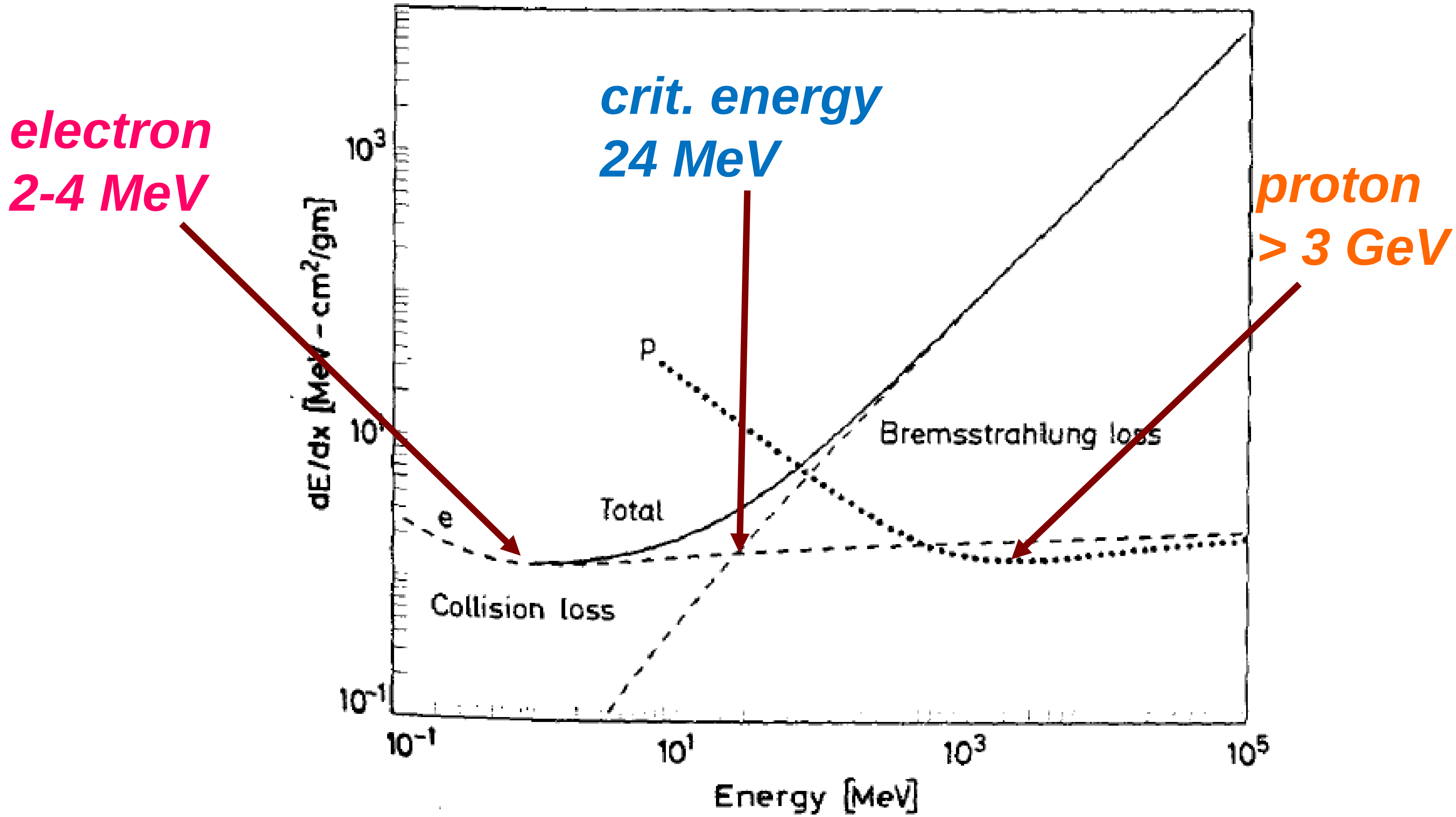
Last lecture: Comparison of interactions

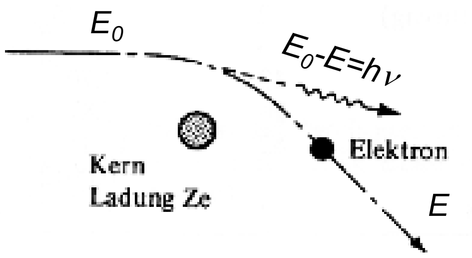
Bremsstrahlung loss and ionisation loss for electrons and protons in copper



Last lecture: Bremsstrahlung

$$d\sigma = 4Z^2 r_e^2 \alpha \frac{d\nu}{\nu}\left[(1+\varepsilon^2)\left(\frac{\phi_1(\xi)}{4} - \frac{1}{3}\ln Z - f(Z)\right) - \frac{2}{3}\varepsilon\left(\frac{\phi_2(\xi)}{4} - \frac{1}{3}\ln Z - f(Z)\right)\right]$$

Differential Bremsstrahlung cross section



$\varepsilon = E/E_0$

E_0 : incoming initial electron energy

E : electron energy after radiation

$h\nu$: photon energy

Important screening Parameter : $\xi = \dfrac{100 m_e c^2 h\nu}{E_0 E Z^{1/3}} = \dfrac{100 m_e c^2 h\nu}{E_0 Z^{1/3} E}$

Screening depends on

(i) initial energy $\zeta \sim$ *12-25* *for 1 MeV electrons, no screening*

$\zeta \sim$ *0,1-0,3* *for 100 MeV electrons, complete screening*

(ii) energy of Bremsstrahlung photon vs. remaining energy varies from small numbers for soft photons to highest numbers for head on collisions causing high energy radiation

Last lecture: Bremsstrahlung

Two solutions for limiting extrem cases:

Low energies

For $\xi >> 1$ no screening :

$$d\sigma = 4Z^2 r_e^2 \alpha \frac{d\nu}{\nu}\left[(1+\varepsilon^2 - \frac{2\varepsilon}{3})\left(\ln\frac{2E_0E}{m_e c^2 h\nu} - \frac{1}{2} - f(Z)\right)\right]$$

high energies

For $\xi \cong 0$ complete screening :

$$d\sigma = 4Z^2 r_e^2 \alpha \frac{d\nu}{\nu}\left[(1+\varepsilon^2 - \frac{2\varepsilon}{3})\left(\ln(183Z^{-1/3}) - f(Z)\right) + \frac{\varepsilon}{9}\right]$$

Radiativ energy loss from integral (xsection $\times$ photon energy) $\times$ N

$$-\left(\frac{dE}{dx}\right)_{rad} = N\int_0^{\nu_0} h\nu \frac{d\sigma}{d\nu}(E_{0,}\nu)d\nu = NE_0\phi_{rad}$$

$$\mathrm{N} = \frac{\rho \mathrm{N_a}}{\mathrm{A}} \text{ and } \nu_0 = \frac{E_0}{h}$$

$$\phi_{rad} = \frac{1}{E_0}\int_0^{\nu_0} h\nu \frac{d\sigma}{d\nu}(E_{0,}\nu)d\nu$$

N: number of atoms/cm³

Last lecture: Bremsstrahlung

For $m_e c^2 << \xi << 137 m_e c^2 Z^{-1/3}, \quad \xi >> 1$ no screening :

$$\phi_{rad} = 4Z^2 r_e^2 \alpha \left(\ln \frac{2E_0}{m_e c^2} - \frac{1}{3} - f(Z) \right)$$

For $E_0 >> 137 m_e c^2 Z^{-1/3}, \quad \xi \cong 0$ complete screening :

$$\phi_{rad} = 4Z^2 r_e^2 \alpha \left(\ln(183 Z^{-1/3}) + \frac{1}{18} - f(Z) \right)$$

High energy limit is independent of electron energy E_0

$$\phi_{rad} = 4Z^2 r_e^2 \alpha \left(\ln(183 Z^{-1/3}) + \frac{1}{18} - f(Z) \right)$$

Last lecture: Collisions vs. Bremsstrahlung

$$-\left(\frac{dE}{dx}\right)_{coll} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{1}{\beta^2}\left[\ln\left(\frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2}\right) + F(\tau) - \delta - 2\frac{C}{Z}\right]$$

Radiativ energy loss from integral (xsection × photon energy) × N

$$-\left(\frac{dE}{dx}\right)_{rad} = N\int_0^{\nu_0} h\nu \frac{d\sigma}{d\nu}(E_{0,}\nu)d\nu = NE_0\phi_{rad}$$

$$\phi_{rad} = \frac{1}{E_0}\int_0^{\nu_0} h\nu \frac{d\sigma}{d\nu}(E_{0,}\nu)d\nu = 4Z^2 r_e^2 \alpha\left(\ln(183 Z^{-1/3}) + \frac{1}{18} - f(Z)\right)$$

Compare collision or ionisation vs Bremsstrahlung:

- *ionisation increases logarithmic in E and linear in Z*
- *bremsstrahlung increases linear in E and quadratic in Z*
- *ionisation: quasi-continuous energy loss*
- *bremsstrahlung: emission of one or two photons can give away full electron energy -> huge fluctuations*

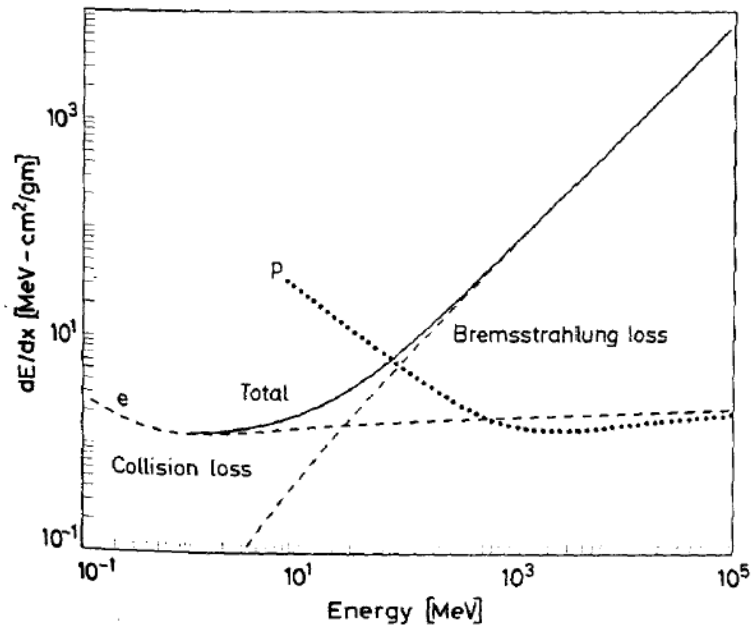
Radiation loss

Critical energy E_c:

$$\left(\frac{dE}{dx}\right)_{rad} = \left(\frac{dE}{dx}\right)_{coll} \quad \text{for } E = E_c \qquad E_c \cong \frac{800 MeV}{Z + 1.2}$$

Typical values:

material	*critical energy E_c*
Pb	*9.51 MeV*
Al	*51.0 MeV*
Fe	*27.4 MeV*
Cu	*24.8 MeV*
Luft	*102 MeV*
NaI	*17.4 MeV*
H2O	*92 MeV*



Radiation length

radiation length
distance in matter related to a radiative energy loss by 1/e

$$-dE/E = N\phi_{rad}dx$$

high energtic limit ($\xi \cong 0$):

(i) coll. loss ignored

(ii) ϕ_{rad} is independen t from energy

Solution of differenti al equation

$$E = E_0 \exp(-\frac{x}{L_{rad}})$$

x is distance in matter and $L_{rad} = 1/N\phi_{rad}$ is radiation length

$$\frac{1}{L_{rad}} \cong \left[4Z(Z+1)\frac{\rho N_a}{A}\right] r_e^2 \alpha \left[\ln\left(183Z^{-1/3}\right) - f(Z)\right]$$

Typical values:

Material	*cm*
Pb	*0.56*
Al	*8.90*
Fe	*1.76*
Cu	*1.43*
NaI	*2.59*
BGO	*1.12*
Org.Scint.	*42.40*
H_2O	*36.10*
Air	*30050.00*

Summary: interactions of electrons

$$\left(\frac{dE}{dx}\right)_{tot} = \left(\frac{dE}{dx}\right)_{rad} + \left(\frac{dE}{dx}\right)_{coll}$$

Energy loss via collisions with electrons:

$$-\left(\frac{dE}{dx}\right)_{coll} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{1}{\beta^2}\left[\ln\left(\frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2}\right) + F(\tau) - \delta - 2\frac{C}{Z}\right]$$

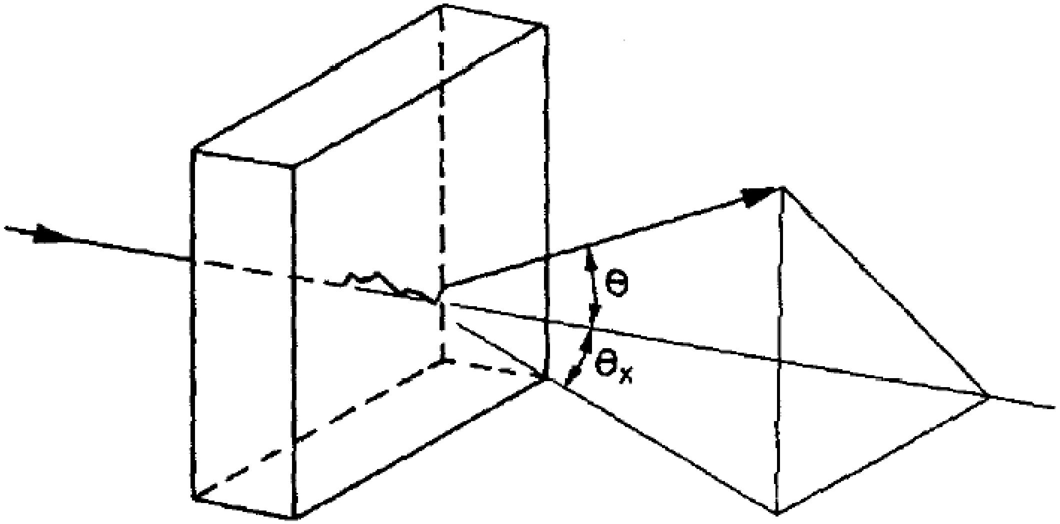
τ : Kin. energy in units of $m_e c^2$

$F(\tau)$: $F(\tau) = 1 - \beta^2 + \ldots$ electron $\quad F(\tau) = 2\ln 2 - \frac{\beta^2}{12}(23 + \ldots)$ positron

Kritical energy E_c:

$$\left(\frac{dE}{dx}\right)_{rad} = \left(\frac{dE}{dx}\right)_{coll} \text{ for } E = E_c \qquad E_c \cong \frac{800 MeV}{Z + 1.2}$$

Multiple Coulomb scattering



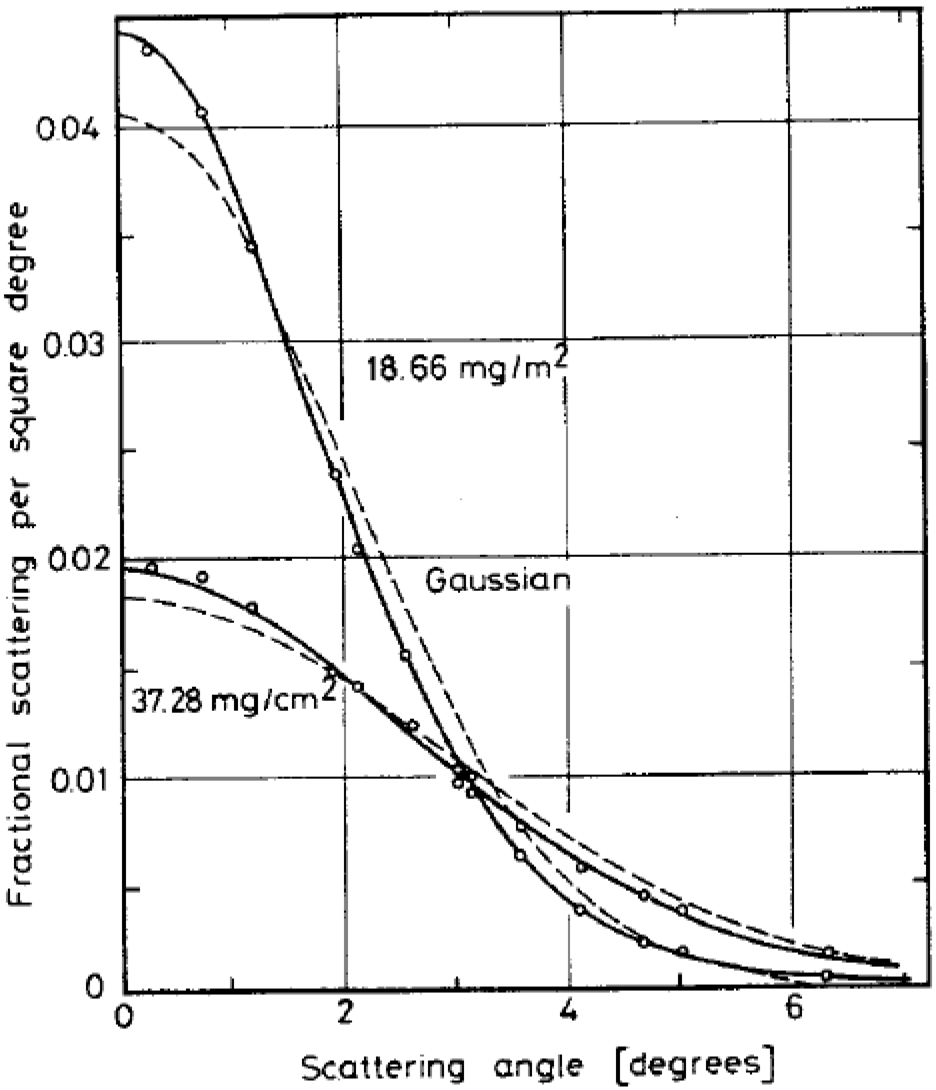
Rutherford scattering at nuclear potential

$$\frac{d\sigma}{d\Omega} = z_2^2 z_1^2 r_e^2 \frac{\left(m_e c / \beta p\right)^2}{4\sin^4(\theta/2)}$$

Three regions:

1. *Single scattering: very thin absorber, probability of more than one scattering very small, Rutherford formula, target like*
2. *Plural scattering, scatterings N<20, complicated calculation, neither Rutherford nor statistical methods.*
3. *Multiple scattering: many independent scatterings N>20, small energy loss -> probability distribution for the net angle of deflection as function of thickness*

Multiple Coulomb scattering



Angular distribution of E=15.7 MeV electrons scattered from thin Au foil

compare:

- experimental data

- Gaussian approximation

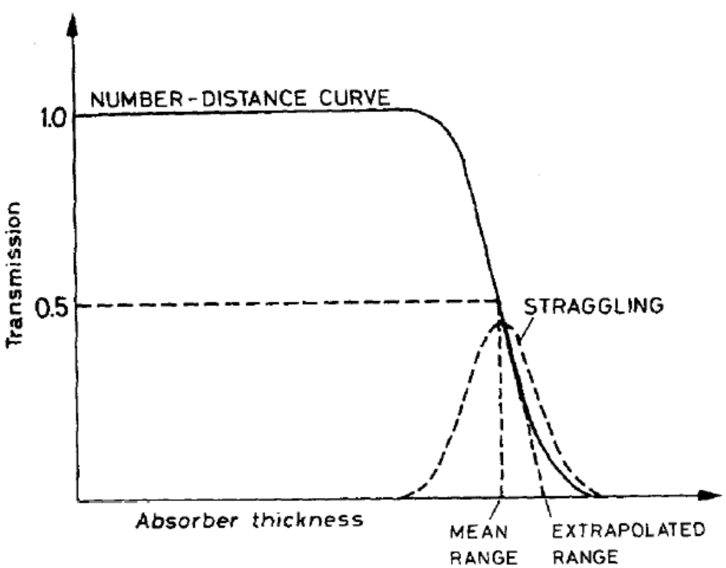
Range and energy loss

Variation in range and energy loss caused by statistics of absorbing collisions.

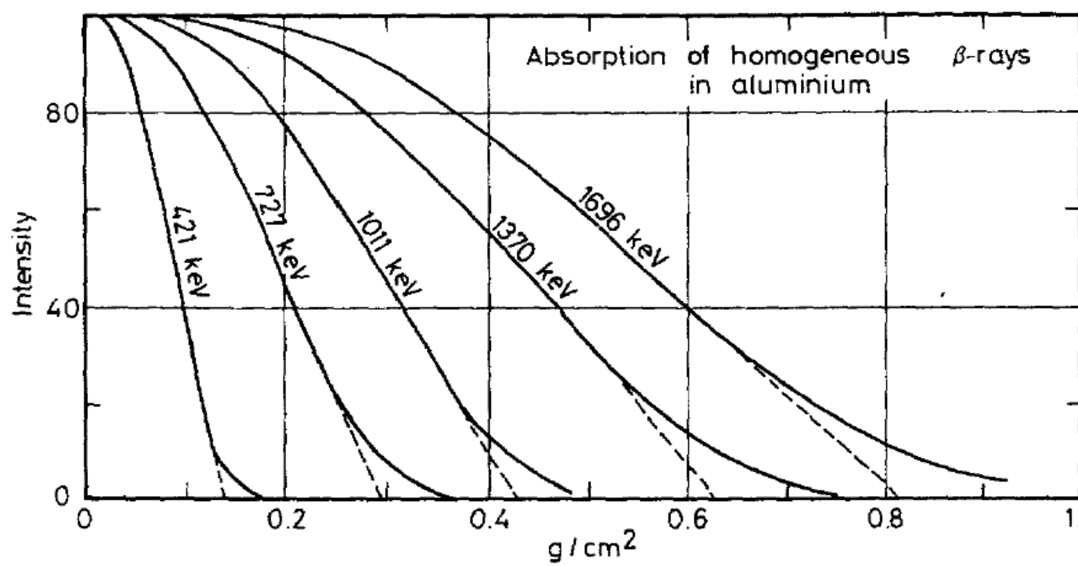
Energy loss distribution
- average energy loss –(dE/dx)
    - energy loss straggling
    - range straggling

Different but related question:
Fluctuation in energy loss –(dE/dx)
for a fixed thickness of absorber?

Heavy charged particle



electron



Thick absorbers: energy loss

thick absorber: *- N number of collisions is large, N→∞*
- small individual energy loss in absorber
- sum of many independent losses causes Gaussian form

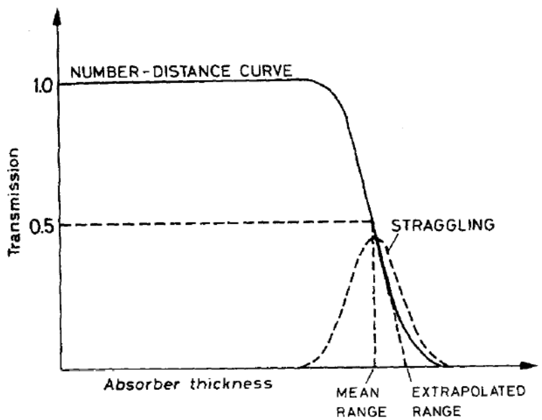
$$f(x,\Delta) \propto \exp\left(\frac{-(\Delta - \overline{\Delta})^2}{2\sigma^2} \right)$$

x : absorber thickness

Δ : energy loss in absorber

$\overline{\Delta}$: mean energy loss

σ : standard deviation



non relativistic heavy particles (Bohr) :

$$\sigma^2 = 4\pi N_a r_e^2 (m_e c^2)^2 \rho \frac{Z}{A} x = 0.1569 \rho \frac{Z}{A} x \,[MeV^2]$$

Energy loss in thin absorbers

distinguishing parameter: $\kappa = \Delta / W_{\max}$

mean E - loss / max E - transfer

very thin absorber material with Z, A

first term of Bethe - Bloch:

$$\Delta \cong \xi = 2\pi N_a m_e r_e^2 c^2 \rho \left(\frac{z}{\beta}\right)^2 \frac{Z}{A} \cdot x \qquad \text{(mean energy loss)}$$

example for z = 1: $\xi = \dfrac{0.1536}{\beta^2} \dfrac{Z}{A} \cdot x$ [keV]   x in mg/cm^2

1 cm thickness Ar gas and $\beta = 1$ $\xi = 0.123$ keV

Landau theory: very thin absorbers

Probabilit y distribution for energy loss Δ of very thin absorber with thickness x (with $\kappa \le 0.01$, thin absorber, $\beta \approx 1$) Landau' s solution :

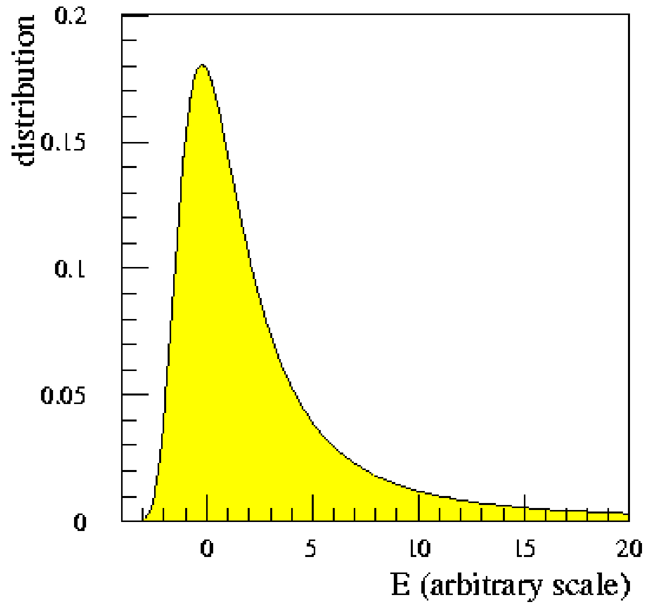
$$f(x,\Delta) = \frac{1}{\xi} \cdot \frac{1}{\pi} \int_0^{\infty} e^{-u \ln u - \lambda u} \sin(\pi u) du$$

λ is normalized deviation from most probable energy loss $\Delta^{m.p.}$ $\quad \lambda = \frac{\Delta - \Delta^{m.p.}}{\xi}$

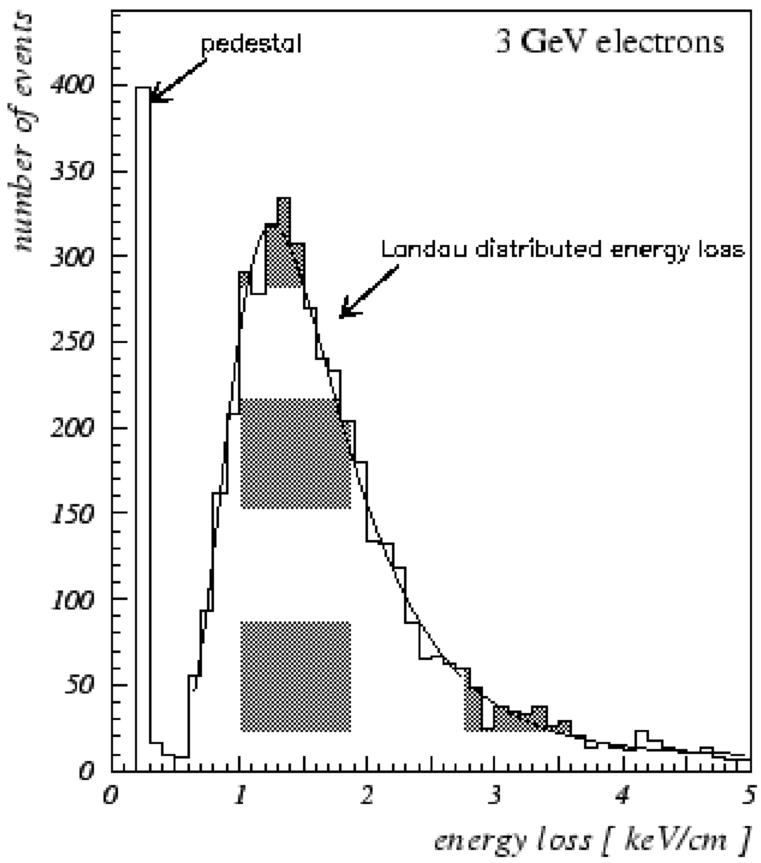
most probable energy loss : $\Delta^{m.p.} = \xi \{ \ln \frac{2 m_e c^2 \beta^2 \gamma^2 \xi}{I^2} - \beta^2 + 1 - \gamma_E \}$ and $\gamma_E = 0.577$

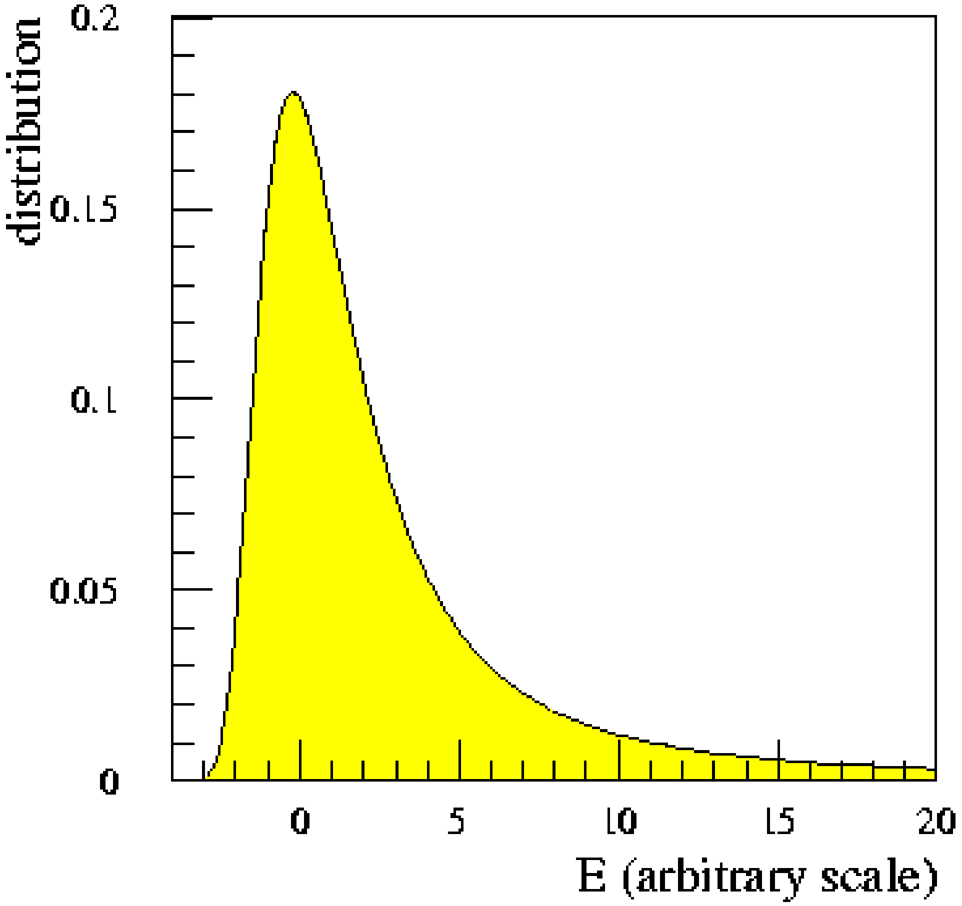
Approximat ion :

$$f(x,\Delta) = \frac{1}{\xi} \cdot \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} (\lambda + e^{-\lambda}) \right)$$

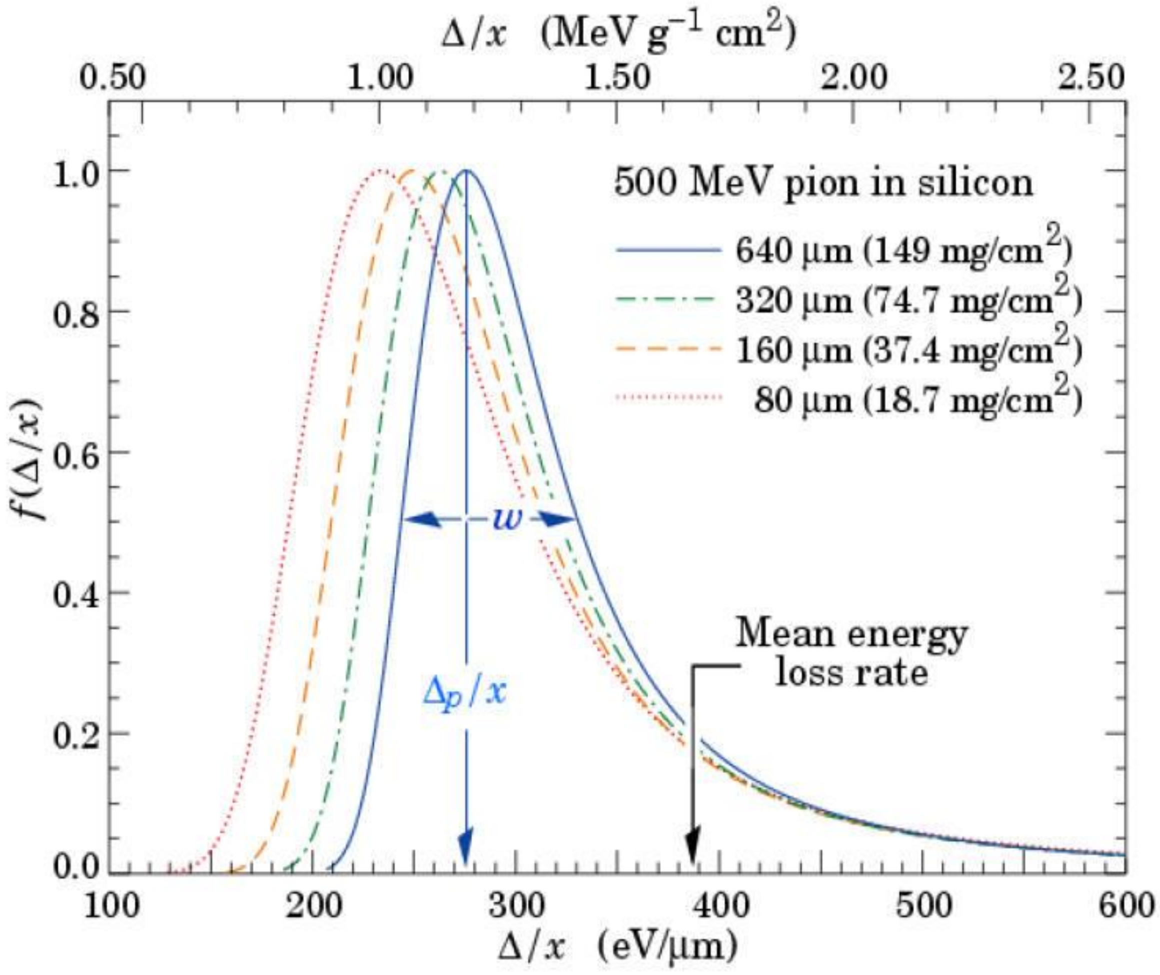


Energy loss of electrons vs Landau distribution





Energy loss of pions in silicon



summary

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radiation length
energy loss by 1/e