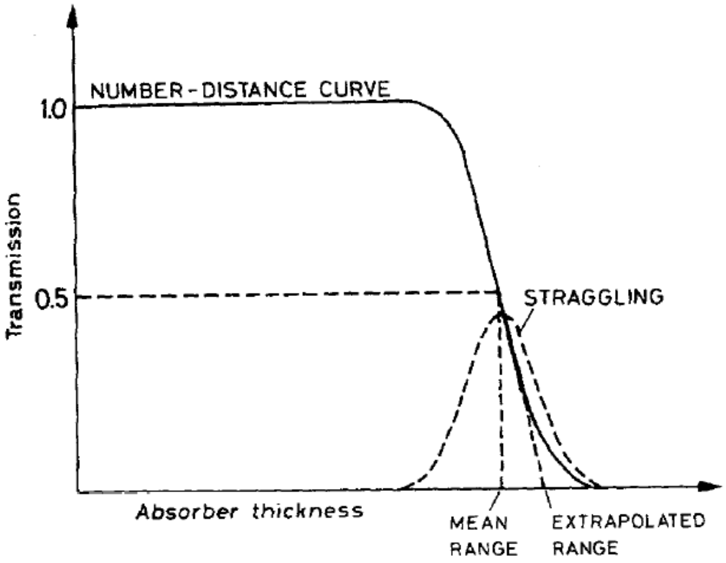
Range and energy loss

Variation in range and energy loss caused by statistics of absorbing collisions.

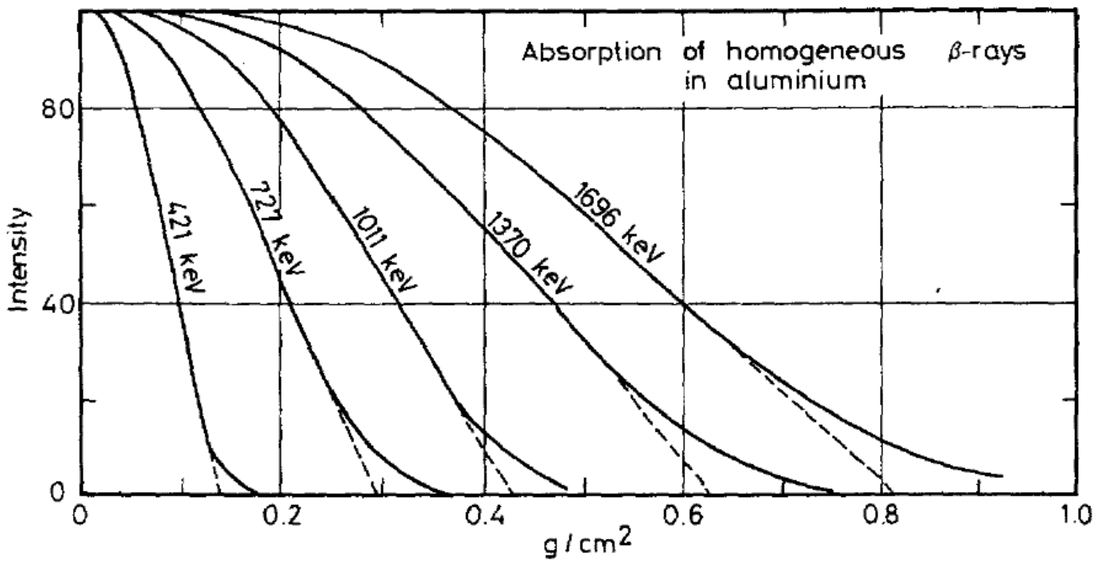
Energy loss distribution
- average energy loss –(dE/dx)
- energy loss straggling
- range straggling

Electron range fluctuates considerably due to multiple scattering.
Large energy transfer via collisions is possible.
Range distribution is smeared out .

heavy charged particle



electrons



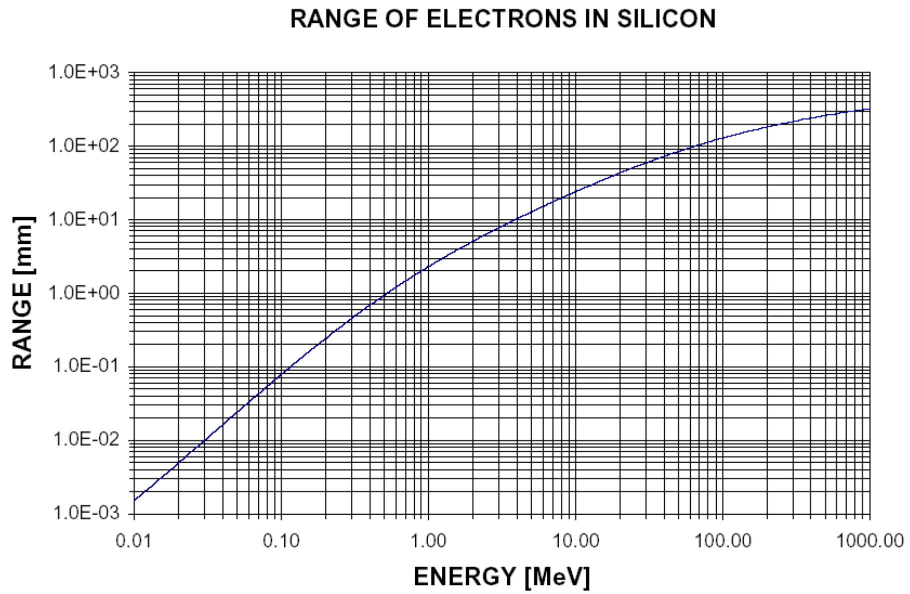
Transmission of electrons in aluminium [1 g/cm^2 = 3.7 mm]

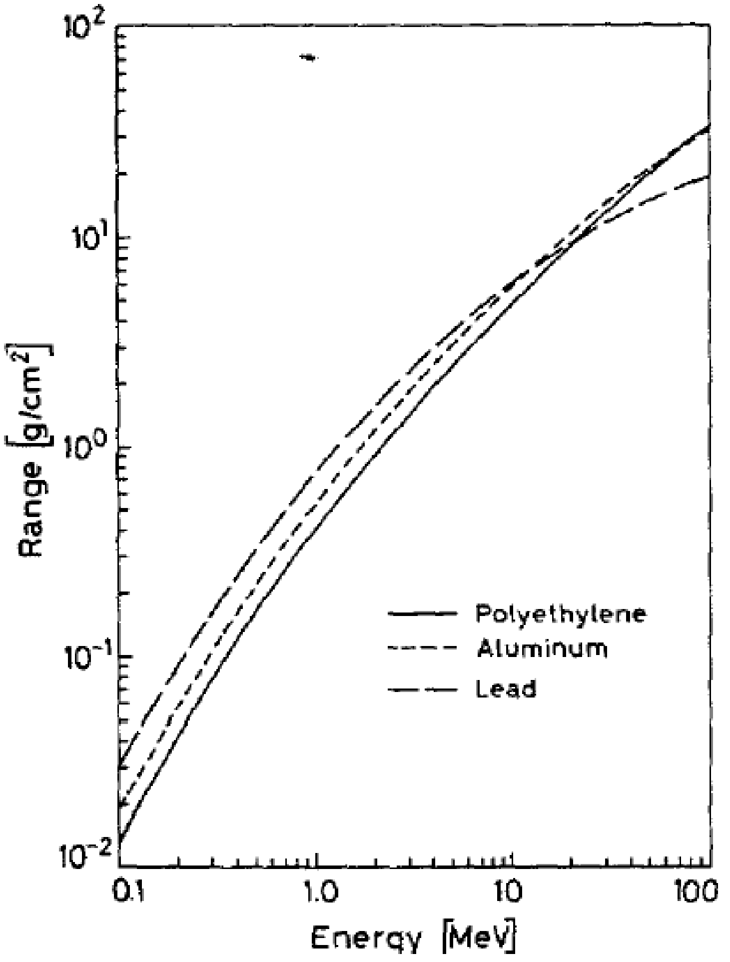
Electron range: low energies

Electron range

Polyethylene [1 g/cm² = 9.0 mm]
Aluminium [1 g/cm² = 3.7 mm]
Lead [1 g/cm² = 0.88 mm]

Range variation below critical energy is very energy dependent.





Absorption of β-electron

β – decay of ^{185}W

$^{185}W \to {}^{185}Re + \beta^- + \nu$

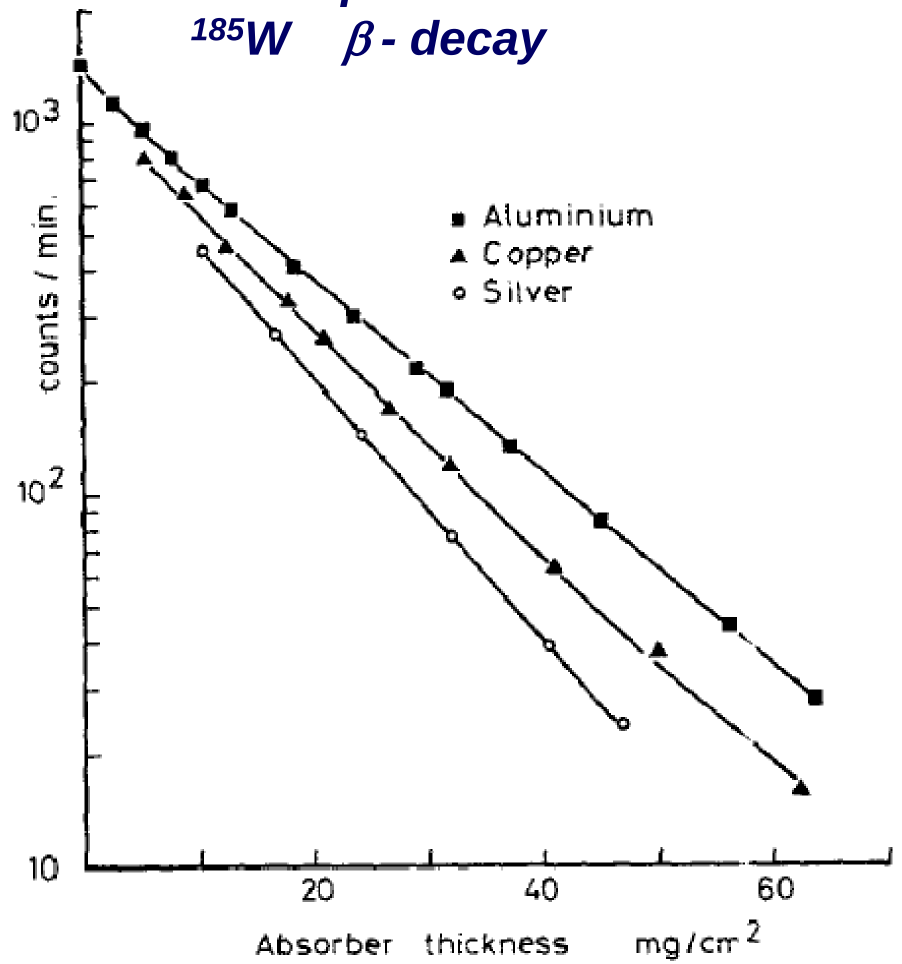
Q = 433 keV, $T_{1/2}$ = 75.1 d

Neutrinos cause continuous electron spectrum

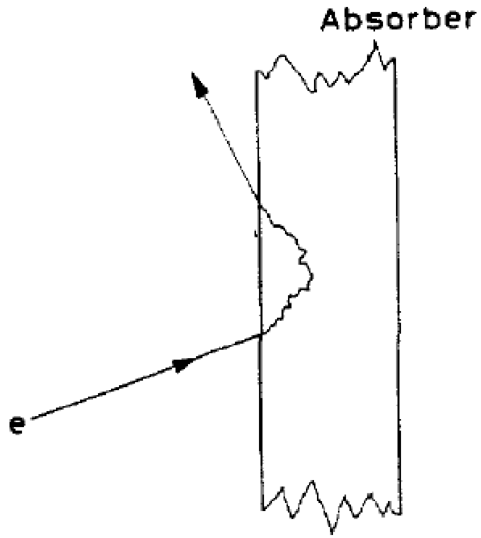
Folding of electron energy spectrum and range in matter shows exponential attenuation
$I = I_0 \exp(-\mu x)$
β – attenuation coefficient μ

Exponential attenuation is not valid in general, only for simple β–decay scheme.

Absorption of electrons from ^{185}W β - decay

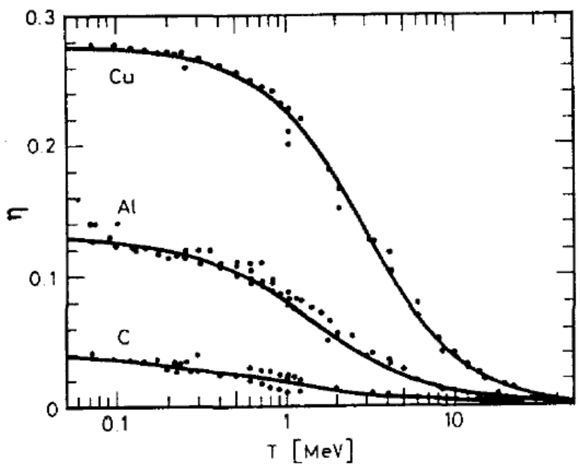


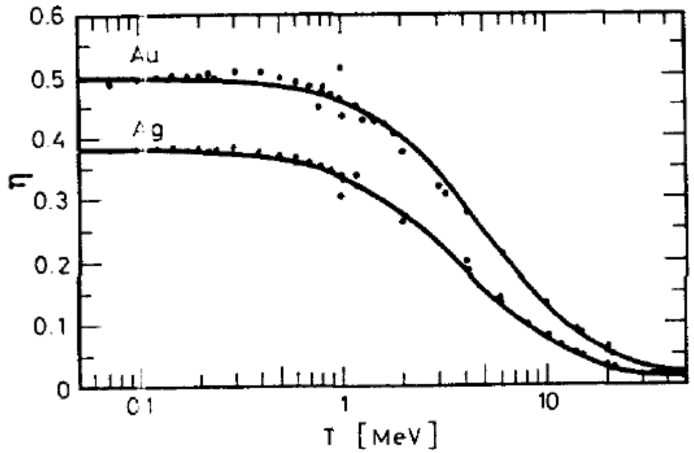
Electron back scattering



Scattering cause large deflection, even Backward angles and small energy losses.
Depending on incident angle!
Electrons leave matter in most cases even below the detection threshold.
Reduced detection efficiency!

Strongly dependent on:
-Electron energy
- *at low energies highest impact*
- *charge number Z of absorbing matter*

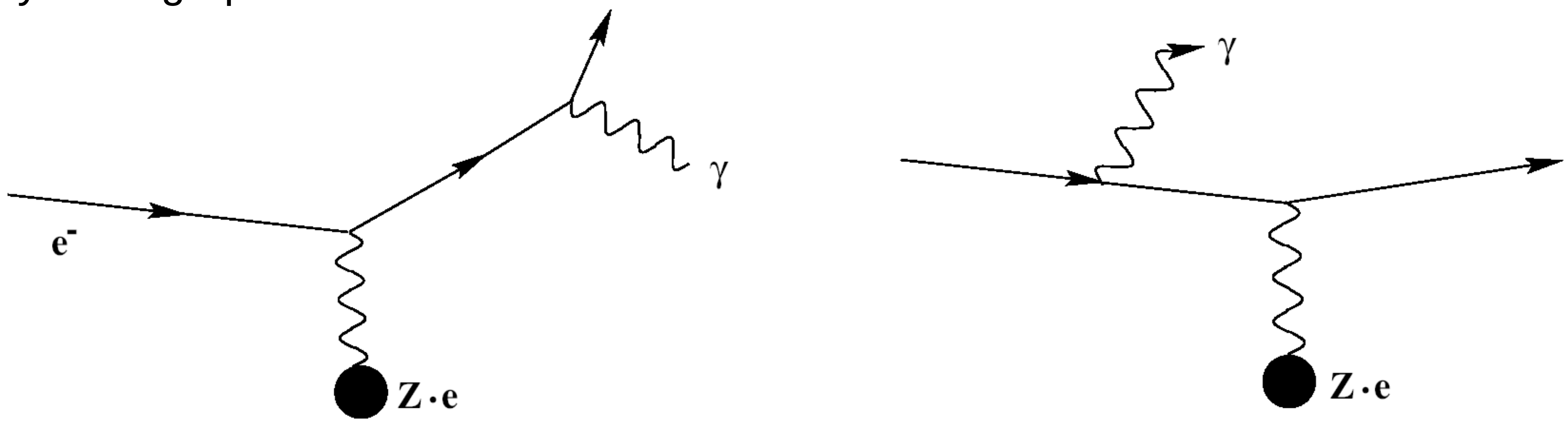




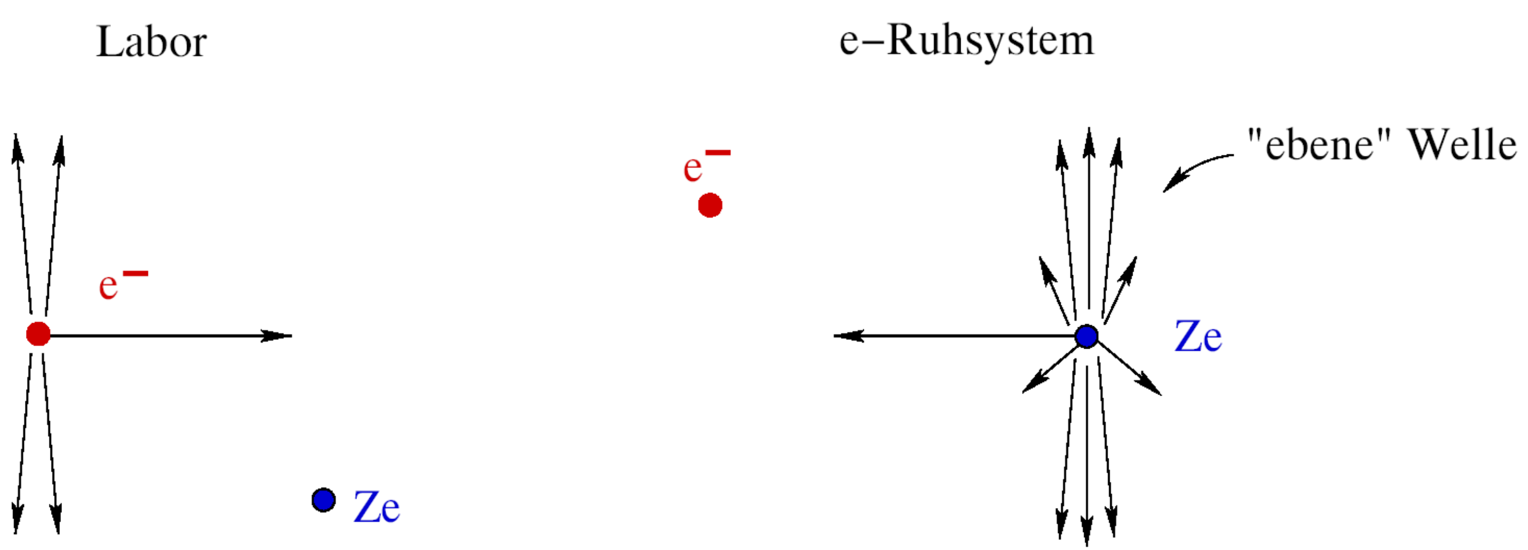
η is backscattering coefficient, given for perpendicular incidence

Bremsstrahlung

- Feynman graphs



- Weizsäcker-Williams approximation: Bremsstrahlung



Bremsstrahlung

Electrons and positrons are particles with considerable energy loss via Bremsstrahlung for electron energies higher than several tens of MeV.

$\sigma \propto r_e^{\,2} = (e^2/mc^2)^2$ *μ mass: m_μ=106 MeV electron mass: m_e=511 keV*

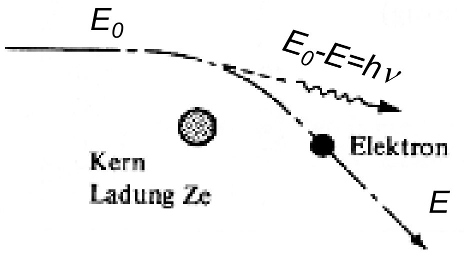
$$d\sigma = 4Z^2 r_e^2 \alpha \frac{d\nu}{\nu}\left[(1+\varepsilon^2)\left(\frac{\phi_1(\xi)}{4} - \frac{1}{3}\ln Z - f(Z)\right) - \frac{2}{3}\varepsilon\left(\frac{\phi_2(\xi)}{4} - \frac{1}{3}\ln Z - f(Z)\right)\right]$$

$$\alpha = \frac{1}{137}$$

Differential Bremsstrahlung cross section

$\varepsilon = \mathrm{E}/\mathrm{E}_0$ E : electron energy after radiation

E_0 : incoming initial electron energy



screening function : $\phi_1(\xi), \phi_2(\xi)$ screening Parameter : $\xi = \dfrac{100 m_e c^2 h\nu}{E_0 E Z^{1/3}}$

$f(Z)$: Coulomb correction, interaction of electrons in field of nuclei

Bremsstrahlung

$$d\sigma = 4Z^2 r_e^2 \alpha \frac{d\nu}{\nu}\left[(1+\varepsilon^2)\left(\frac{\phi_1(\xi)}{4} - \frac{1}{3}\ln Z - f(Z)\right) - \frac{2}{3}\varepsilon\left(\frac{\phi_2(\xi)}{4} - \frac{1}{3}\ln Z - f(Z)\right)\right]$$

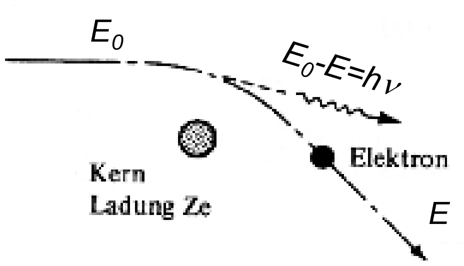
Differential Bremsstrahlung cross section

$\varepsilon = \mathrm{E}/\mathrm{E}_0$

E_0 : incoming initial electron energy

E : electron energy after radiation

$h\nu$: photon energy



Important screening Parameter: $\xi = \dfrac{100 m_e c^2 h\nu}{E_0 E Z^{1/3}} = \dfrac{100 m_e c^2 h\nu}{E_0 Z^{1/3} E}$

Screening of nuclear charge depends on
(i) initial energy $\zeta \sim$ *12-25 for 1 MeV electrons, no screening*
$\zeta \sim$ *0,1-0,3 for 100 MeV electrons, complete screening*
(ii) energy of Bremsstrahlung photon vs. remaining energy varies from small numbers for soft photons to highest numbers for head on collisions causing high energy radiation

Electron interaction: Bremsstrahlung

Two solutions for limiting extrem cases:

Low energies

For $\xi >> 1$ no screening:

$$d\sigma = 4Z^2 r_e^2 \alpha \frac{d\nu}{\nu}\left[(1+\varepsilon^2 - \frac{2\varepsilon}{3})\left(\ln\frac{2E_0 E}{m_e c^2 h\nu} - \frac{1}{2} - f(Z)\right)\right]$$

high energies

For $\xi \cong 0$ complete screening:

$$d\sigma = 4Z^2 r_e^2 \alpha \frac{d\nu}{\nu}\left[(1+\varepsilon^2 - \frac{2\varepsilon}{3})\left(\ln(183 Z^{-1/3}) - f(Z)\right) + \frac{\varepsilon}{9}\right]$$

Radiativ energy loss from integral (xsection × photon energy) × N

$$-\left(\frac{dE}{dx}\right)_{rad} = N\int_0^{\nu_0} h\nu \frac{d\sigma}{d\nu}(E_{0,}\nu)\,d\nu = N E_0 \phi_{rad}$$

$$\phi_{rad} = \frac{1}{E_0}\int_0^{\nu_0} h\nu \frac{d\sigma}{d\nu}(E_{0,}\nu)\,d\nu$$

$$\mathrm{N} = \frac{\rho \mathrm{N_a}}{\mathrm{A}} \text{ and } \nu_0 = \frac{E_0}{h}$$

N: number of atoms/cm³

Bremsstrahlung

For $m_e c^2 \ll \xi \ll 137 m_e c^2 Z^{-1/3}, \quad \xi \gg 1$ no screening :

$$\phi_{rad} = 4Z^2 r_e^2 \alpha \left(\ln \frac{2E_0}{m_e c^2} - \frac{1}{3} - f(Z) \right)$$

For $E_0 \gg 137 m_e c^2 Z^{-1/3}, \quad \xi \cong 0$ complete screening :

$$\phi_{rad} = 4Z^2 r_e^2 \alpha \left(\ln(183 Z^{-1/3}) + \frac{1}{18} - f(Z) \right)$$

High energy limit is independent of electron energy E_0

$$\phi_{rad} = 4Z^2 r_e^2 \alpha \left(\ln(183 Z^{-1/3}) + \frac{1}{18} - f(Z) \right)$$

Collisions vs. Bremsstrahlung

$$-\left(\frac{dE}{dx}\right)_{coll} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{1}{\beta^2}\left[\ln\left(\frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2}\right) + F(\tau) - \delta - 2\frac{C}{Z}\right]$$

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Compare collision or ionisation vs Bremsstrahlung:

- *ionisation increases logarithmic in E and linear in Z*
- *bremsstrahlung increases linear in E and quadratic in Z*
- *ionisation: quasi-continuous energy loss*
- *bremsstrahlung: emission of one or two photons can give away full electron energy -> huge fluctuations*

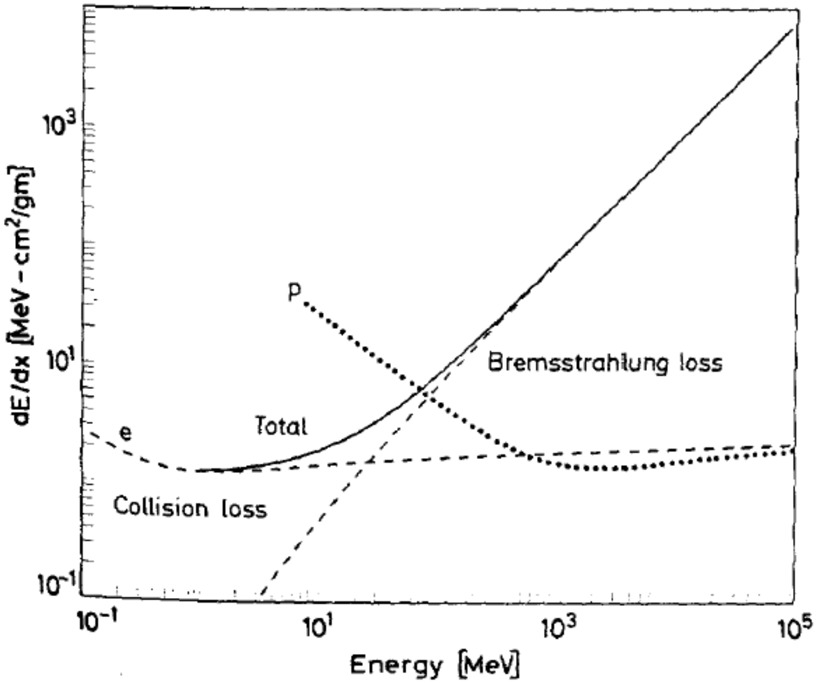
Radiation loss

Critical energy E_c:

$$\left(\frac{dE}{dx}\right)_{rad} = \left(\frac{dE}{dx}\right)_{coll} \text{ for } E = E_c \qquad E_c \cong \frac{800MeV}{Z+1.2}$$

Typical values:

material	*critical energy E_c*
Pb	*9.51 MeV*
Al	*51.0 MeV*
Fe	*27.4 MeV*
Cu	*24.8 MeV*
Luft	*102 MeV*
NaI	*17.4 MeV*
H2O	*92 MeV*



Radiation length

radiation length
Distance in matter related to a radiative energy loss by 1/e

$$-dE/E = N\phi_{rad}dx$$

high energtic limit ($\xi \cong 0$):

(i) coll. loss ignored

(ii) ϕ_{rad} is independent from energy

Solution of DEQ

$$E = E_0 \exp(-\frac{x}{L_{rad}})$$

x is distance in matter and $L_{rad} = 1/N\phi_{rad}$ is radiation length

$$\frac{1}{L_{rad}} \cong \left[4Z(Z+1)\frac{\rho N_a}{A}\right] r_e^2 \alpha \left[\ln\left(183Z^{-1/3}\right) - f(Z)\right]$$

Radiation length

Radiation length

Typical values:

Material	cm
Pb	0.56
Al	8.90
Fe	1.76
Cu	1.43
NaI	2.59
BGO	1.12
Org.Scint.	42.40
H_2O	36.10
Air	30050.00

Summary: interactions of electrons

$$\left(\frac{dE}{dx}\right)_{tot} = \left(\frac{dE}{dx}\right)_{rad} + \left(\frac{dE}{dx}\right)_{coll}$$

Energy loss via collisions with electrons:

$$-\left(\frac{dE}{dx}\right)_{coll} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{1}{\beta^2}\left[\ln\left(\frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2}\right) + F(\tau) - \delta - 2\frac{C}{Z}\right]$$

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summary

$$-\left(\frac{dE}{dx}\right)_{coll} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{1}{\beta^2}\left[\ln\left(\frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2}\right) + F(\tau) - \delta - 2\frac{C}{Z}\right]$$

Radiativ energy loss from integral (xsection $\times$ photon energy) $\times$ N

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radiation length
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