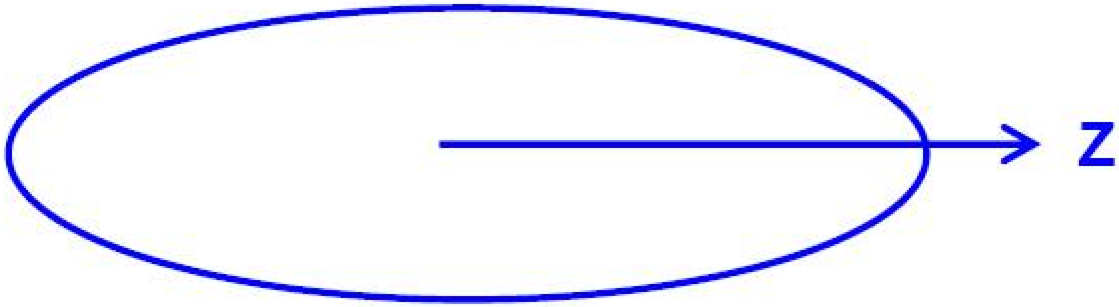
Deformed Shell Model

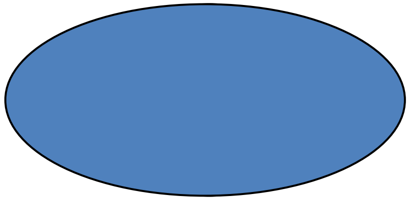
Nilsson Model



With five Appendices at the end

Deformed nuclei

So far, we have dealt with spherical nuclei only. How do we describe deformed nuclei?



We need two parameters to describe shape: the amount of deformation, and the axial symmetry.

What is different about deformed nuclei?

OK, sure, the fact that they aren't spherical !! But what else?

Ans: They can ROTATE

How to describe deformation?

Spherical nucleus: Radius R_0

Deformed nucleus: $R = R_0 \left[1 + \sum_{\lambda,\mu} \alpha_{\lambda\mu} Y_{\lambda\mu} \right]$

Quadrupole Def:

$$R = R_0 \left[1 + \sum_{\mu} \alpha_{2\mu} Y_{2\mu} \right]$$

Do usual (see later discussion of e-e nuclei) change of coordinates from the 5 $\alpha_{2\mu}$'s

Separate <u>body-fixed from lab coords</u>

Intrinsic shape: β , γ

Ellipsoid Def. Axial asymmetry

Can have higher multipoles: β_4, β_6, ….

Often drop subscript if “2” is clear.

$$\delta R_z = \sqrt{\frac{5}{4\pi}} R_0 \beta \cos\gamma$$

$$\delta R_x = \sqrt{\frac{5}{4\pi}} R_0 \beta \cos\left[\gamma - \frac{2}{3}\pi\right]$$

$$\delta R_y = \sqrt{\frac{5}{4\pi}} R_0 \beta \cos\left[\gamma - \frac{4}{3}\pi\right]$$

Table 6.6 *Changes in the radius of a quadrupole ellipsoid in the x, y, z directions for several γ values and fixed β.**

	Prolate (+1, -2)	Max. Asymm.	γ Oblate (+2, -1)	
	0°	30°	60°	180°
δR_z	+1	+0.866	+1/2	−1
δR_x	−1/2	0	+1/2	+1/2
δR_y	−1/2	−0.866	−1	+1/2

*All numbers are in units of $\sqrt{5/4\pi}\, R_0\, \beta$.

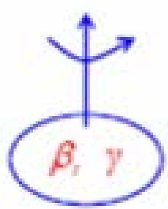
Unfortunate historical note:

Notation for deformation:

$$\underbrace{\beta, \quad \varepsilon, \quad \delta,}_{\approx \text{ equal}} \quad \underset{\sim 20\,\beta}{\underset{\downarrow}{\eta}}$$

Reasons for these, especially ε

For now, treat β, ε, δ as interchangeable



Position of nucleus in space:

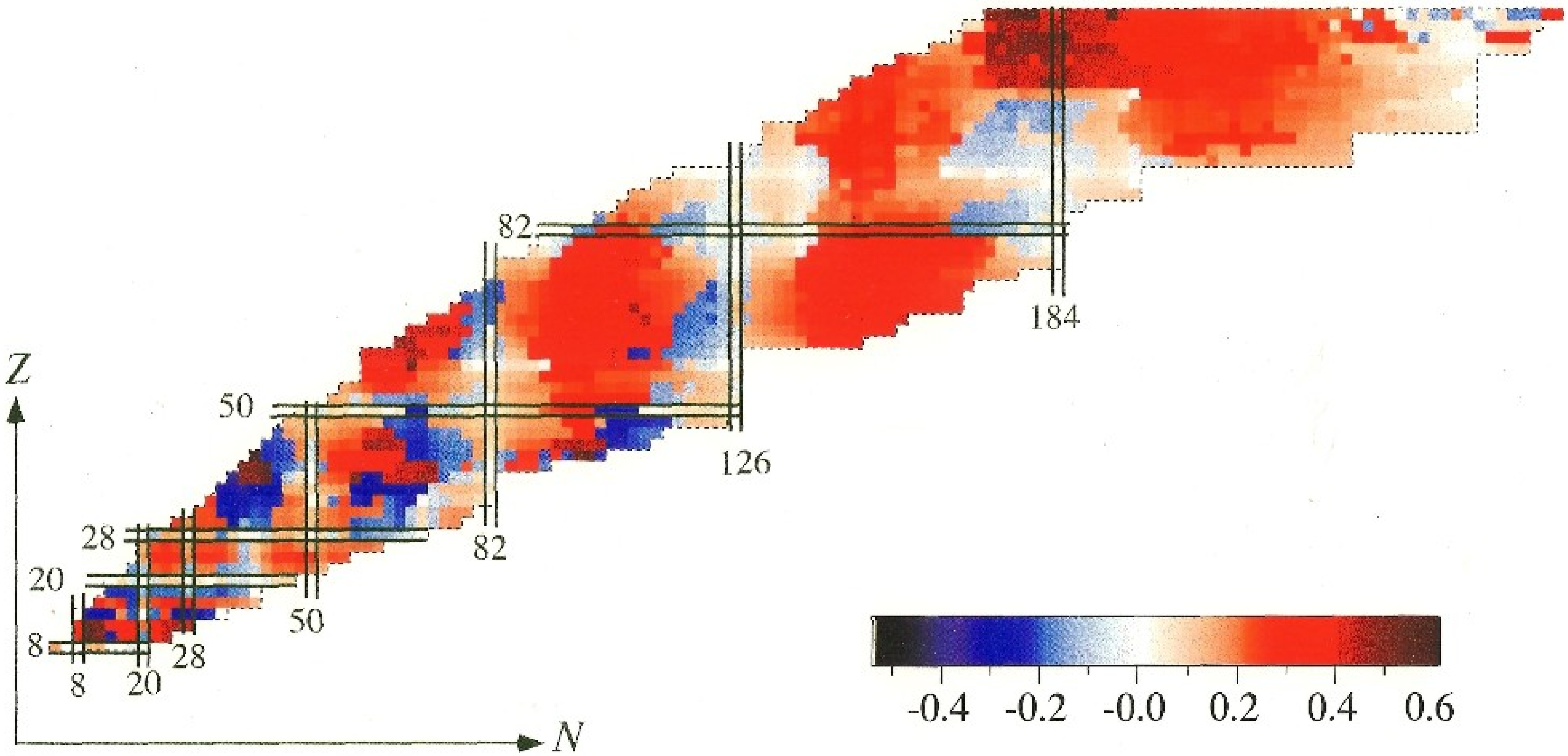
3 Euler angles

$$\therefore \quad 5 \quad \alpha_{2\mu} \rightarrow \underbrace{\beta, \gamma,}_{\substack{\text{Intrinsic} \\ \text{frame}}} \quad + \quad \underbrace{\text{3 Euler angles}}_{\substack{\text{lab} \\ \text{frame}}}$$

Intrinsic frame $\longrightarrow$ lab frame

We will assume axial sym. $\Rightarrow$ ignore γ ($\gamma \equiv 0^\circ$)

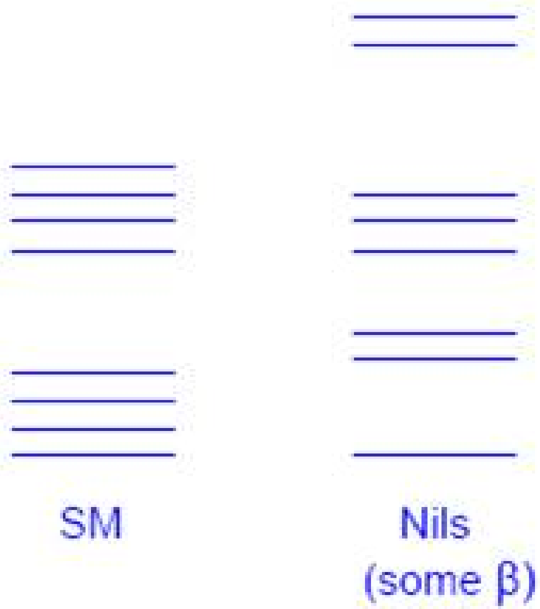
β_{2p} deformation



Hirata, Sumiyoshi, Tanihata, Sugahara and Toki '98

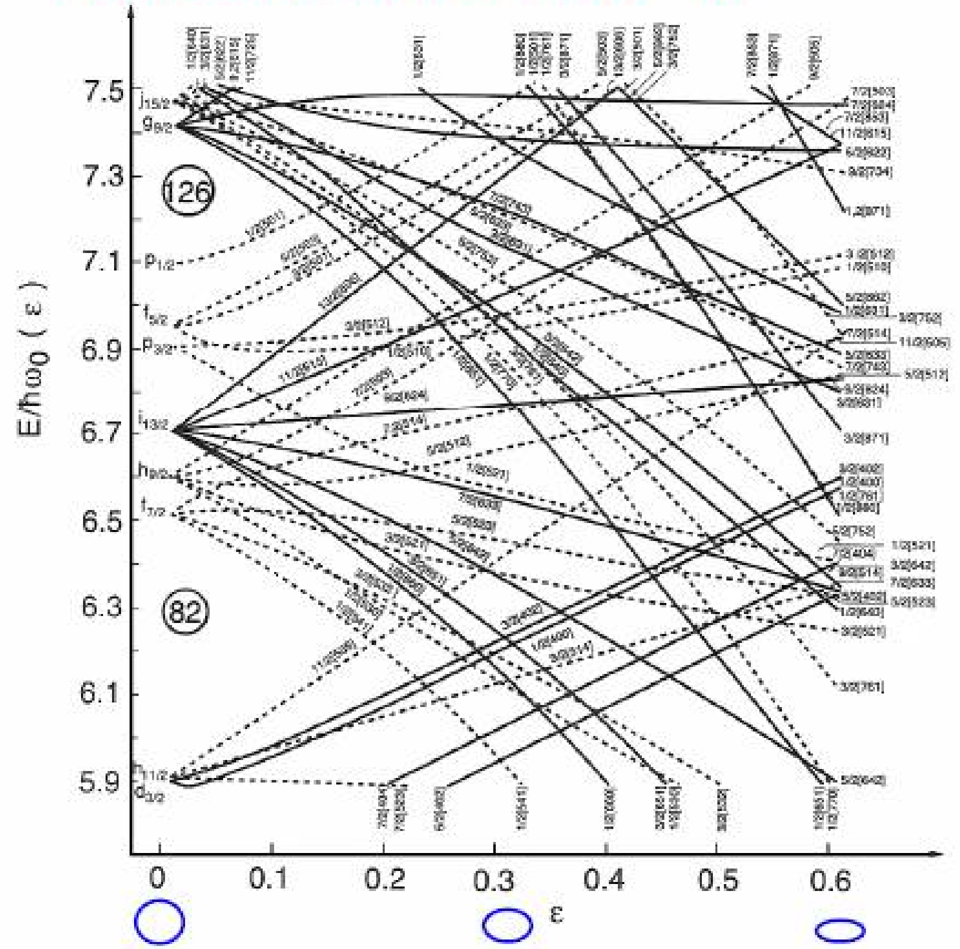
So, Nilsson Model

is just a specification of s.p.e.'s (and Ψ's) for any β (or ε, or, more generally, for β_2, β_4, ...)



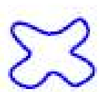
Usually, one plots SM and Nils energies for <u>all</u> β in a single plot called a <u>Nilsson Diagram</u>

The Nilsson Diagram (neutron 82 – 126)



Goal: to understand this diagram

to be able to "derive" it with*out* any calculation

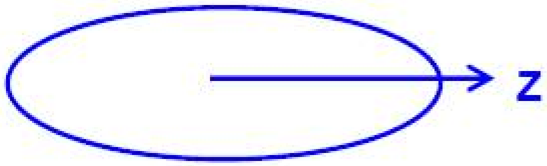


to be able to do same for *any* shape

!

$\boxed{\text{All}}$ we need is:

Nilsson Model

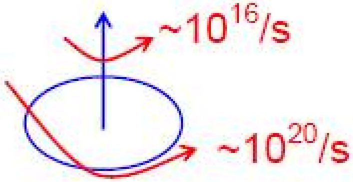


New features

- Non-spherical $\Rightarrow$ can **rotate**

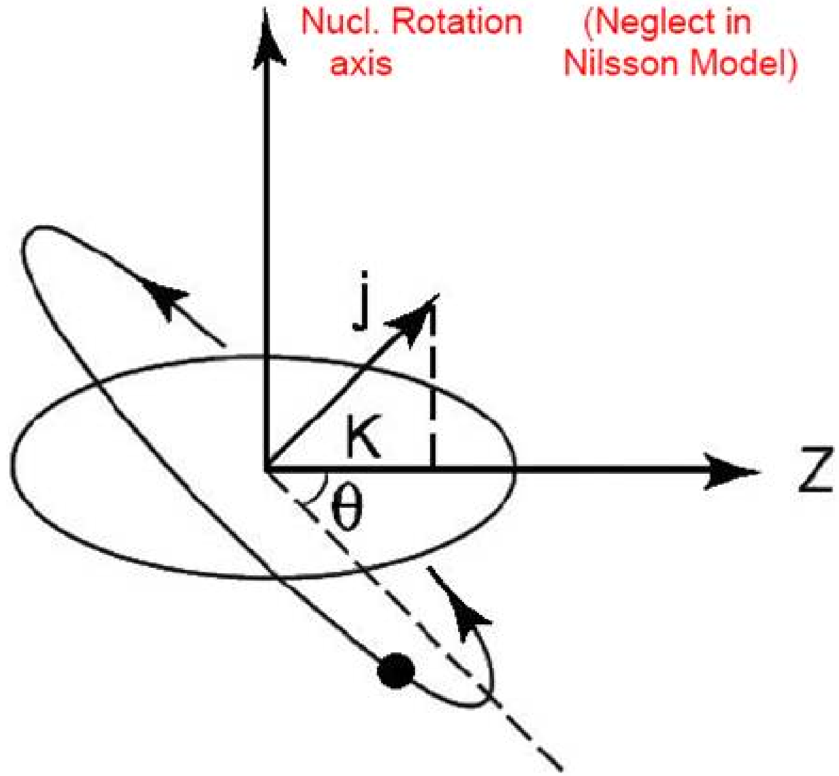
Useful to separate lab-fixed from body-fixed systems

Can do this because frequencies of orbits (nucle<u>on</u>, nucle<u>us</u>) are so different.



So, nucleus is effectively stationary while particle orbits. (Careful—this can break down, and does, at very high rotational frequency).

Define coordinate system and quantum numbers



$$\sin\theta \sim K / j$$

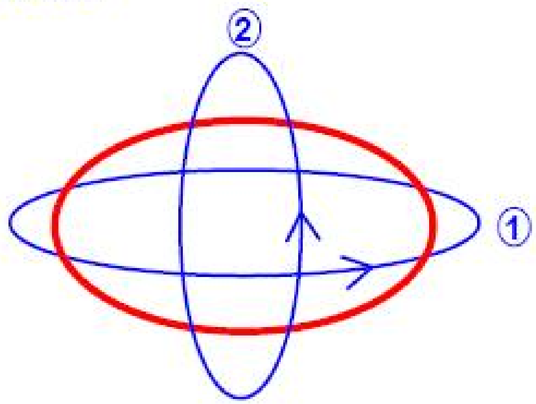
K – Projection of s.p. Angular Momentum on symmetric axis.
(Note: Rotation is perpendicular sym. axis so does not change *K*).

θ – Angle of orbit to "equator."

Nilsson Model

What happens to Shell Model energies in a deformed nucleus?

Consider 2 orbits



For orbits of same principal quantum number N, *i.e.*,

same *radius*, orbit 1 is closer to bulk of nucleus than

orbit 2.

Nuclear force is attractive

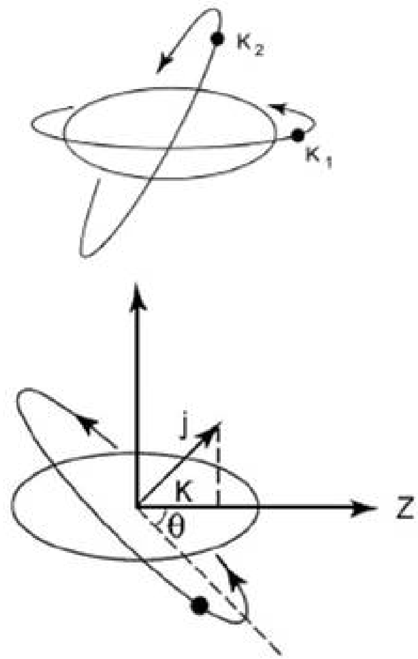
∴ Orbit 1 is lower in energy

This, plus 2-state mixing, is all that is needed to produce Nilsson diagram

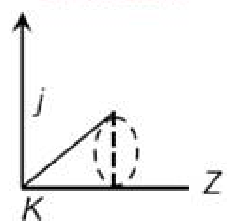
Conceptual Approach:

- Nucl. force attractive
  nucleons “like” to be near others
- 2-state mixing

K is projection of *j* along symm. axis *Z*.



K is constant of motion



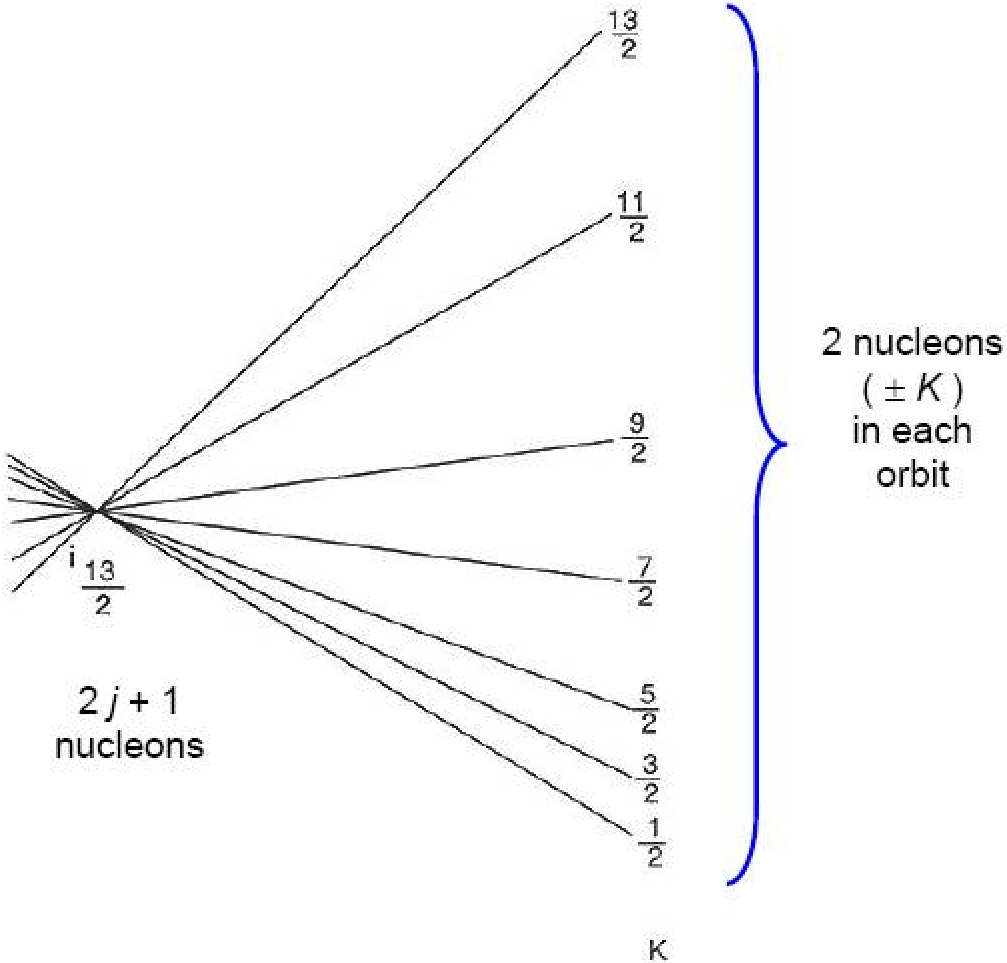
$\sin\theta \sim K / j$

Classical orbit angles, relative to the nuclear equator, for $j = 13/2$.

K	1/2		3/2		5/2		7/2		9/2		11/2		13/2
θ(deg)	4.4		13.3		22.6		32.6		43.8		57.8		90
$\Delta\theta$(deg)		8.9		9.3		10.0		11.2		14.0		32.2	

$\therefore\ \Delta\theta$ increases with K, $\therefore\ \Delta E$ spacings increase with K

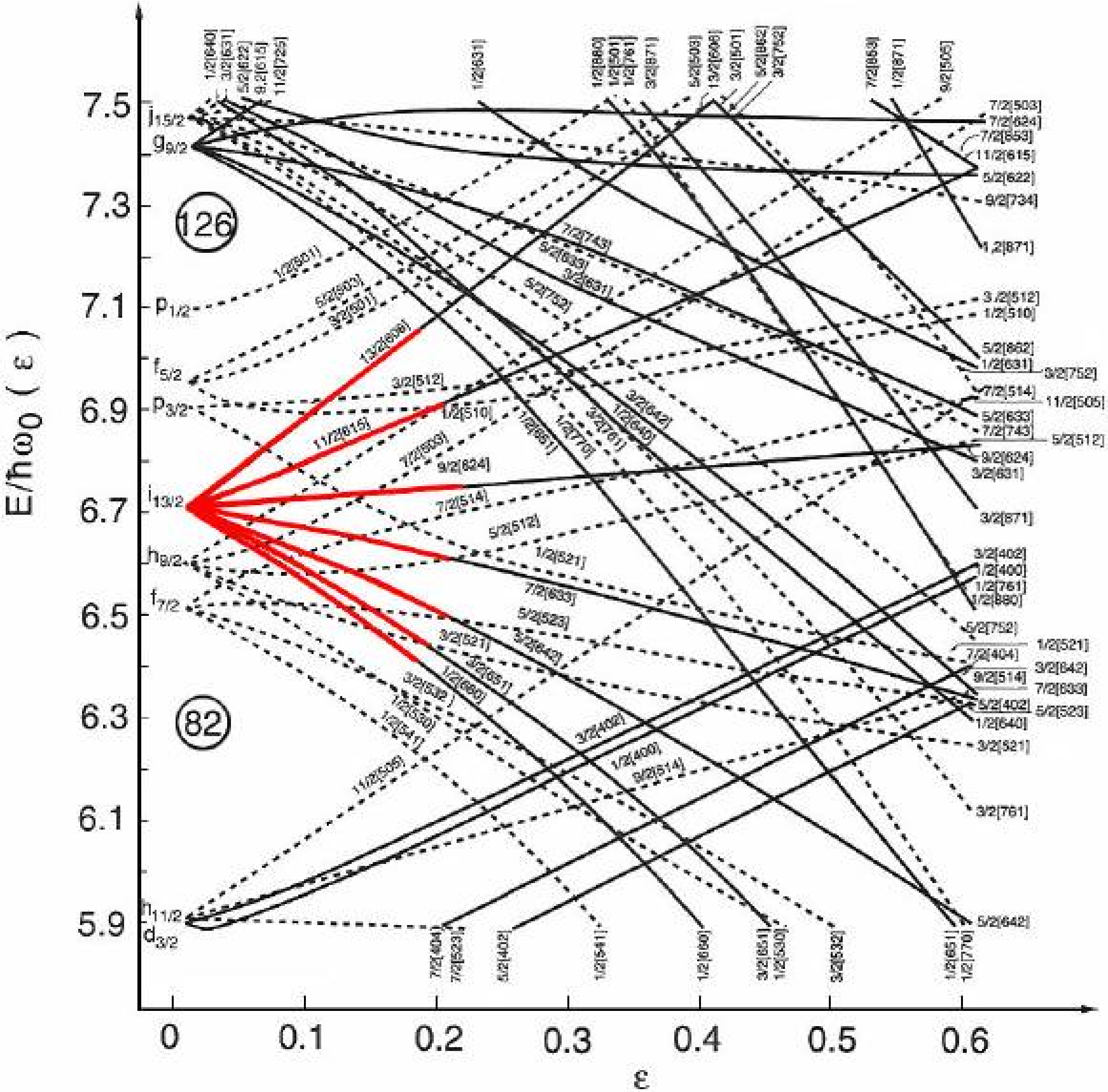
Splitting of levels of given *j* in deformed field.



Variation of single-particle energies of $i_{13/2}$ orbits with different projections K (orientation θ) as a function of deformation (β > 0, *prolate*, to the right).

Structure of Nils. Diag:

s.p. level splitting for each orbit



This is the essence of the Nilsson model and diagram. Just repeat this idea for EACH j-orbit of the spherical shell model. Look at the left and you will see these patterns.

There is only one other ingredient needed. Note that some of the lines are curved ! What does this mean? Where does that come from?

We have discussed effects of shapes on the energy of a particle in a given *j*-orbit.

This is analogous to our earlier discussions of effects of interactions on energies: *i.e.*, it is a <u>diagonal</u> effect.

But the interaction (quadrupole in this case) also <u>mixes</u> $\varphi_{sp.}$ (off-diagonal).

$$\left\langle n_1 l_1 \; j_1 \middle| \, Q \, \middle| n_2 \; l_2 \; j_2 \right\rangle$$

Detour on 2-state mixing

Suppose we have a Hamiltonian

$$H = H_0 + H_{resid.}$$

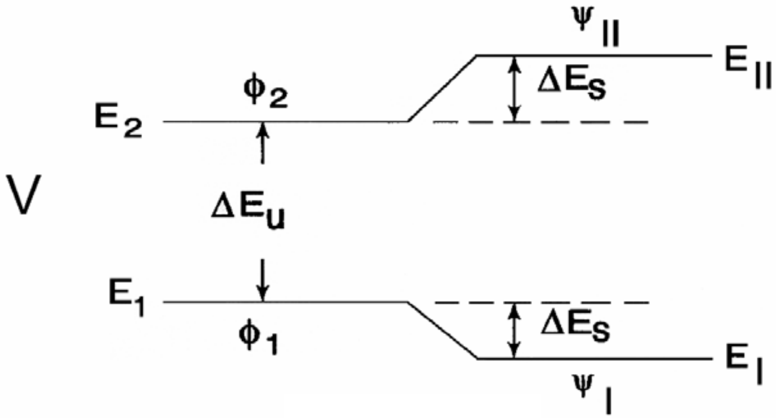
where H_0 has a set of eigenstates X_i

The eigenstates of H will therefore be **mixtures** of those of H_0, that is linear combinations of the X_i with coefficients C_i. (In our case, H_0 will be the Hamiltonian of the **SPHERICAL** shell model that we have been discussing. $H_{resid.}$ will be the terms that change things because the nucleus is deformed.)

To understand the effects of these terms, we need to briefly discuss a topic of much wider and fundamental importance:

2- state mixing –with a brief excursion to multi-state mixing

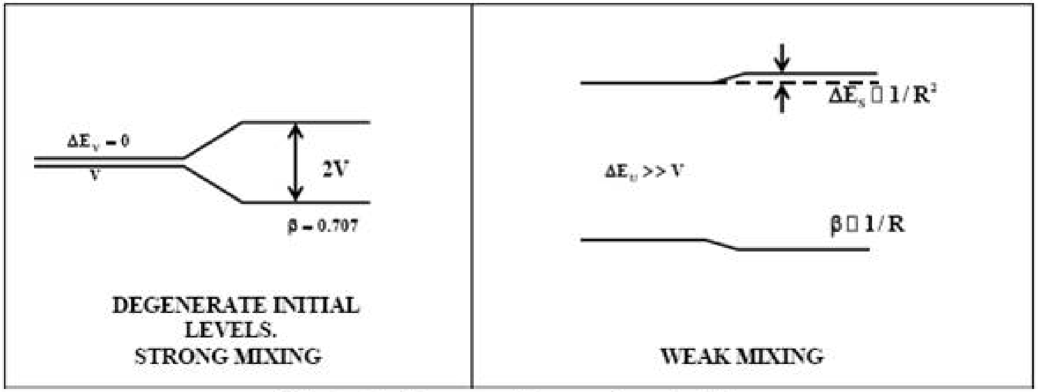
TWO-STATE MIXING



$$\boxed{E_{\mathrm{I,II}} = \frac{1}{2}(E_1 + E_2) \pm \frac{1}{2}\sqrt{(E_2 - E_1)^2 + 4V^2}}$$

This is the main result we will need but I include a bit more here and a fuller discussion in an Appendix because the topic is of such fundamental importance.

Limiting Cases



The two limiting cases of strong and weak mixing

$\boxed{\text{Strong}}$ **Mixing (Degenerate levels)** $E_1 = E_2 = E_0$

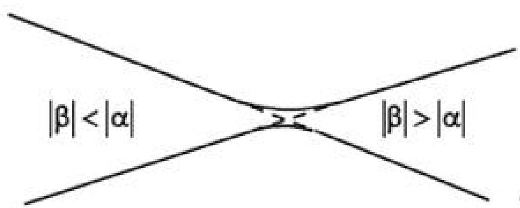
$R \rightarrow 0$

$$E_{I,II} = \frac{1}{2}[(E_1 + E_2) \pm 2V] = E_0 \pm V$$

$\rightarrow$ $E_{II} - E_I = 2V$ **Two levels can never be closer than <u>twice the mixing matrix ele.</u>**

$\rightarrow$ **Also** $\beta \rightarrow \frac{1}{\sqrt{2}} = 0.707$ **<u>Complete mixing</u>**

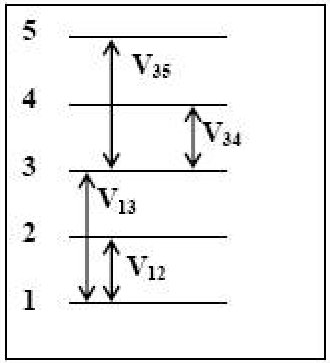
Level Crossing



$\psi_I = \alpha\phi_1 + \beta\phi_2$

$\psi_{II} = -\beta\phi_1 + \alpha\phi_2$

Multi-State Mixing



Need to diagonalize

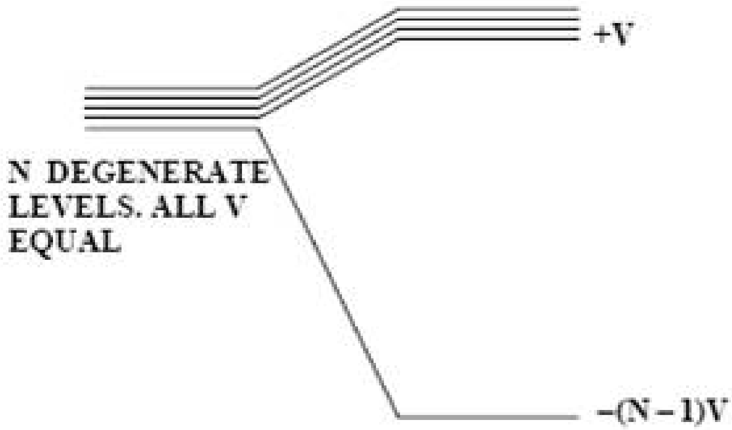
$$\begin{array}{c c} & \begin{array}{ccccc} ① & ② & ③ & ④ & ⑤ \end{array} \\ \begin{array}{c} ① \\ ② \\ ③ \\ ④ \\ ⑤ \end{array} & \begin{pmatrix} E_1 & V_{12} & V_{13} & 0 & 0 \\ V_{12} & E_2 & 0 & 0 & 0 \\ V_{13} & 0 & E_3 & V_{34} & V_{35} \\ 0 & 0 & V_{34} & E_4 & 0 \\ 0 & 0 & V_{35} & 0 & E_5 \end{pmatrix} \end{array}$$

$$\psi_k = \sum_i \alpha_i \varphi_i$$

Details depend on V_{ij}'s, E_i's

Important special case: degenerate levels

Special case of the utmost importance. Mixing of degenerate levels



$$\Psi_{\mathrm{LOWEST}} = \frac{1}{\sqrt{N}}\left[\phi_1 + \phi_2 + \ldots + \phi_N\right]$$

This is the origin of collectivity in nuclei. Essential also for understanding masses

Not much is more important than this idea.

Please remember it and think about it often (and try to develop a deep love for it).

$$\Psi_{\text{Nils}} = \alpha\,\varphi_1 + \beta\,\varphi_2$$

α, β, . . . depend on < | Q | > and on unperturbed energy spacing of s.p.e.'s.

Recall 2-state mixing:

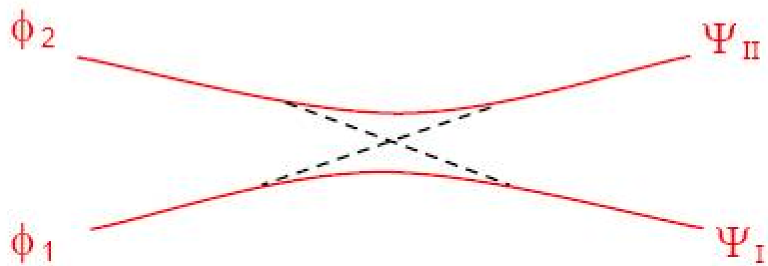
Universal results depend

ONLY on $R \equiv \dfrac{\Delta E_u}{V}$

Usual notation:

$$\Psi_{\text{Nils}} = \sum_j C_j\,\varphi_j$$

2-State Mixing



After inflection point, Ψ's interchange

Inflection point $\rightarrow$ max mixing

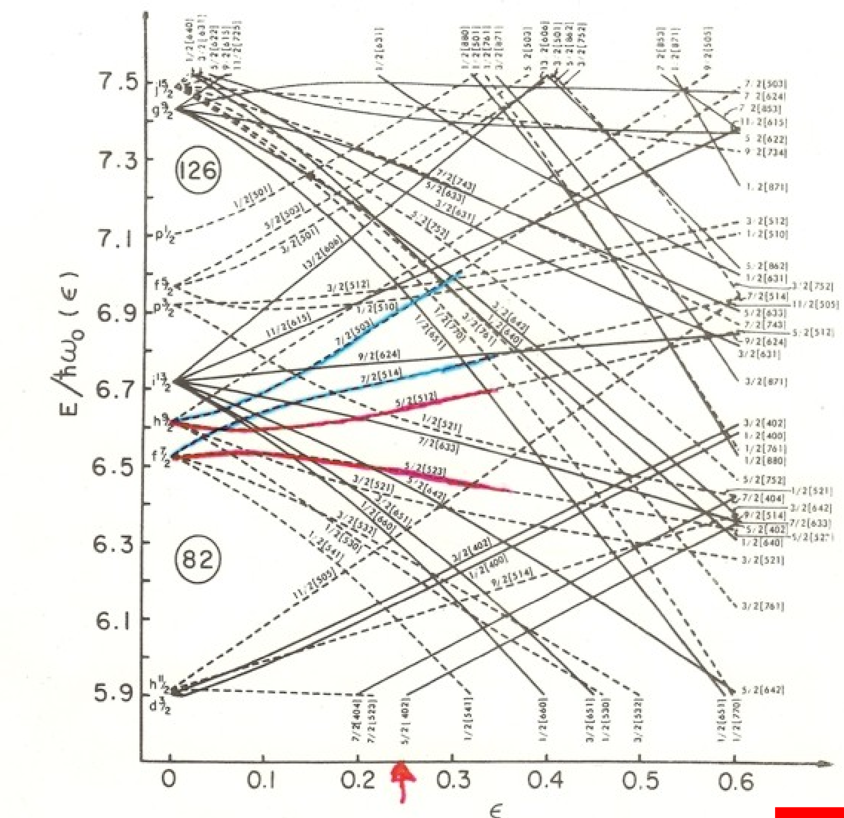
$$\Psi_I = 0.707\,\phi_1 + 0.707\,\phi_2$$

$$\Psi_{II} = -\,0.707\,\phi_2 + 0.707\,\phi_1$$

$\Delta E_{1,2}$ (inflection point) = $2V$

Ψ_I on right is predominantly ϕ_2

Ψ_{II} on right is predominantly ϕ_1



Can even "guesstimate" wave function $\boxed{\Psi_{Nils} = \sum_j C_j \phi_j}$

Table 7.2. Nilsson wave functions ($\boxed{C_j}$ coefficients) for some N = 5 orbits

	j					
$K\pi[Nn_z\Lambda]$	1/2	3/2	5/2	7/2	9/2	11/2
3/2⁻[532]		0.234	0.369	–0.560	–0.651	0.268
5/2⁻[523]			0.237	–0.472	–0.826	–0.196
7/2⁻[514]				0.323	0.938	0.128
1/2⁻[521]	–0.510	0.345	0.473	0.431	0.444	0.120
5/2⁻[512]			–0.023	0.836	–0.515	0.157
1/2⁻[510]	0.021	–0.676	0.586	–0.343	0.277	0.067
3/2⁻[512]		0.379	0.815	0.283	0.327	0.063
7/2⁻[503]				0.937	–0.336	0.099
9/2⁻[505]					0.998	0.071
1/2⁻[501]	–0.821	–0.361	–0.411	–0.122	–0.104	–0.019

*$\delta = 0.22$, $\kappa = 0.0637$, $\mu = 0.42$.

Uh-oh, is something wrong with all this?

Nilsson Model

Deformed Shell Model

Uses Shell Model basis states

Mixed j states

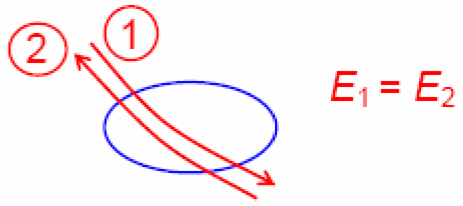
Implies j is not a good quantum number

Huh ??

How do we use Nilsson Model?

Similarly to Independent Particle Model except

- Orbits have 2-fold degeneracy



rather than $2j + 1$ degeneracy

- Need to know, or guess, deformation to use

- Nilsson Model is in intrinsic system—for prediction need to go to lab.

Add rotation

Discussed in 9 added slides in the Appendix at the end

- So, take Nilsson diagram for proper kind of particle (n or p)
- Assume all except <u>last odd nucleon</u> are coupled in pairs to $J = 0^+$
- Mentally place 2 nucleons (of the “odd” type) into each Nilsson state starting from 0, filling up to the number of the last odd nucleon. (Use magic gaps to simplify this)

∴ Identify “ground state” Nilsson orbital

Identify excited particle and hole excitation relative to ground state

For each such state, add a rotational band

To use the Nilsson Model, need to know deformation

Note: Several def. parameters in use

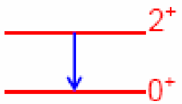
$\beta, \quad \varepsilon, \quad \delta$

In first order: $\beta \sim \varepsilon \sim \delta$

(interchangeable at 5% level)

How to estimate deformation:

a) From transition rates in neighboring e-e nucleus.



$$\left\langle 2^+ \left\| E2 \right\| 0^+ \right\rangle \propto Q^2 \propto \beta^2 \text{ (later)}$$

b) From rotational energies in neighboring e-e.

$$E\left(2_1^+\right) = \frac{\hbar^2}{2\mathcal{I}}\left[J\left(J+1\right) - J_{\mathrm{g.s.}}\left(J_{\mathrm{g.s.}}+1\right)\right]$$

$$= \frac{\hbar^2}{2\mathcal{I}}\left[2 \cdot 3\right] = 6\left(\frac{\hbar^2}{2\mathcal{I}}\right)$$

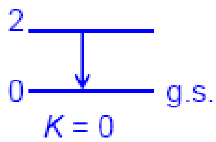
$$\Rightarrow \boxed{\mathcal{I} = 6\hbar^2 / 2E\left(2_1^+\right) \quad \text{or} \quad \frac{\hbar^2}{2\mathcal{I}} = \frac{E(2_1^+)}{6}}$$

Use $\mathcal{I} = \frac{2}{5} A M R_0^2 \left[1 + 0.31\beta\right]$ Classical for spheroid

(to first order in β)

Quadrupole Moment of Ellipsoid

$$Q_0 = \frac{3e}{\sqrt{5\pi}} Z R^2 \beta \left[1 + \underbrace{0.16\,\beta}_{\text{higher order term } (\propto \beta^2)} \right]$$



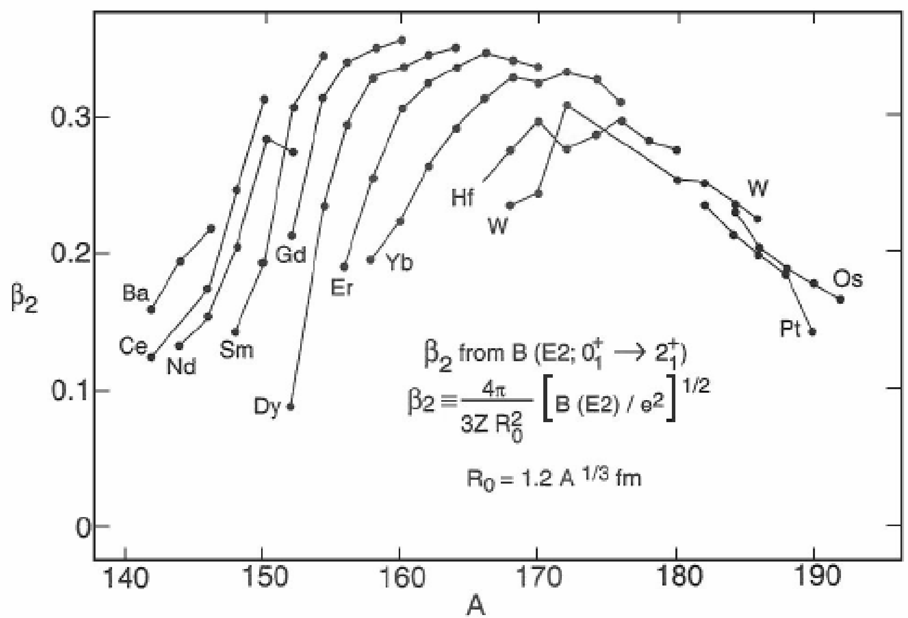
$$B\left(E2; J_i \rightarrow J_f\right) = \frac{5}{16\pi} Q_0^2 \left\langle J_i K\, 20 \middle| J_f K \right\rangle^2 \; e^2 b^2$$

For $0_1^+ \rightarrow 2_1^+$:

$$B\left(E2; 0_1^+ \rightarrow 2_1^+\right) = \frac{5}{16\pi} \; Q_0^2$$

$$\frac{B(E2)}{e^2} = \frac{5}{16\pi} \left(\frac{3}{\sqrt{5\,\pi}} Z R^2 \beta \right)^2$$

or $\quad \beta = \dfrac{4\,\pi}{3\,Z R^2} \sqrt{B(E\,2)/e^2}$

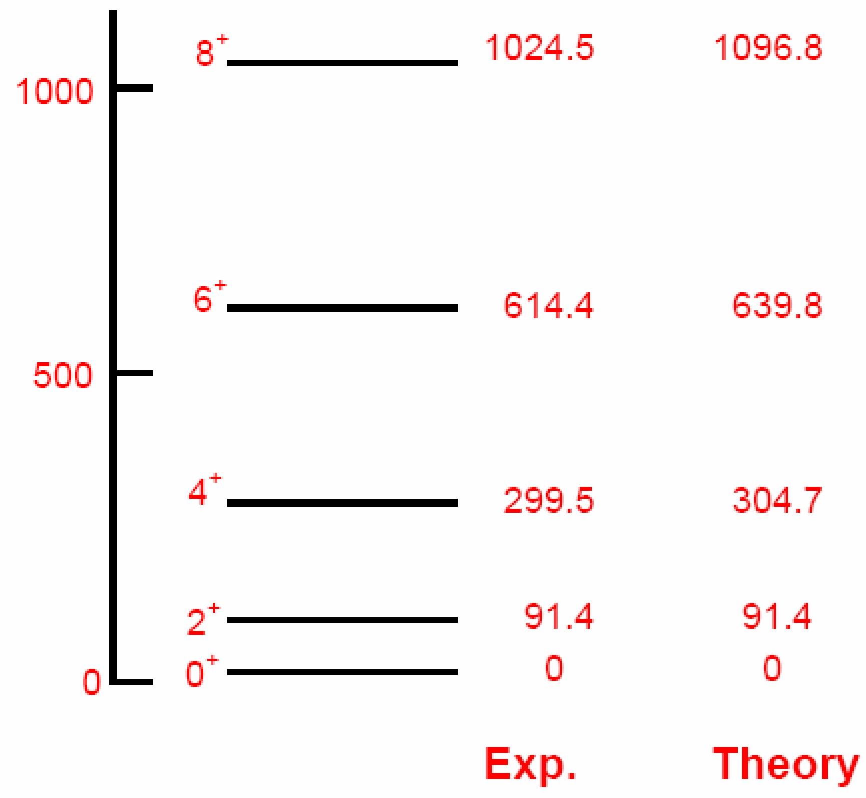


Now add rotation as well for full description of odd-A deformed nuclei. Look at e-e nuclei first.

$$H = \frac{\hbar^2}{2I}\mathbb{R}^2$$

$$E_{\text{rot}}(J) = \frac{\hbar^2}{2I} J(J+1)$$

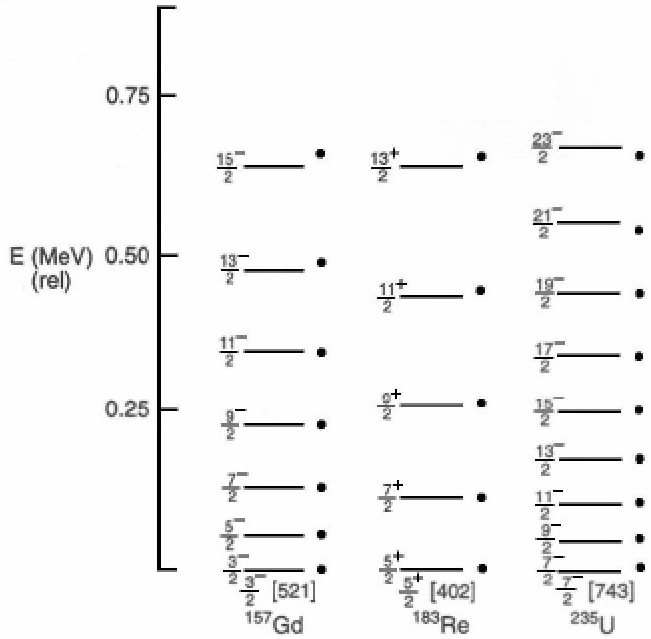
Example: ^{164}Er



Now use this idea for odd-A nuclei:

$$E_{\text{rot}}(J) = \frac{\hbar^2}{2I}[J(J+1) - K(K+1)]$$

Rotational Formula works well

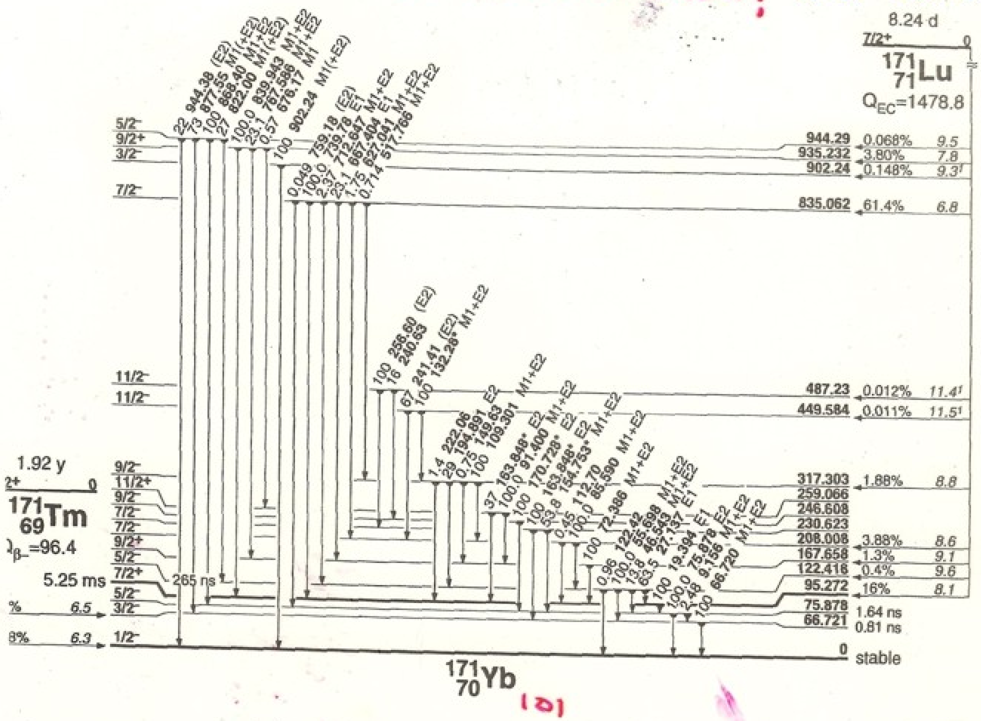


Dots: Rotational Formula

$$E_{\text{Level } J} - E_{\text{Bandhead}} = \frac{\hbar^2}{2\mathcal{I}}\left[J(J+1) - J_0(J_0+1)\right]$$

$\frac{\hbar^2}{2\mathcal{I}}$ fitted from 1st rotational level

171 Yb: Large number of low lying states in no obvious order! Not Id Pattern!



Ignore paired off 70 protons and 100 neutrons

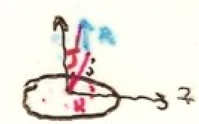
Look only at last odd neutron

$\epsilon \sim 0.25$ or so

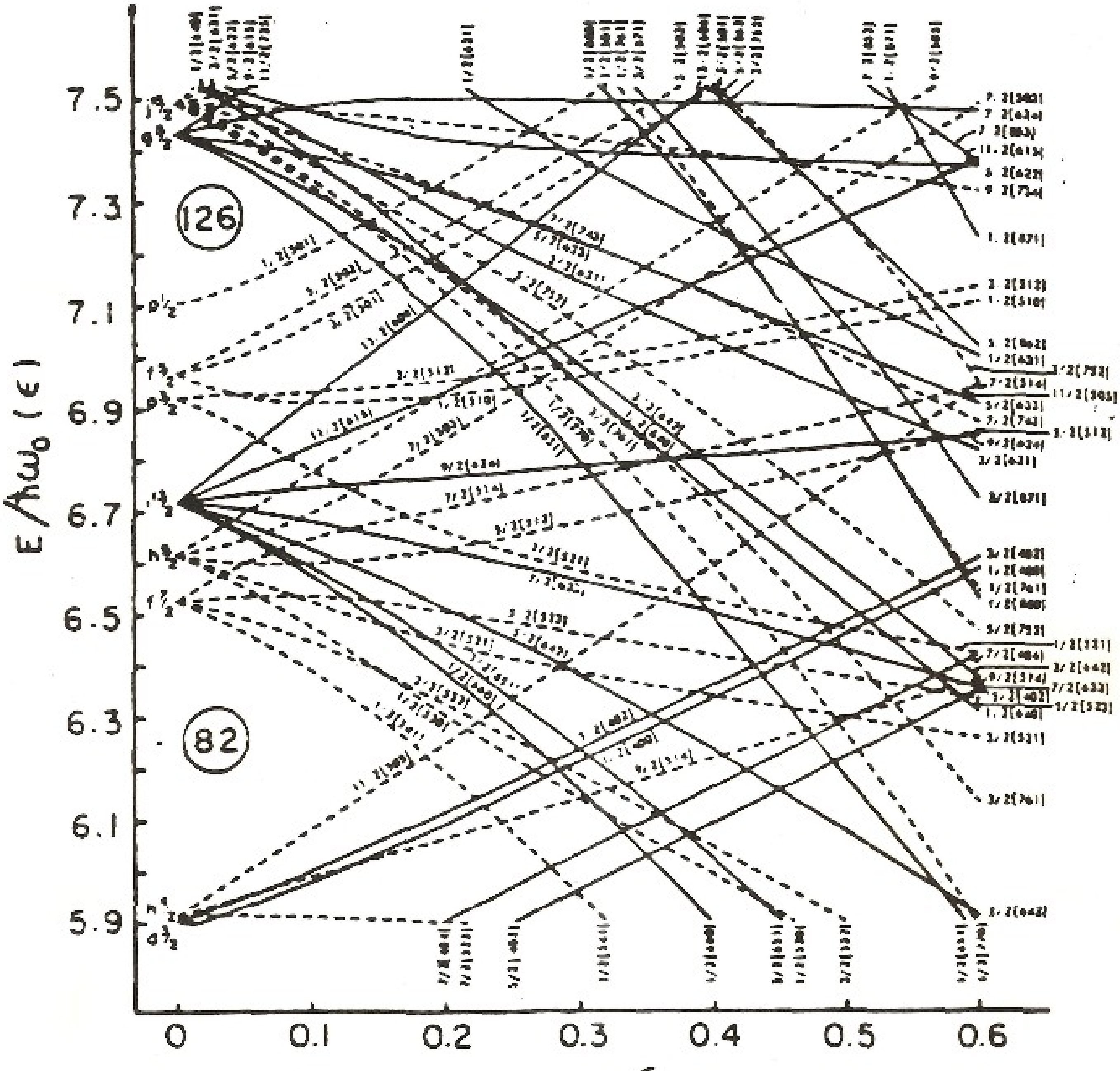
Look up what Nilsson orbits are at/near the 101^{th} position

Assign Nilsson q. #'s

Add Rotations: $E_{rot} \sim \frac{\hbar^2}{2\mathcal{J}}[J(J+1) - K(K+1)]$

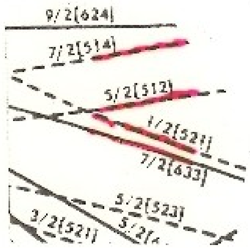


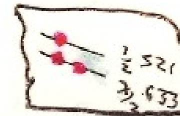
R does not change K



E/ħω0(ε)
7.5
7.3
7.1
6.9
6.7
6.5
6.3
6.1
5.9
126
82
0
0.1
0.2
0.3
0.4
0.5
0.6
ε

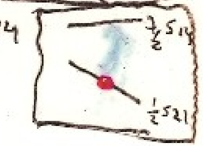
So, expect g.s. is $\frac{1}{2}^-$ 521, low lying $\frac{7}{2}^+$ 633



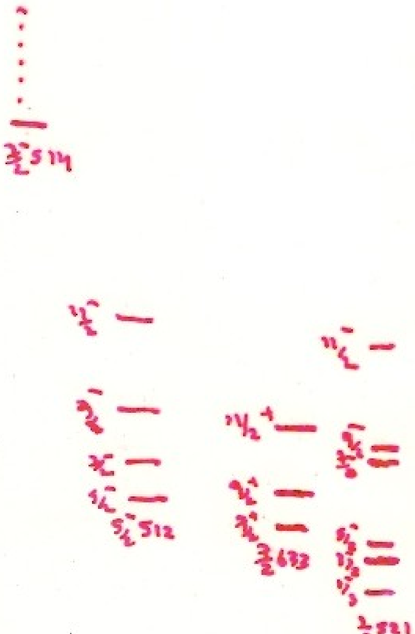


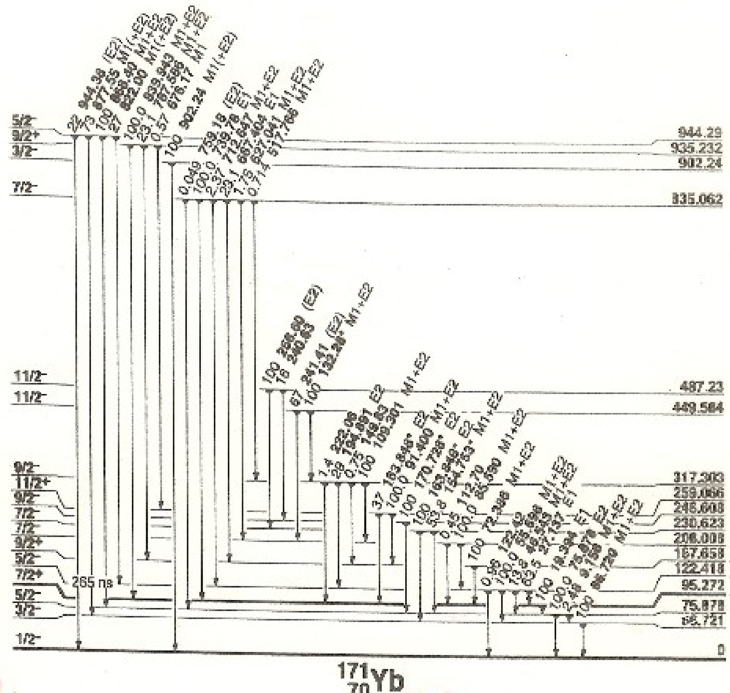
$\frac{5}{2}^-$ 512

, and $\frac{7}{2}^-$ 514



excitations





101

Nilsson intrinsic excitations

→

rotational bands

More on rotation later

Nilsson Model

Another example of paradigm shift

Consider nucleus ${}^{157}_{64}Gd_{93}$

$$\left.\begin{array}{l}\text{14 valence protons}\\ \text{11 valence neutrons}\end{array}\right\}\begin{array}{l}\text{100's of trillions}\\ \text{of configurations}\end{array}$$

But – trivial in Nilsson scheme

$\Rightarrow$ Why – shift from point of view of closed shell

to

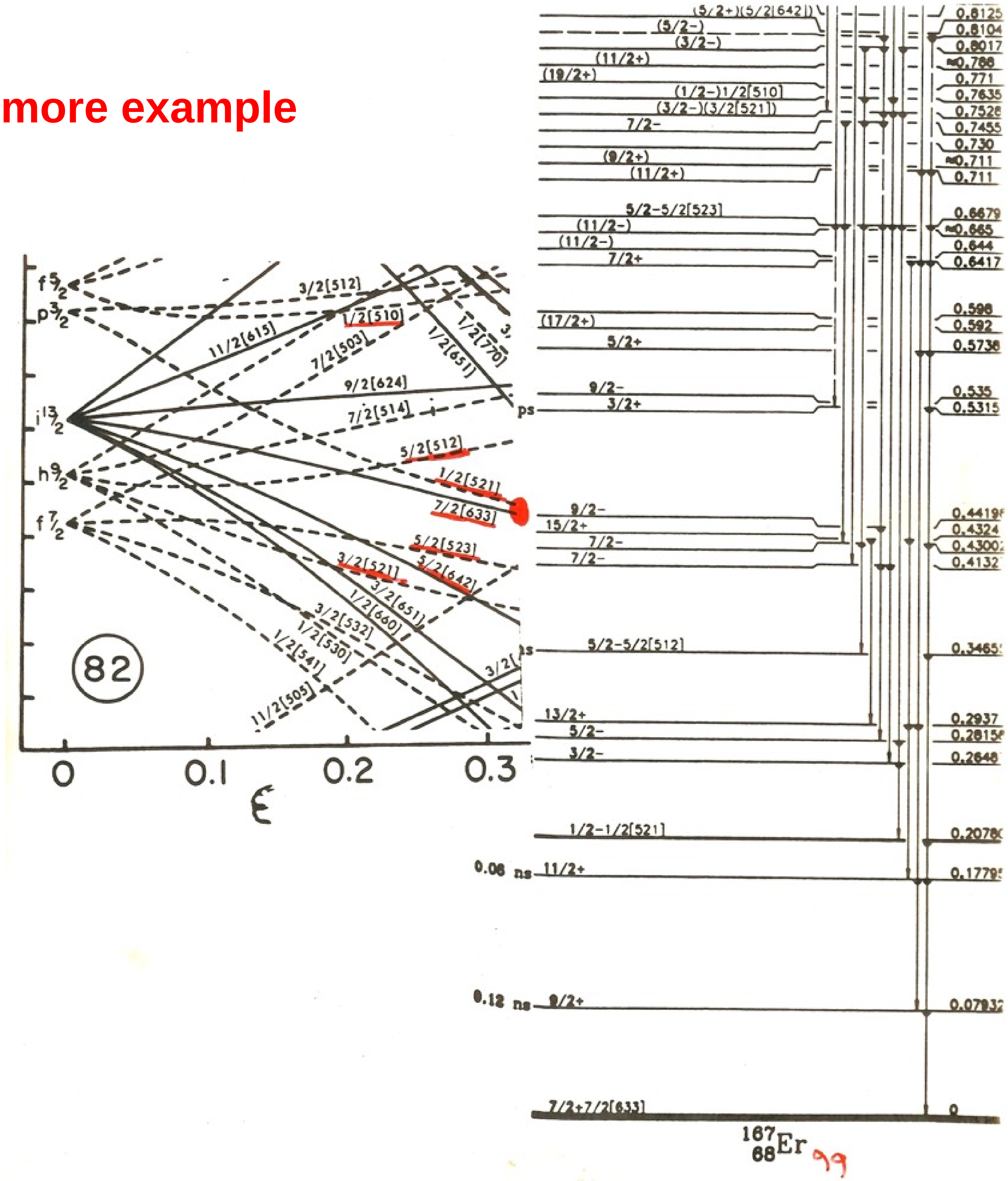
Fermi surface

All correlations and complexity of nucleus absorbed into

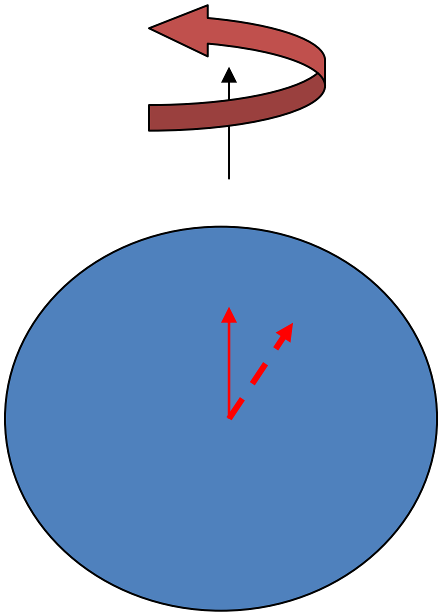
$\Psi_{\text{Nils g.s.}}$ All excitations relative to g.s.

1-quasi-particle excitations

One more example



Coriolis effects in a rotating body (e.g., earth, nucleus)



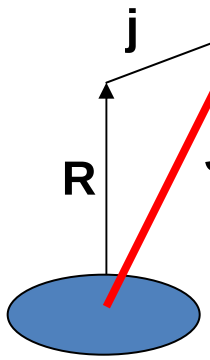
Notice that the Coriolis effect is equivalent to a tilting of the orbit axis

Examples

- Rivers are eroded more on one side than the other
- Astronauts fixing things in space
- Continental drift
- Nuclei --- Yaaaaay !!!


North America
Central America
South America
Europe
Eastern Europe
& North Asia
Africa
East Asia
Australia

Rotational motion in odd-A nuclei and the origin of the Coriolis force



$$\mathbf{H} = \frac{\hbar^2}{2I}\mathbf{R}^2 = \frac{\hbar^2}{2I}(\mathbf{J}-\mathbf{j})^2 = \frac{\hbar^2}{2I}(\mathbf{J}^2+\mathbf{j}^2-2\mathbf{J}\bullet\mathbf{j})$$

$$\mathbf{J}_\pm = \mathbf{J}_1 \pm i\mathbf{J}_2$$

$$\mathbf{j}_\pm = \mathbf{j}_1 \pm i\mathbf{j}_2$$

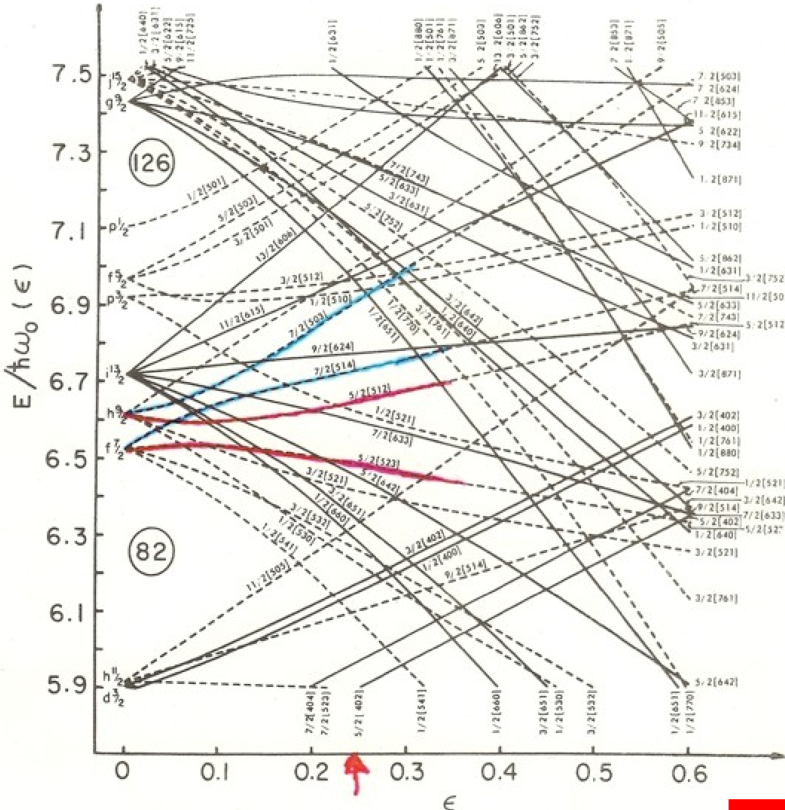
$$\mathbf{J}\bullet\mathbf{j} = \mathbf{J}_1\mathbf{j}_1 + \mathbf{J}_2\mathbf{j}_2 + \mathbf{J}_3\mathbf{j}_3 = \frac{1}{2}(\mathbf{J}_+\mathbf{j}_- + \mathbf{J}_-\mathbf{j}_+) + \mathbf{J}_3\mathbf{j}_3$$

and hence

$$H = \frac{\hbar^2}{2I}\left[\mathbf{J}^2 + \mathbf{j}^2 - 2\mathbf{J}_3\mathbf{j}_3 - (\mathbf{J}_+\mathbf{j}_- + \mathbf{J}_-\mathbf{j}_+)\right]$$

$$E(J) = \frac{\hbar^2}{2I}\left[J(J+1) - 2K^2 + \langle\mathbf{j}^2\rangle - (\mathbf{J}_+\mathbf{j}_- + \mathbf{J}_-\mathbf{j}_+)\right]$$

$$V_{\text{Coriolis}} = -\frac{\hbar^2}{2I}(\mathbf{J}_+\mathbf{j}_- + \mathbf{J}_-\mathbf{j}_+)$$



Can even "guesstimate" wave function $\psi_{Nils} = \sum_j C_j \phi_j$

Table 7.2. Nilsson wave functions ($\boxed{C_j}$ coefficients) for some N = 5 orbits

	j					
$K\pi[Nn_z\Lambda]$	1/2	3/2	5/2	7/2	9/2	11/2
$3/2^-[532]$		0.234	0.369	−0.560	−0.651	0.268
$5/2^-[523]$			0.237	−0.472	−0.826	−0.196
$7/2^-[514]$				0.323	0.938	0.128
$1/2^-[521]$	−0.510	0.345	0.473	0.431	0.444	0.120
$5/2^-[512]$			−0.023	0.836	−0.515	0.157
$1/2^-[510]$	0.021	−0.676	0.586	−0.343	0.277	0.067
$3/2^-[512]$		0.379	0.815	0.283	0.327	0.063
$7/2^-[503]$				0.937	−0.336	0.099
$9/2^-[505]$					0.998	0.071
$1/2^-[501]$	−0.821	−0.361	−0.411	−0.122	−0.104	−0.019

*$\delta = 0.22$, $\kappa = 0.0637$, $\mu = 0.42$.

Some formalities

$$\langle K \,|\mathbf{V}_{\mathrm{Cor}}|\, K+1\rangle = \frac{-\hbar^2}{2I}\sqrt{(J-K)(J+K+1)}\,\langle K\,|\mathbf{j}_-|\,K+1\rangle\,(U_1U_2+V_1V_2)$$

$$\langle K\,|\mathbf{j}_-|\,K+1\rangle = \sum_j C_j^K C_j^{K+1}\sqrt{(j-K)(j+K+1)}$$

Very roughly: $V_{\mathrm{Cor}} \rightarrow \frac{-\hbar^2}{2I} J j$

Increases with angular momentum, J

UPO

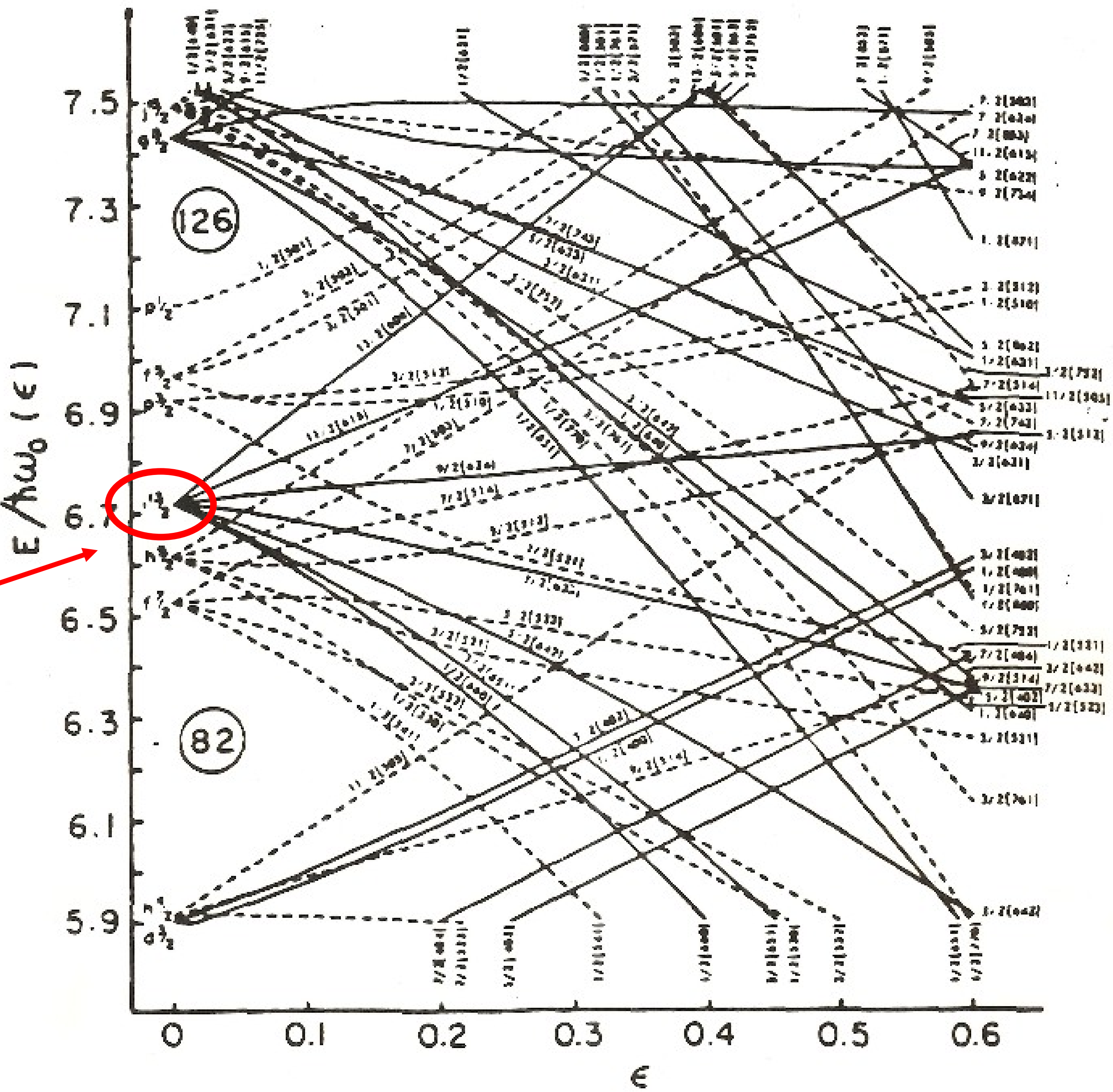


Table 9.3 *Theoretical values* $\langle K | j_- | K+1 \rangle^*$

	$9/2^-[505]$	$7/2^-[503]$	$3/2^-[512]$	$1/2^-[510]$	$5/2^-[512]$	$1/2^-[521]$	$7/2^-[514]$
$9/2^-[505]$		−0.973					2.847
$7/2^-[503]$	−0.973				2.858		
$3/2^-[512]$				0.951	0.045	2.546	
$1/2^-[510]$			0.951			−2.541	
$5/2^-[512]$		2.858	0.045				−1.151
$1/2^-[521]$			2.546	−2.541			
$7/2^-[514]$	2.847				−1.151		

* The Nilsson model parameters are $\delta = 0.2$, $\kappa = 0.0637$, $\mu = 0.42$.

J matrix elements are also typically 6 for the UPOs

Estimate size of $\langle\, |V_{\text{cor}}|\, \rangle$

$$\underbrace{\langle\, |V_{\text{cor}}|\, \rangle = \frac{\hbar^2}{2\mathcal{J}}}\ (J_+\, j_- + J_-\, j_+) \times P$$

only 1 is non-zero
according as $\Delta K \pm 1$

$$= \frac{E(2_1^+)}{6} \sqrt{(J-K)(J+K+1)}\ \langle\, |j_-|\, \rangle \times P$$

$$\sim (15) \times (\sim 2-3) \times \left\{ \begin{array}{ll} \sim N & \text{UPO} \\ \sim 1\text{-}3 & \text{Other} \\ & \Delta n_z = -\Delta\Lambda \\ & = \pm 1 \end{array} \right\}$$

$\lesssim 1$ others

$P \sim U_1 V_1 + U_2 V_2 \sim 0.7$

$$\therefore \langle\, |V_{\text{cor}}|\, \rangle \sim 40 \times 0.7 \times (\text{extra } 0.7) \times (0 \leftrightarrow N)$$

"Cor. red. factor"

$\boxed{\textbf{\sim 150 keV --- few keV}}$

huge effect   minor perturbation

Two effects of Coriolis force

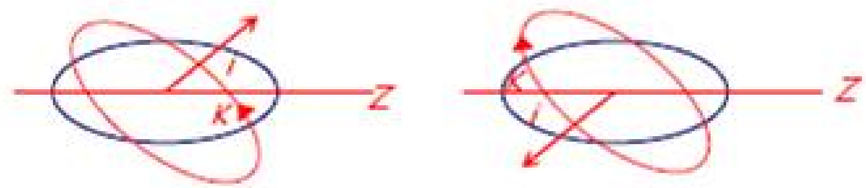
- Diagonal --- energies
- Off-diagonal --- mixing of Nilsson wave functions

Diagonal Coriolis Effects

Recall deformed wave functions

$$\Psi\,(J\,K) = N\,(D_{JMK}\,\chi_K + (-1)^{J-1/2}\,D_{JM-K}\,\chi_{-K})$$

The 2 terms reflect the degeneracy associated with orbits in opposite directions



For $K = ½$

$$\Psi\,(J,\,K = ½) = N\,(D_{JM\,1/2}\,\chi_{1/2} + (-1)^{J-1/2}\,D_{JM-1/2}\,\chi_{-1/2})$$

But V_{cor} has ME that satisfy $\Delta K = 1$

$\Delta K_{K=1/2} = ½ - (-½) = 1$ So, self mixing

This gives an extra, diagonal, term in the energy expression for rotational bands with K = ½.

$$E(J) = \frac{\hbar^2}{2I}\left[J(J+1) + \delta_{K\frac{1}{2}}\, a(-1)^{J+\frac{1}{2}}\left(J+\frac{1}{2}\right)\right]$$

$$a = \sum_j (-1)^{j-\frac{1}{2}}\left(j+\frac{1}{2}\right)C_j^2$$

Note that the effect alternates with spin, leading to a staggering of rotational energies

Decoupling Parameter

Suppose $a = 1$

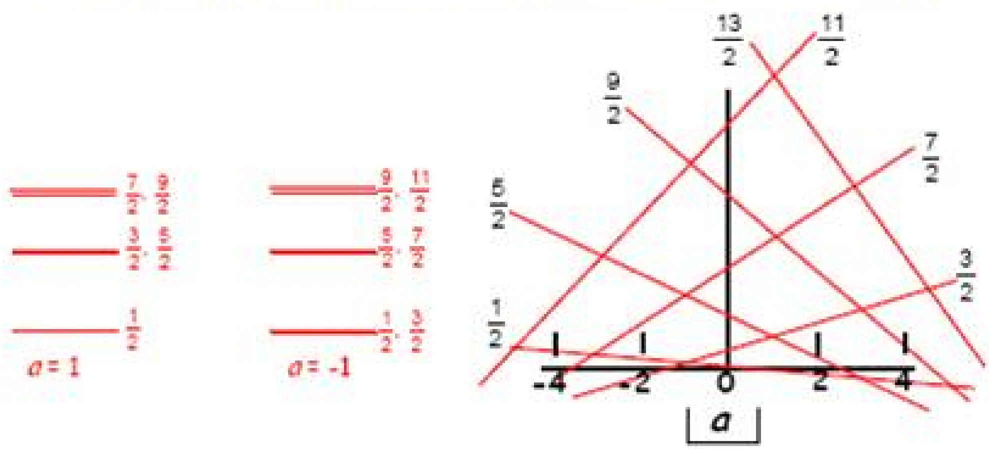
$$E^J_{rot}\left(K=\frac{1}{2}\right)_{a=1} = \frac{\hbar^2}{2\mathscr{I}}\left[J(J+1)+(-1)^{J+\frac{1}{2}}\left(J+\frac{1}{2}\right)\right]$$

Consider $E\left(\frac{3}{2}\right)$, $E\left(\frac{5}{2}\right)$

$$E\left(\frac{3}{2}\right) = \frac{\hbar^2}{2\mathscr{I}}\left[\frac{3}{2}\left(\frac{5}{2}\right)+2\right] = \frac{\hbar^2}{2\mathscr{I}}\left(\frac{23}{4}\right)$$

$$E\left(\frac{5}{2}\right) = \frac{\hbar^2}{2\mathscr{I}}\left[\frac{5}{2}\left(\frac{7}{2}\right)-3\right] = \frac{\hbar^2}{2\mathscr{I}}\left(\frac{23}{4}\right)$$

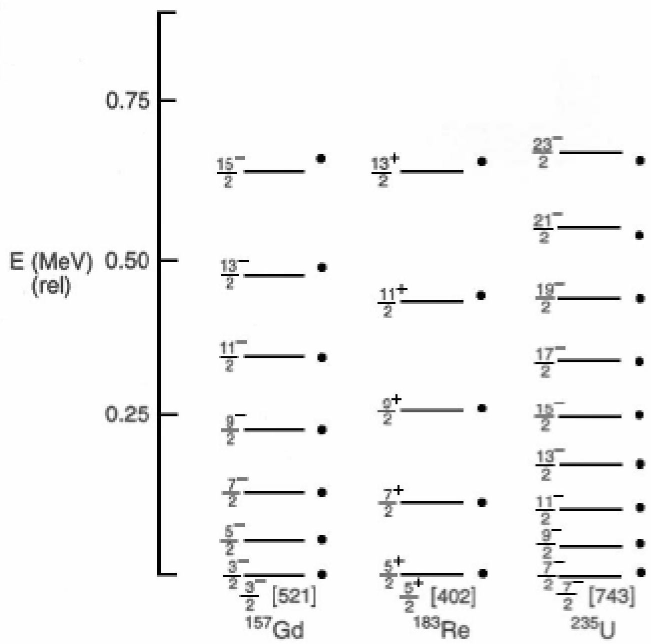
Degenerate. $\therefore$ Diagonal Coriolis force contributes enough to energies to completely upset normal rotational spacings.

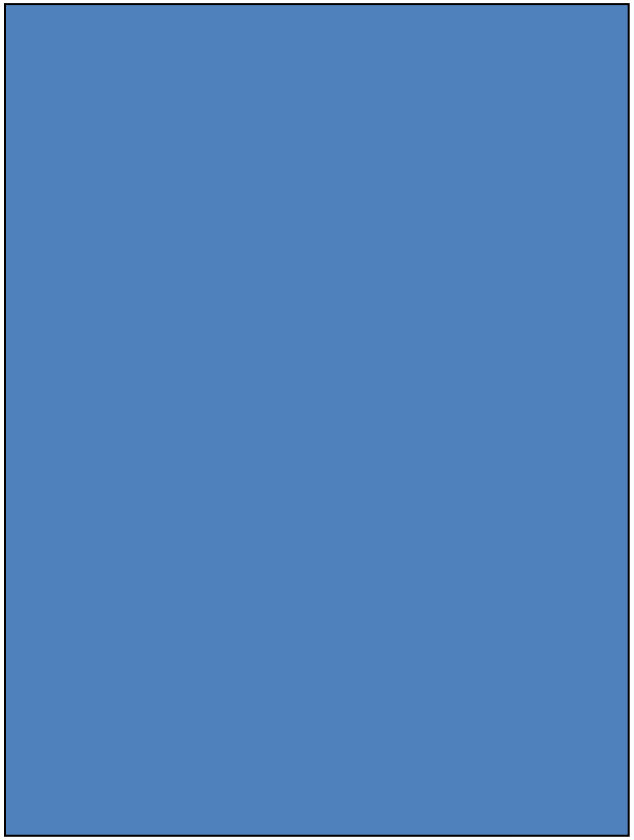


Degeneracies at $a = \pm 1$

For $|a| > 1$, even the order of the states J is not monotonic.

Rotational Formula works well



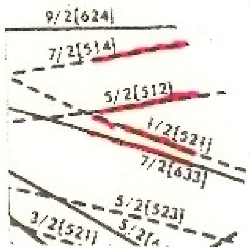


Dots: Rotational Formula

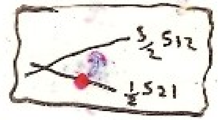
$$E_{\text{Level } J} - E_{\text{Bandhead}} = \frac{\hbar^2}{2\mathcal{I}}\left[J(J+1) - J_0(J_0+1)\right]$$

$\frac{\hbar^2}{2\mathcal{I}}$ fitted from 1st rotational level

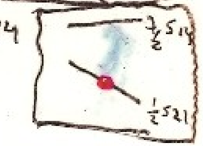
So, except g.s. is $\frac{1}{2}^-521$, low lying $\frac{7}{2}^+633$



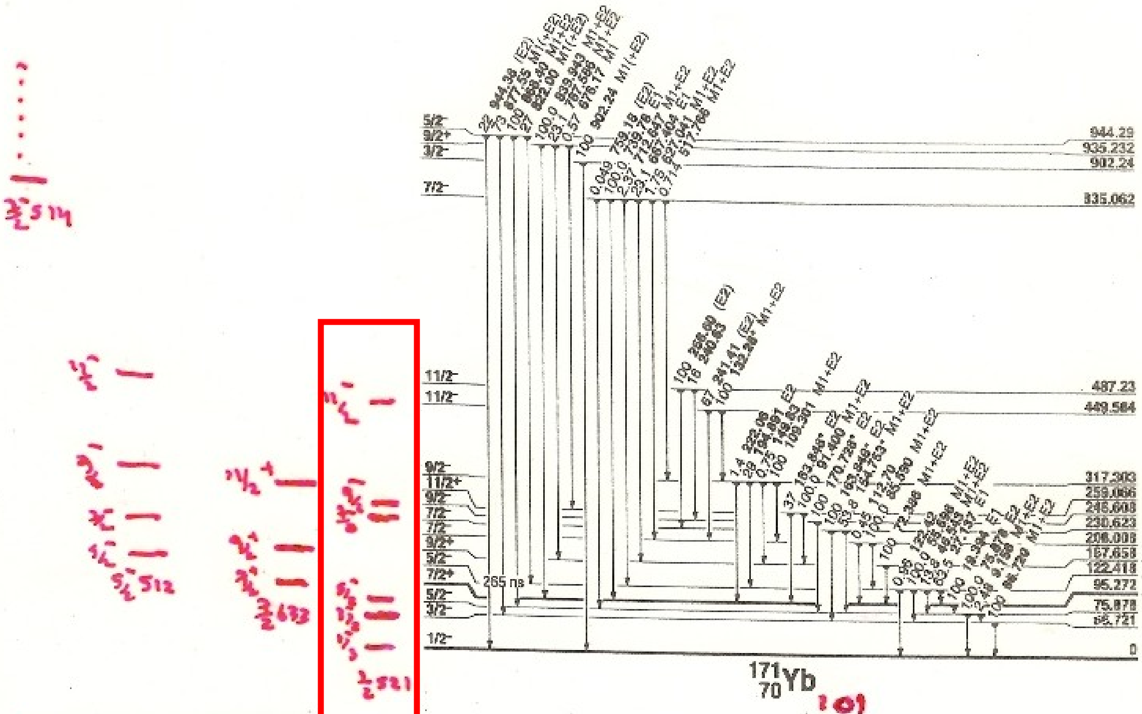
$\frac{5}{2}^-512$



, and $\frac{7}{2}^-514$



excitations



Nilsson intrinsic excitations
→
rotational bands

More on rotation later

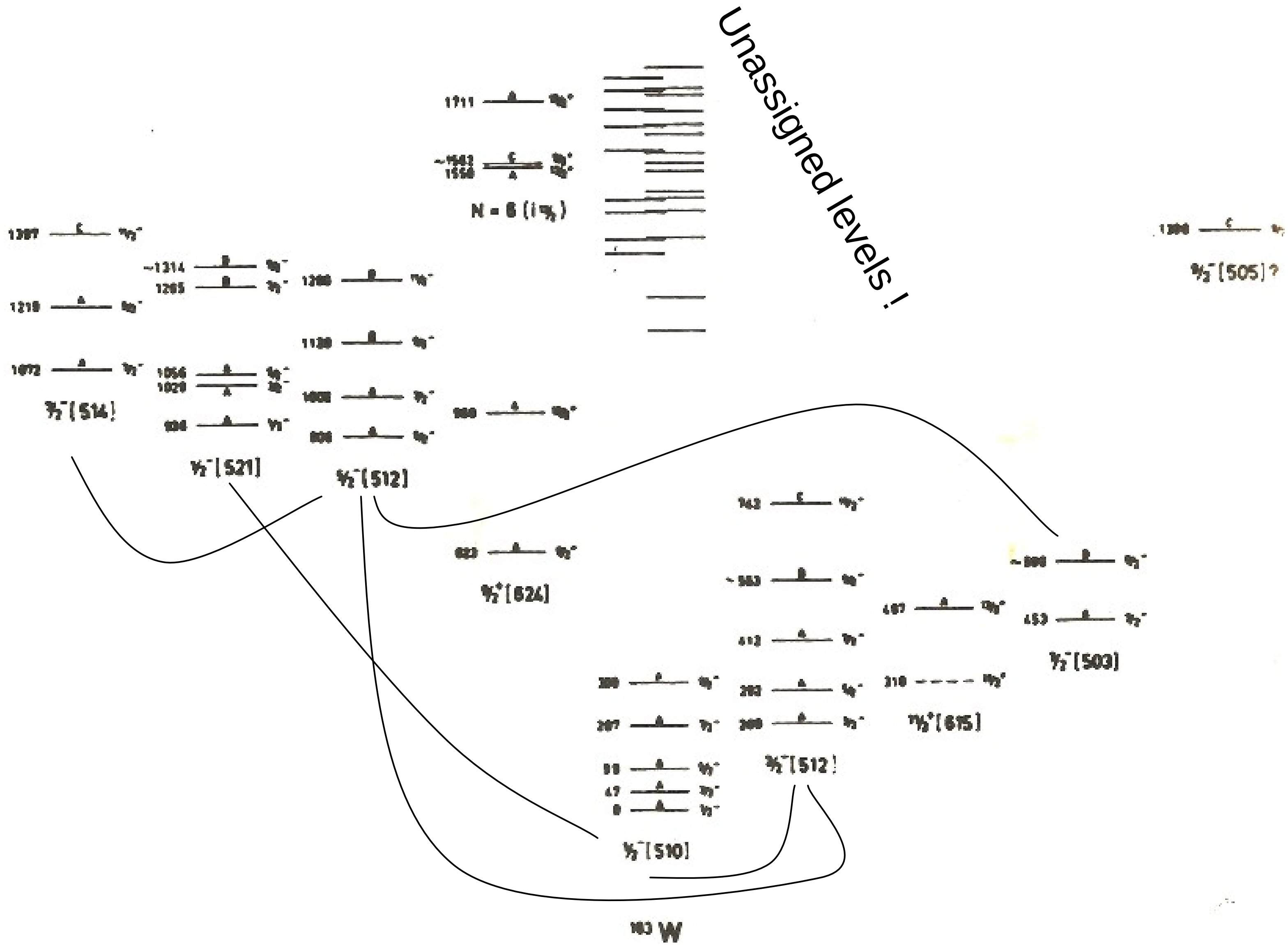
Off diagonal mixing effects of the Coriolis interaction

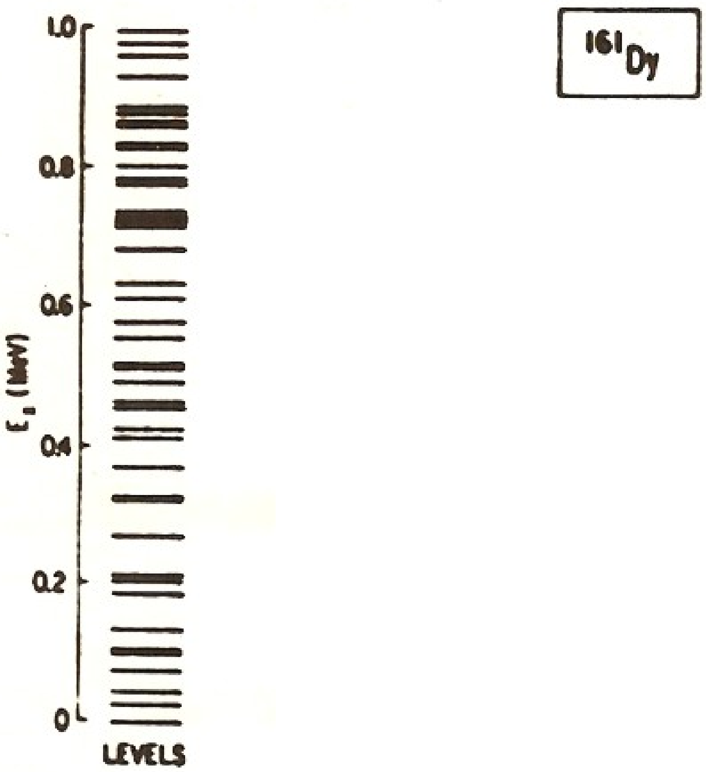
Effect of Coriolis force is a "tilting" of orbit (towards equator)

But—orbit angles are quantized by projection quantum number K

$$\boxed{\textbf{Coriolis} \text{ ---- } \Delta K = \pm 1}$$

183W - Nilsson Assignments





161Dy
1.0
0.8
0.6
E_i (MeV)
0.4
0.2
0
LEVELS

Summary

Nilsson Model: Deformed Ind. Part. Model

Application to nuclei: Nils. + Rotation

$\downarrow$

Coriolis

So: with Nils. + Rotation (including Coriolis) + Pairing

Can account quantitatively for most low-lying levels of most deformed odd-A nuclei

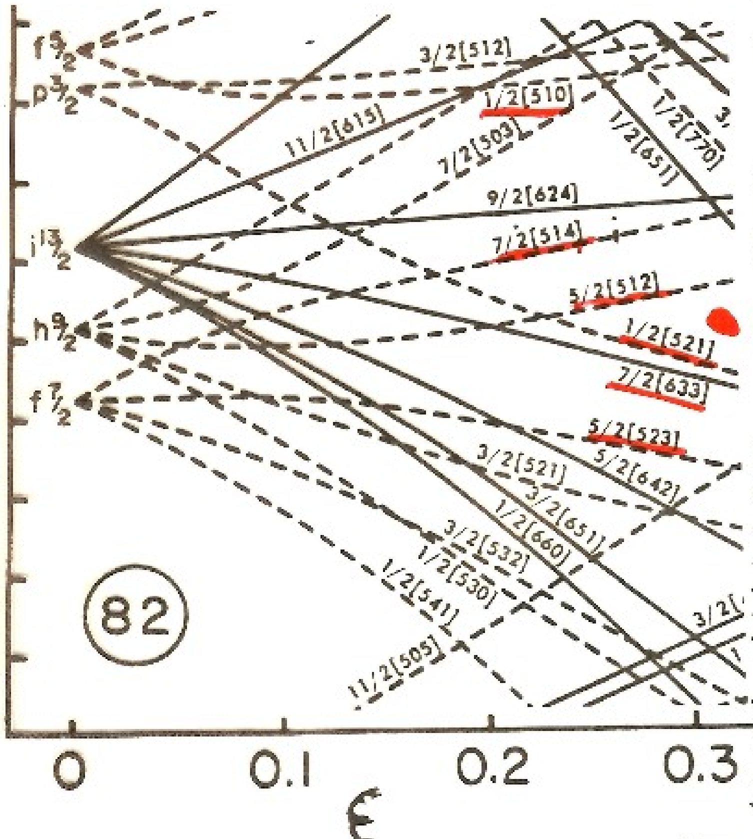
Remarkably simple, successful model

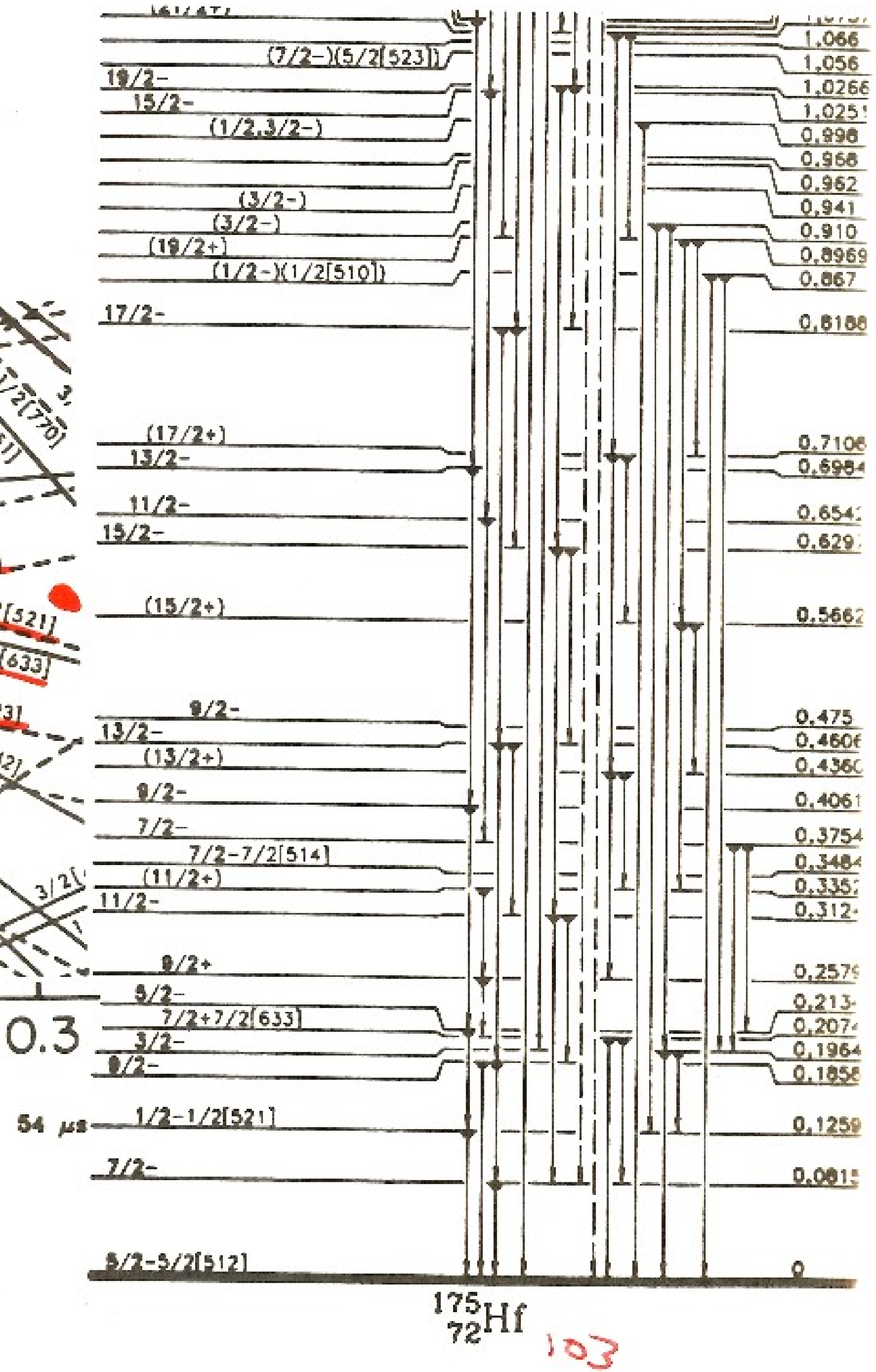
Five Appendices

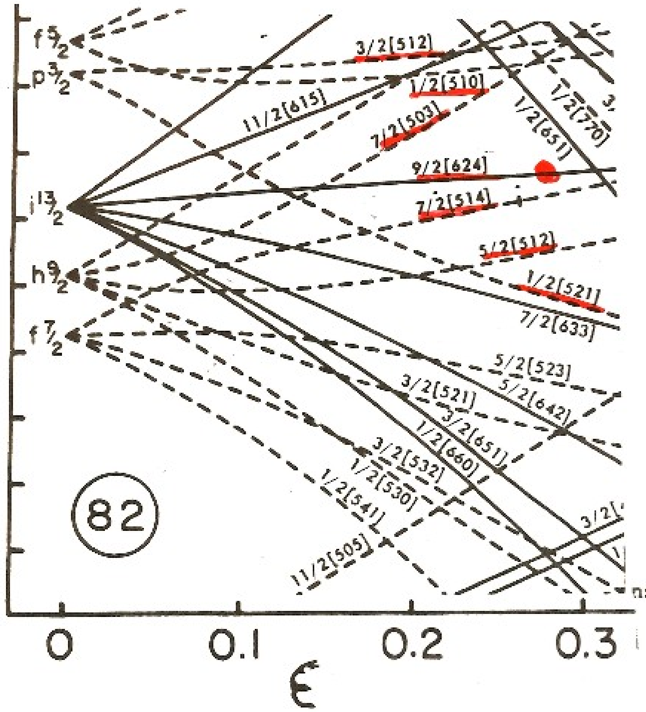
Appendix A

with more examples of how to use the Nilsson Model in practice

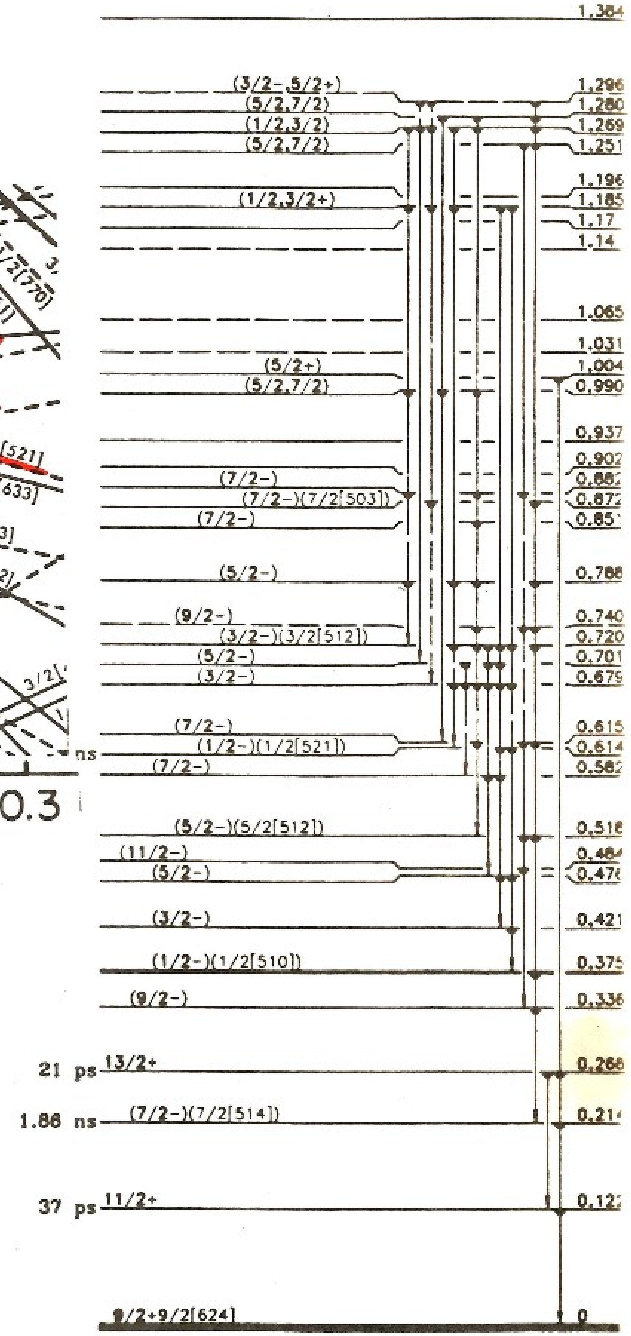
On the next slides are other that you can look through and see if you understand.





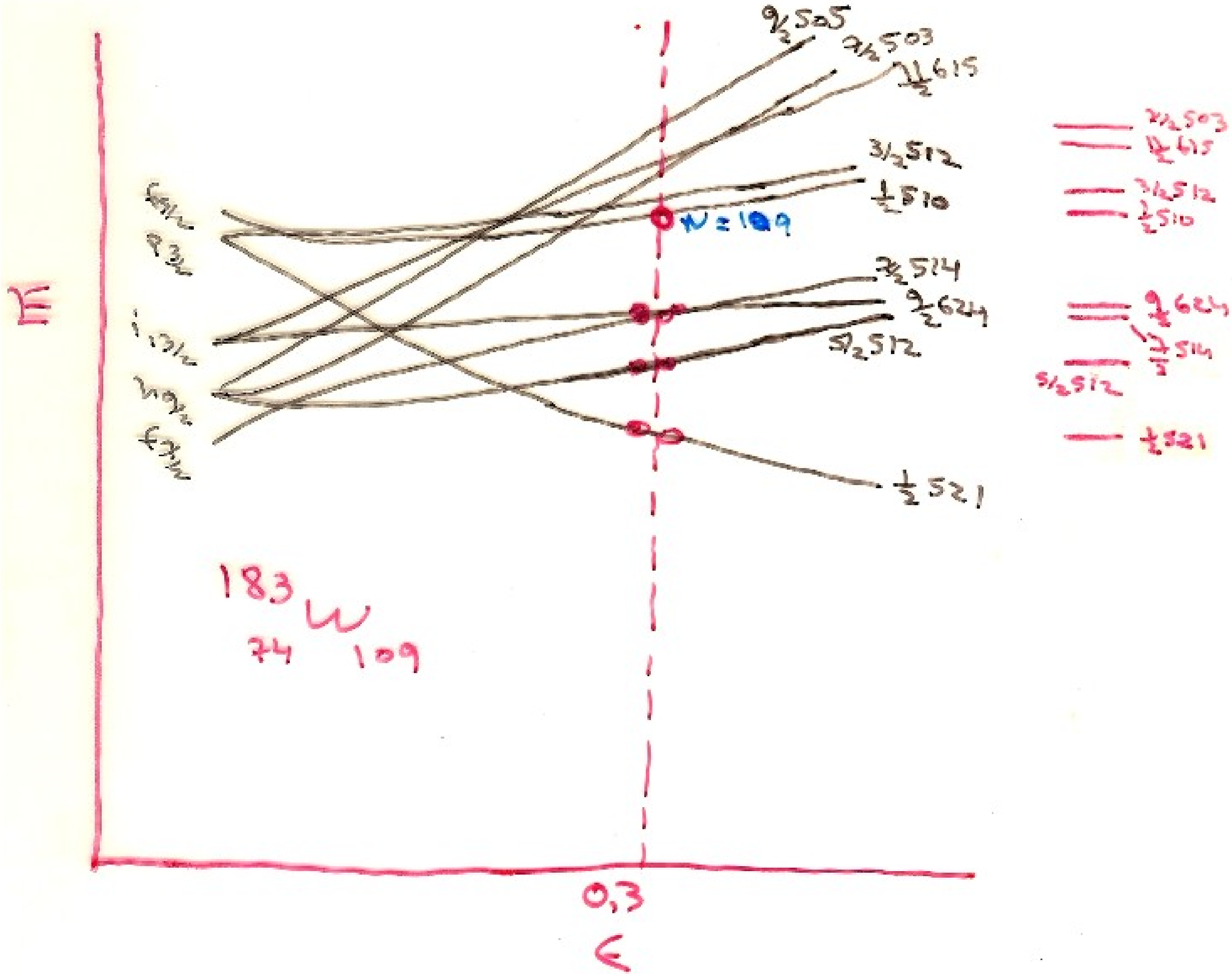


f5/2
p3/2
i13/2
h9/2
f7/2
3/2[512]
1/2[510]
11/2[615]
7/2[503]
9/2[624]
1/2[651]
1/2[770]
7/2[514]
5/2[512]
1/2[521]
7/2[633]
5/2[523]
5/2[642]
3/2[521]
3/2[651]
1/2[660]
3/2[532]
1/2[530]
1/2[541]
11/2[505]
82
0
0.1
0.2
0.3
ε



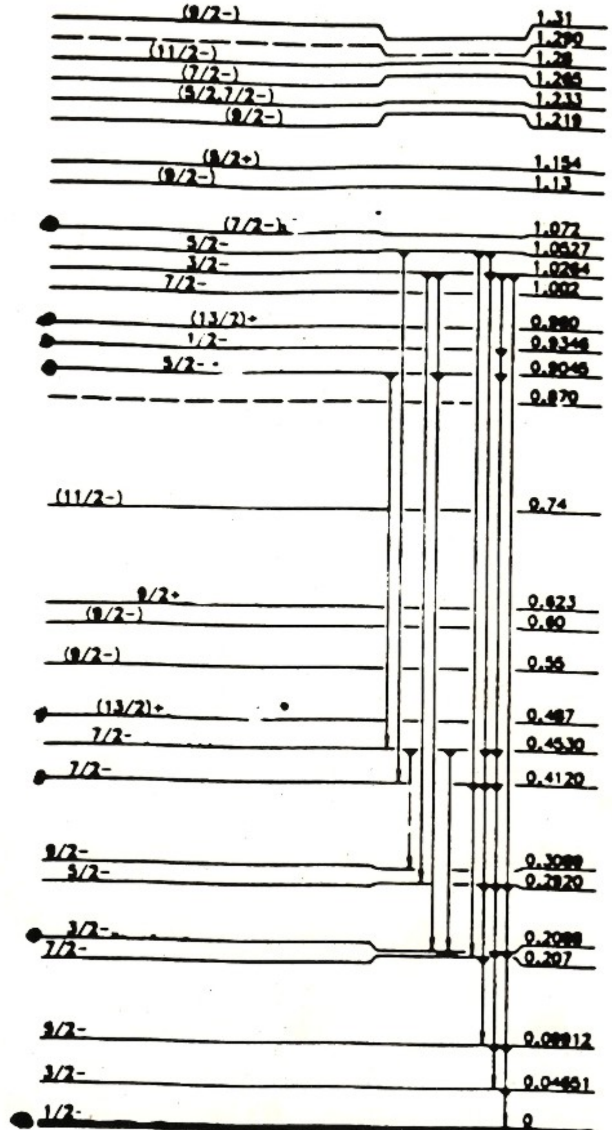
1,384
(3/2-,5/2+) 1,296
(5/2,7/2) 1,280
(1/2,3/2) 1,269
(5/2,7/2) 1,251
1,196
(1/2,3/2+) 1,185
1,17
1,14
1,065
1,031
(5/2+) 1,004
(5/2,7/2) 0,990
0,937
0,902
(7/2-) 0,882
(7/2-)(7/2[503]) 0,872
(7/2-) 0,851
(5/2-) 0,788
(9/2-) 0,740
(3/2-)(3/2[512]) 0,720
(5/2-) 0,701
(3/2-) 0,679
(7/2-) 0,615
(1/2-)(1/2[521]) 0,614
(7/2-) 0,582
(5/2-)(5/2[512]) 0,516
(11/2-) 0,484
(5/2-) 0,478
(3/2-) 0,421
(1/2-)(1/2[510]) 0,375
(9/2-) 0,336
21 ps 13/2+ 0,266
1.86 ns (7/2-)(7/2[514]) 0,214
37 ps 11/2+ 0,122
9/2+9/2[624] 0

$^{179}_{72}$Hf 107



9/2 505
7/2 503
11/2 615
3/2 512
1/2 510
N = 109
7/2 514
9/2 624
5/2 512
1/2 521
7/2 503
11/2 615
3/2 512
1/2 510
9/2 624
7/2 514
5/2 512
1/2 521
E
0,3
ε
183 W 74 109

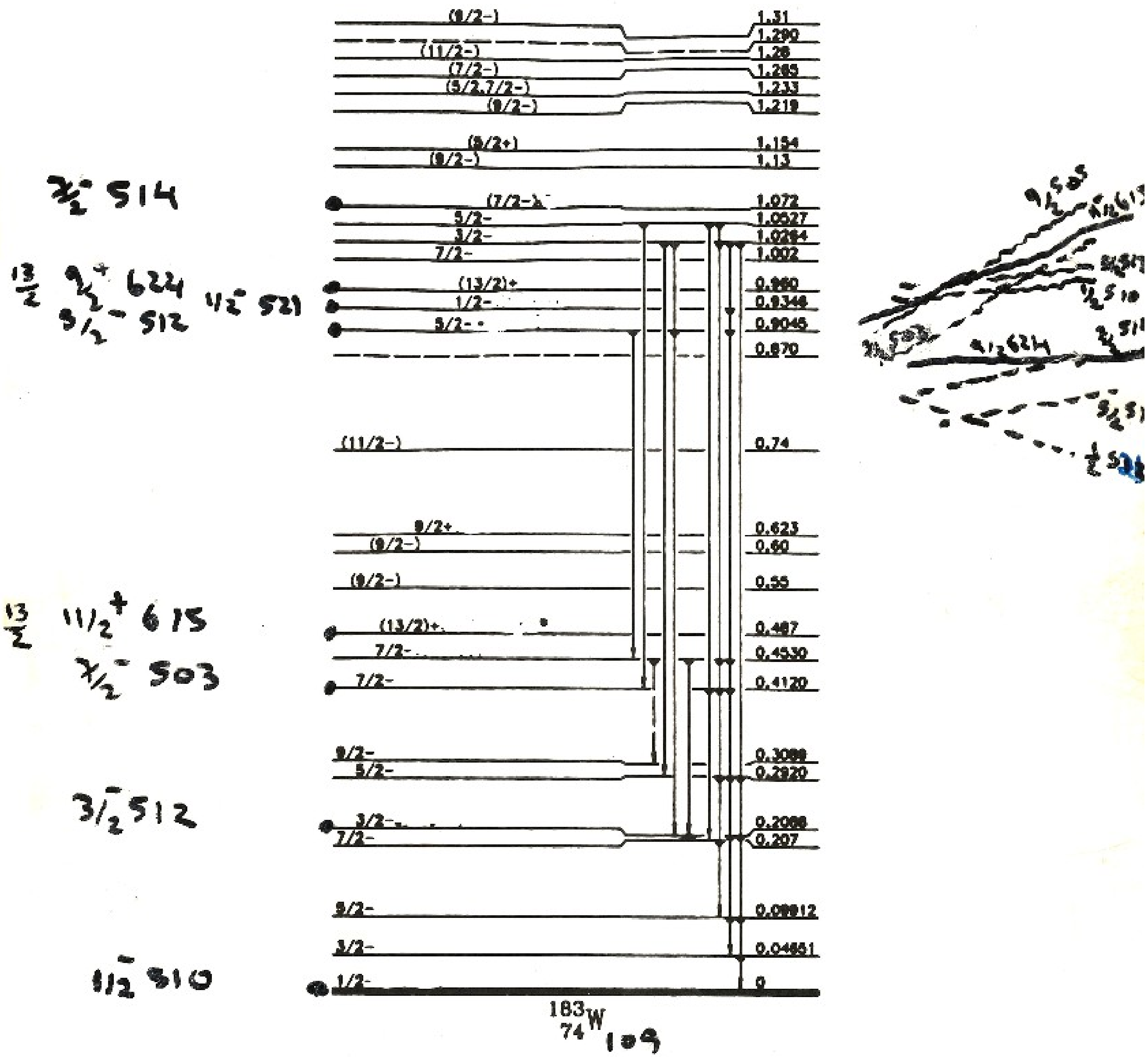
183 W
74 109



$^{183}_{74}W_{109}$

$^{183}_{74}W_{109}$

24 protons, 27 neutrons from (50,82), and non-spherical!



APPENDIX B
on 2-state mixing

Suppose we have a Hamiltonian

$$H = H_0 + H_{resid.}$$

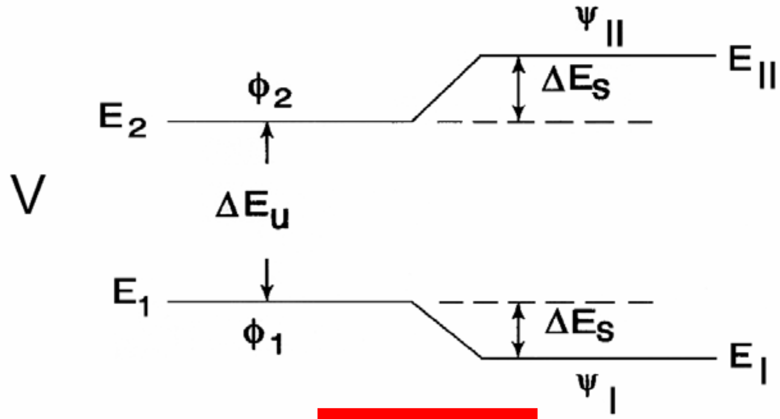
where H_0 has a set of eigenstates X_i

The eigenstates of H will be **mixtures** of those of H_0 – linear combinations of the X_i with coefficients C_i. (In our case, H_0 will be the Hamiltonian of the **SPHERICAL** shell model . $H_{resid.}$ will be the terms that change things because the nucleus is deformed.)

To understand the effects of these terms, we need to briefly discuss a topic of much wider and fundamental importance:

2- state mixing –with a brief excursion to multi-state mixing

TWO-STATE MIXING



$$\boxed{R = \frac{\Delta E_u}{V}}$$

$$\boxed{E_{\mathrm{I,II}} = \frac{1}{2}(E_1 + E_2) \pm \frac{1}{2}\sqrt{(E_2 - E_1)^2 + 4V^2}}$$

$$= \frac{1}{2}(E_1 + E_2) \pm \frac{\Delta E_u}{2}\sqrt{1 + \frac{4V^2}{\Delta E_u^2}}$$

$$= \frac{1}{2}(E_1 + E_2) \pm \frac{\Delta E_u}{2}\sqrt{1 + \frac{4}{R^2}}$$

$$\Delta E_p = E_{\mathrm{II}} - E_{\mathrm{I}} = \Delta E_u\sqrt{1 + \frac{4}{R^2}}$$

$$\frac{\Delta E_p}{\Delta E_u} = \frac{E_{\mathrm{II}} - E_{\mathrm{I}}}{\Delta E_u} = \sqrt{1 + \frac{4}{R^2}}$$

$$|\Delta E_s| = |E_{\mathrm{II}} - E_2| = |E_{\mathrm{I}} - E_1| = \frac{\Delta E_u}{2}\left[\sqrt{1 + \frac{4}{R^2}} - 1\right]$$

$$\frac{|\Delta E_s|}{\Delta E_u} = \frac{|E_{\mathrm{II}} - E_2|}{\Delta E_u} = \frac{|E_{\mathrm{I}} - E_1|}{\Delta E_u} = \frac{1}{2}\left[\sqrt{1 + \frac{4}{R^2}} - 1\right]$$

This is the main result we will need but I include a more general discussion because the topic is of such fundamental importance. I will skip several of the next slides but I URGE you to study them.

The mixed wave functions are

$$\psi_I = \alpha\phi_1 + \beta\phi_2$$

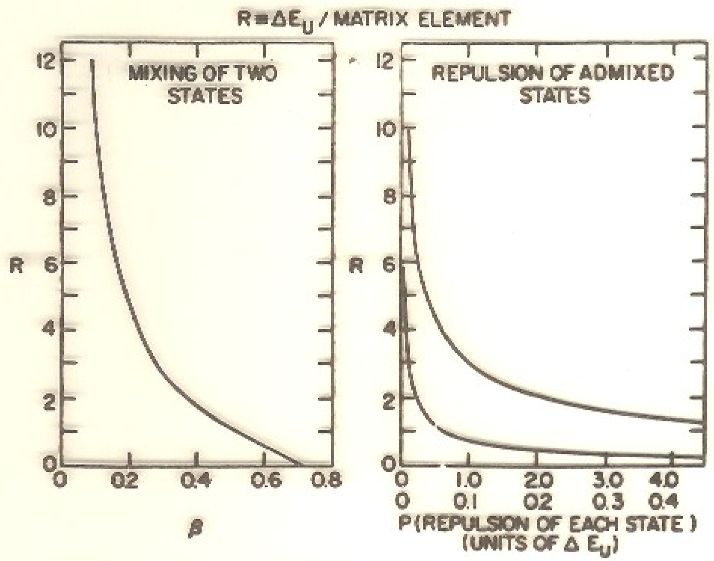
$$\alpha^2 + \beta^2 = 1$$

$$\psi_{II} = -\beta\phi_1 + \alpha\phi_2$$

Depends only on R

where the smaller amplitude β is given by

$$\beta = \frac{1}{\left\{1 + \left[R/2 + \sqrt{1 + R^2/4}\right]^2\right\}^{\frac{1}{2}}}$$



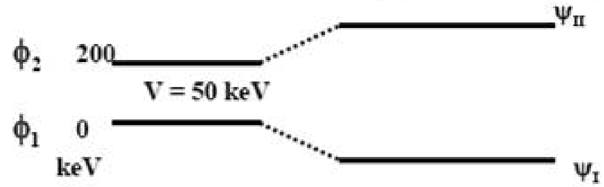
Universal two-state mixing curves. The one on the left gives the smaller of the two mixing amplitudes, β, while the curves on the right give the energy shift of each level in units of the unperturbed energy separation. Here the lower curve goes with the upper abscissa scale, while the upper curve goes with the lower scale.

Examples of two-state mixing energy shifts and mixing amplitudes

R	$\Delta E_s/\Delta E_u$	β	Specific case: $\Delta E_u = 100$ keV V (keV)	ΔE_s (keV)
0.2	4.52	0.67	500	452
0.5	1.56	0.61	200	156
1	0.62	0.53	100	62
2	0.207	0.38	50	20.7
3	0.101	0.29	33.3	10.1
5	0.0385	0.19	20	3.85
10	0.0099	0.099	10	0.99
20	0.0025	0.050	5	0.25

*For $R = 0$, $\beta = 0.707$, and $\Delta E_s = V$.

Example of 2-state Mixing



R = 4

$$\frac{|\Delta E_s|}{\Delta E_u} = \frac{1}{2}\left[\sqrt{1+\frac{4}{R^2}} - 1\right]$$

$$= \frac{1}{2}\left[\sqrt{1+\frac{4}{16}} - 1\right] = \frac{1}{2}\left[\sqrt{1.25} - 1\right]$$

$$= 0.059$$

$$\therefore \quad \Delta E_s = 0.059 \times 200 = 11.8\ \text{keV}$$

$$\psi_I = \gamma\phi_1 + \beta\phi_2$$

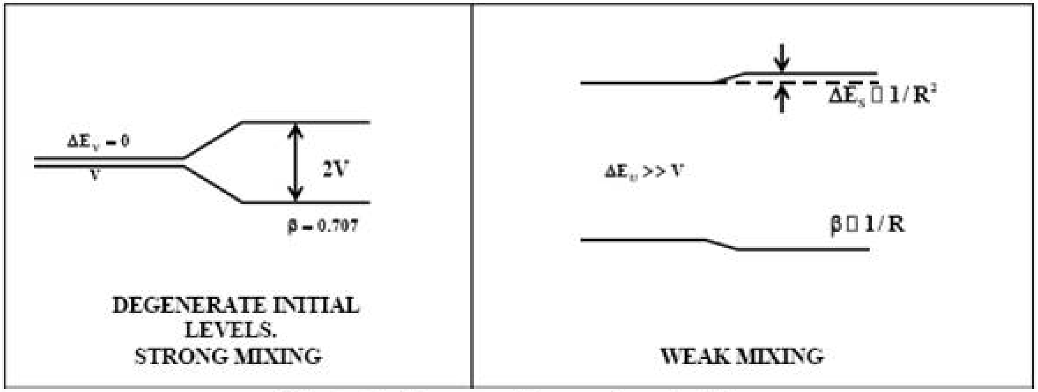
$$\beta = \frac{1}{\left[1 + \left\{\frac{R}{2} + \sqrt{1+\frac{R^2}{4}}\right\}^2\right]^{\frac{1}{2}}}$$

$$= \frac{1}{\left[1+\{2+2.24\}^2\right]^{\frac{1}{2}}} = \frac{1}{\left[1+18\right]^{\frac{1}{2}}} = \frac{1}{\sqrt{19}} = 0.23$$

$$\beta = 0.23$$

$$\therefore \quad \psi = 0.97\phi_1 + 0.23\phi_2$$

Limiting Cases



The two limiting cases of strong and weak mixing

$\boxed{\text{Strong}}$ Mixing (Degenerate levels) $\quad E_1 = E_2 = E_0$

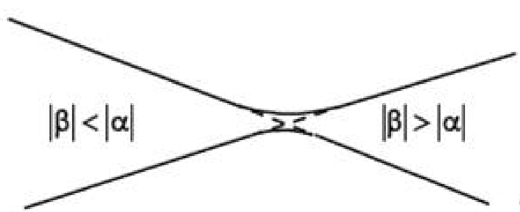
$R \to 0$

$$E_{I,II} = \frac{1}{2}[(E_1 + E_2) \pm 2V] = E_0 \pm V$$

$\rightarrow \quad E_{II} - E_I = 2V$ $\quad$ **Two levels can never be closer than <u>twice the mixing matrix ele.</u>**

$\rightarrow$ **Also** $\quad \beta \rightarrow \frac{1}{\sqrt{2}} = 0.707$ $\quad$ **<u>Complete mixing</u>**

Level Crossing



$\psi_I = \alpha\phi_1 + \beta\phi_2$

$\psi_{II} = -\beta\phi_1 + \alpha\phi_2$

$\boxed{\text{Weak}}$ Mixing: $R \gg 1$

$$\beta = \frac{1}{\left[1+\left\{\frac{R}{2}+\sqrt{1+\frac{R^2}{4}}\right\}^2\right]^{\frac{1}{2}}} = \frac{1}{\left[1+\left\{\frac{R}{2}+\frac{R}{2}\right\}^2\right]^{\frac{1}{2}}} = \frac{1}{\left[1+R^2\right]^{\frac{1}{2}}}$$

$$\boxed{\beta = \frac{1}{R}}$$

$$V = \beta \Delta E_u = \beta \Delta E_{final}$$

Also: $$\frac{|\Delta E_s|}{\Delta E_u} = \frac{1}{2}\left[\sqrt{1+\frac{4}{R^2}}-1\right]$$

or $$\frac{|\Delta E_s|}{\Delta E_u} = \frac{1}{2}\left[1+\frac{2}{R^2}-1\right] = \frac{1}{R^2} = \beta^2$$

$\therefore$ $$|\Delta E_s| = \frac{\Delta E_u}{R^2}$$

An example is useful. Suppose R=10.

$\beta = 0.1$ and $\Delta E_s / \Delta E_u = 0.01$

The exact results are $\beta = 0.0985$ and $\Delta E_s / \Delta E_u = 0.0099$

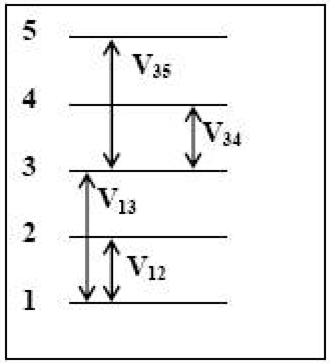
This is a good approximation even for R as small as 3.

$\boxed{R = 3}$ Above approximation:

$\beta = 0.33$ $\quad |\Delta E_s| / \Delta E_u = \frac{1}{9} = 0.11$

Exact values: $\beta = 0.29$ $\quad |\Delta E_s| / \Delta E_u = 0.101$

Multi-State Mixing



Need to diagonalize

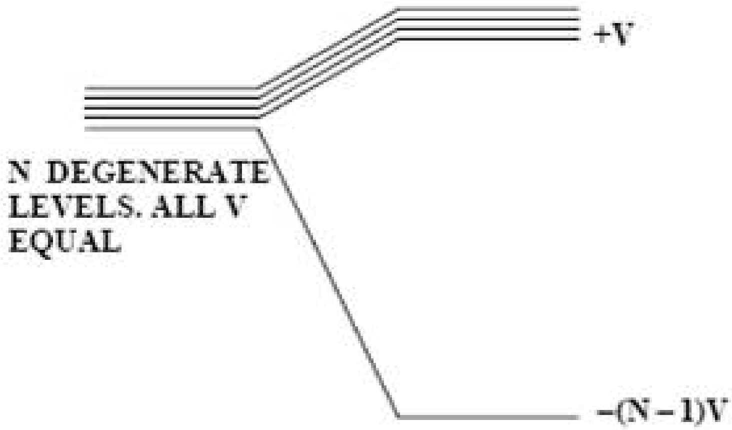
$$
\begin{array}{c|ccccc|}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & E_1 & V_{12} & V_{13} & 0 & 0 \\
2 & V_{12} & E_2 & 0 & 0 & 0 \\
3 & V_{13} & 0 & E_3 & V_{34} & V_{35} \\
4 & 0 & 0 & V_{34} & E_4 & 0 \\
5 & 0 & 0 & V_{35} & 0 & E_5
\end{array}
$$

$$\psi_k = \sum_i \alpha_i \varphi_i$$

Details depend on V_{ij}'s, E_i's

Important special case: degenerate levels

Special case of the utmost importance. Mixing of degenerate levels



$$\Psi_{LOWEST} = \frac{1}{\sqrt{N}}[\phi_1 + \phi_2 + \ldots + \phi_N]$$

This is the origin of collectivity in nuclei. Essential also for understanding masses

Not much is more important than this idea.

Please remember it and think about it often (and try to come to love it).

Appendix C

on one of the mysterious (i.e., almost unknown) aspects of rotational motion in odd-A deformed nuclei

Transformation to Lab system

(*i.e.*, to what's measurable)

Note interesting aspect of Nilsson Ψ's.

$$\Psi = \sum_j C_j \varphi_j$$

$\Rightarrow$ j is not a good quantum # ! !

But angular momentum must be conserved

$\Rightarrow$ There must be another compensating angular momentum in systems

Answer: Rotational angular momentum

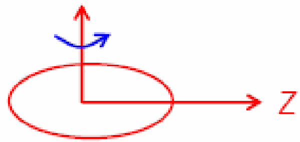
$$\bar{J} = \bar{j} + \bar{R}$$

When transformed to Lab system, a nucleus with last odd particle in a given Nilsson orbit can rotate

$$H_{\text{Lab}} = \frac{\hbar^2}{2I}\bar{R}^2$$

Rotational Motion in Deformed Nuclei

Consider



States with projections K and $-K$ will be degenerate. The nuclear wave function must reflect this and thus one has the symmetrized product form for wave functions in rotational nuclei:

$$\psi_{JM} = \left(\frac{2J+1}{16\pi^2}\right)^{\frac{1}{2}} \left[\underbrace{\overset{\text{Rotation}}{D_{JMK}}\,\overset{\text{Intrinsic}}{\chi_K}}_{K} + (-1)^{J-K} \underbrace{D_{JM-K}\,\chi_{-K}}_{-K} \right]$$

Note that for $K = 0$, only even J values are allowed, so the wave function collapses to a single term

$$\psi_{JM} = \left(\frac{2J+1}{8\pi^2}\right)^{\frac{1}{2}} D_{J0}\,\chi_0$$

With these wave functions in hand, we consider axially symmetric nuclei in which the rotation has equal frequencies around the x or y axes. The rotational Hamiltonian is simply

$$H = \frac{\hbar^2}{2\mathcal{I}} R^2$$

where $\mathcal{I}$ is the moment of inertia and R is the rotational angular momentum operator. If we assume that the ground state is $J^\pi = 0^+$, $K = 0$, and if all the angular momentum can be ascribed to rotation (as is normally true for the low-lying, low-spin, positive parity states in deformed even-even nuclei) then the total angular momentum $J = R$ and we obtain the famous symmetric top rotational energy expression

$$E_{\text{rot}}(J) = \frac{\hbar^2}{2\mathcal{I}} J(J+1)$$

where only even J are allowed.

Odd A Nuclei

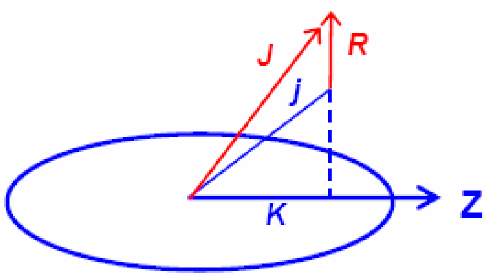
How do we add rotation?

$$H_{\text{Lab}} = H_{\text{Nils}} + H_{\text{Rot}} = H_{\text{Nils}} + \underbrace{\frac{\hbar^2}{2\mathcal{I}} R(R+1)}_{\substack{\text{gives rotational} \\ \text{energies relative} \\ \text{to intrinsic state.}}}$$

Intrinsic state has K

$\therefore\ J_{\text{min}} = K$

$\therefore\ J = K, K+1, K+2, K+3, \ldots$

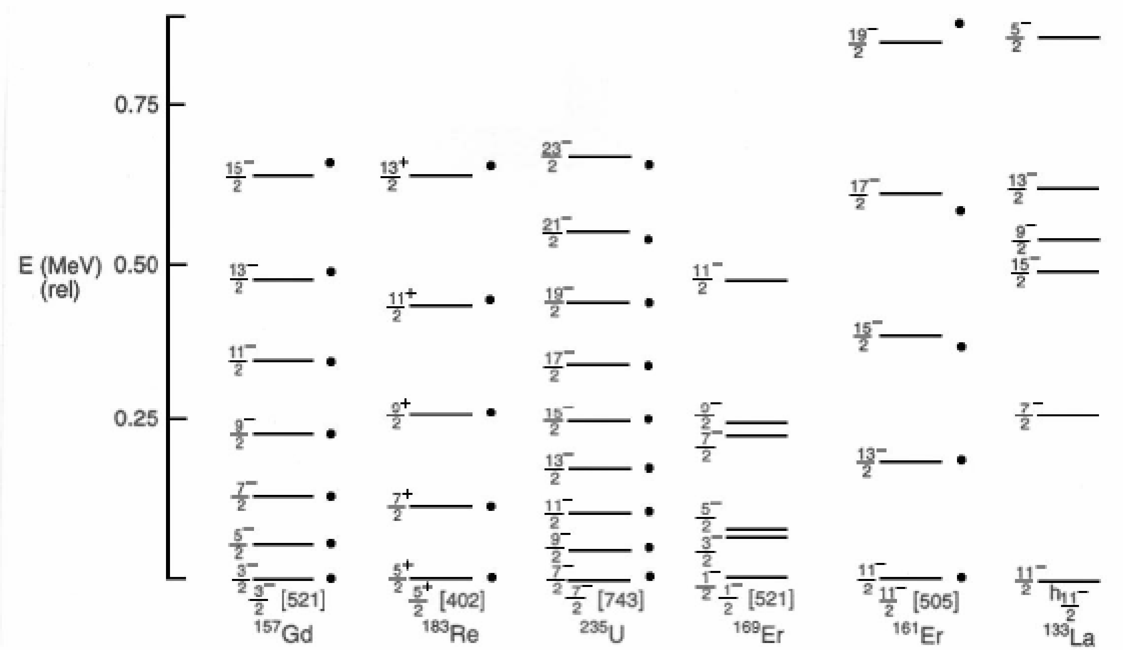


$$\bar{J} = \bar{j} + \bar{R}$$

$\therefore$ Usually write:

$$H_{\text{Lab}} = H_{\text{Nils}} + \underbrace{\frac{\hbar^2}{2\mathcal{I}} \left[J(J+1) - K(K+1) \right]}_{\substack{\text{give rotational “bands”} \\ \text{built on each intrinsic state}}}$$

Rotational Formula works well



Dots: Rotational Formula

$$E_{\text{Level } J} - E_{\text{Bandhead}} = \frac{\hbar^2}{2\mathcal{I}}\left[J\left(J+1\right) - J_0\left(J_0+1\right)\right]$$

$\frac{\hbar^2}{2\mathcal{I}}$ fitted from 1st rotational level

Trouble on the horizon.
Rotational formula doesn't explain all bands.

Dots: Rotational Formula

$$E_{\text{Level } J} - E_{\text{Bandhead}} = \frac{\hbar^2}{2\mathcal{I}}\left[J(J+1) - J_0(J_0+1)\right]$$

$\frac{\hbar^2}{2\mathcal{I}}$ fitted from 1st rotational level

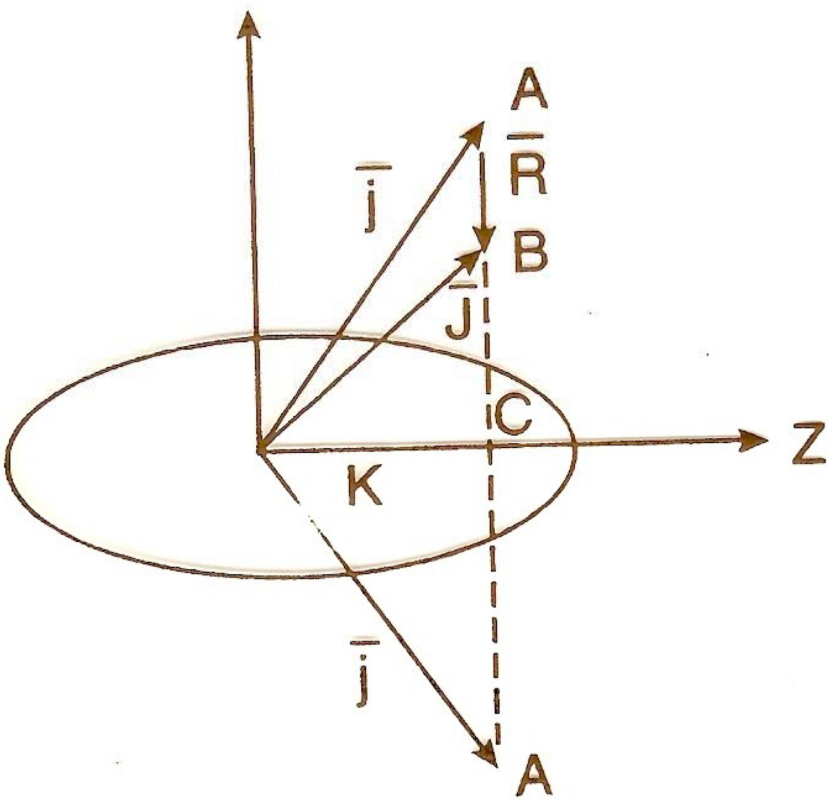
Return to this shortly

These disagreements must tell us something. They do.

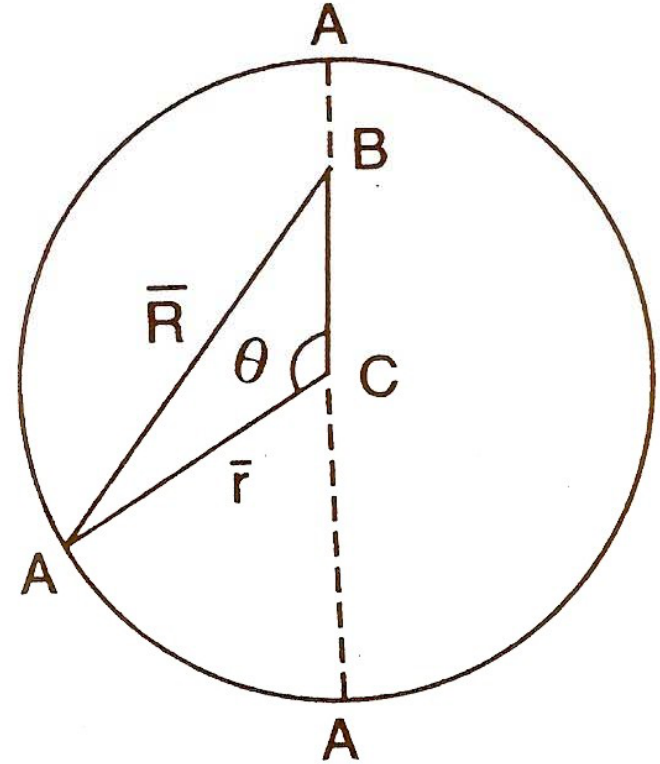
½ 521 – effect of Coriolis force

$h_{11/2}$ – ditto, plus a new coupling scheme

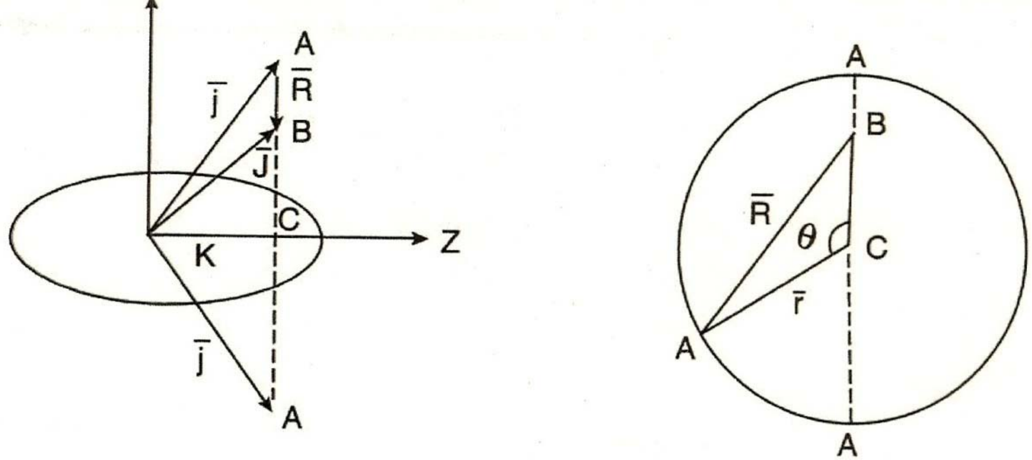
New ideas about rotation



A
$\bar{R}$
$\bar{j}$
B
$\bar{J}$
C
Z
K
$\bar{j}$
A



A
B
$\bar{R}$
θ
C
$\bar{r}$
A
A



$$\mathbf{R}^2 = r^2 + (BC)^2 + 2r(BC)\cos\theta$$

Integrating this expression over θ gives

$$R^2_{\text{ave}} = r^2 + (BC)^2$$

but

$$r^2 = (AC)^2 = j^2 - K^2$$

and

$$(BC)^2 = J^2 - K^2$$

Thus

$$R^2_{\text{ave}} = j^2 - K^2 + J^2 - K^2$$

or, taking the quantum mechanical expectation values

$$R(R+1) = J(J+1) + j(j+1) - 2K^2$$

Appendix D

on a more formal derivation of the Nilsson Model with emphasis on the small and large deformation limits and quantum numbers

Formal Treatment of the Deformed Shell Model—Nilsson Model

$$H = T + V$$

$$V = \underbrace{V_0(r)}_{\text{sph.}} + \underbrace{V_2(r)}_{\text{Quad. shape}} P_2(\cos\theta)$$

$$\therefore \quad H = T + V = \frac{P^2}{2m} + \underbrace{\frac{1}{2}m\omega r^2}_{x,\,y,\,z \text{ no longer equivalent}} + Cl \bullet s + Dl^2$$

$$\therefore \quad H = \frac{P^2}{2m} + \frac{1}{2}m\left[\omega_x \underbrace{(x^2 + y^2)}_{\text{assume axial symmetry}} + \omega_z Z^2\right] + Cl \bullet s + Dl^2 \quad \text{A}$$

Useful to look at this in the

$$\left.\begin{array}{c}\text{large}\\ \text{and}\\ \text{small}\end{array}\right\} \text{deformation limits}$$

Nilsson Quantum Numbers

K^{π} $[\,N n_z\,\Lambda\,]$

N – prin. q. #

n_z – # nodes in Z direction
(extension of Ψ in Z)

Λ – projection of l in Z axis

Σ – projection of l_z in Z axis

$K = \Lambda + \Sigma$ – projection of total ang. momentum on Z

$n_z + \Lambda$ even or odd as N is even or odd

Assignment of q. #'s

Lowest orbit is most aligned with bulk of nucleus—*i.e.*,
most extended in Z direction, hence largest n_z
(also smallest K as we saw)

For smaller def., useful to write

$$\omega_x^2 = \omega_y^2 = \omega_0^2\left(1 + \frac{2}{3}\delta\right)$$

$$\omega_z^2 = \omega_0^2\left(1 - \frac{4}{3}\delta\right)$$

(ω_z decreases for pos. δ -- larger circumference)

Volume condition

$$\omega_0 = \left(1 - \frac{4}{3}\delta^2 - \frac{16}{27}\delta^3\right)^{-\frac{1}{6}} = \text{constant}$$

Rewriting A in terms of δ

$$H = \frac{P^2}{2m} + \frac{1}{2}m\omega_0^2 r^2 - \underbrace{m\omega_0^2 r^2 \delta \frac{4}{3}\sqrt{\frac{\pi}{5}}\, Y_{20}\left(\theta,\phi\right)} + C\, l \bullet s + D\, l^2$$

Isotropic osc. (with $l.s.$ and l^2) and $\delta r Y_{20}$ pert.

$\kappa = C/2\hbar\omega_0$ and $\mu = 2D/C$.

κ typically takes on values around 0.06 and μ varies from 0 to $\approx$ 0.7.

$\delta r Y_{20}$ splits S.M. E's

$$\Delta E\left(N l j m\right)=-\frac{4}{3}\sqrt{\frac{\pi}{5}}\, m\omega_0^2\,\delta\left\langle N l j m\left|r^2 Y_{20}(\theta,\phi)\right|N l j m\right\rangle$$

Separate radial, angular parts as usual.

$$\frac{1}{2}m\omega_0^2\left\langle N l j m\left|r^2\right|N l j m\right\rangle=\frac{1}{2}\hbar\omega_0\left(N+\frac{3}{2}\right)$$

Using ME's of Y_{20} gives

$$\boxed{\Delta E\left(N l j m\right)=-\frac{2}{3}\hbar\omega_0\left(N+\frac{3}{2}\right)\delta\frac{\left[3K^2-j(j+1)\right]\left[\frac{3}{4}-j(j+1)\right]}{(2j+1)j(j+1)(2j+3)}}$$ B

- There is a proportionality to δ, the quadrupole deformation.
- The shifts display a dependence on K^2. Lowest K lowest
- They depend linearly on the oscillator quantum number N.

B negative if $K<\sqrt{\frac{j(j+1)}{3}}\approx\frac{j}{1.8}=0.65\,j$

e.g. $j=\frac{13}{2}$ neg. for $j=\frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2}$

pos. for $j=\frac{9}{2},\frac{11}{2},\frac{13}{2}$

Large Deformation Limit

$H = T + V$

$$= \frac{P^2}{2m} + \frac{1}{2}m\left[\omega_x\left(x^2 + y^2\right) + \omega_z z^2\right] + Cl \bullet s + Dl^2$$

l^2, $l.s.$ negligible

Anisotropic Oscillator—separation of motion:

$(n_x + n_y)$ and n_z are good quantum numbers.

$$\therefore \quad E\left(n_x, n_y, n_z\right) = \hbar\,\omega_x\left(N - n_z + 1\right) + \hbar\,\omega_z\left(n_z + \frac{1}{2}\right) \qquad \text{C}$$

Note: Still have $E \propto \delta \; (\omega \propto \delta)$

E's separate according to distribution of motion in x-y vs. z

H is independent of ϕ

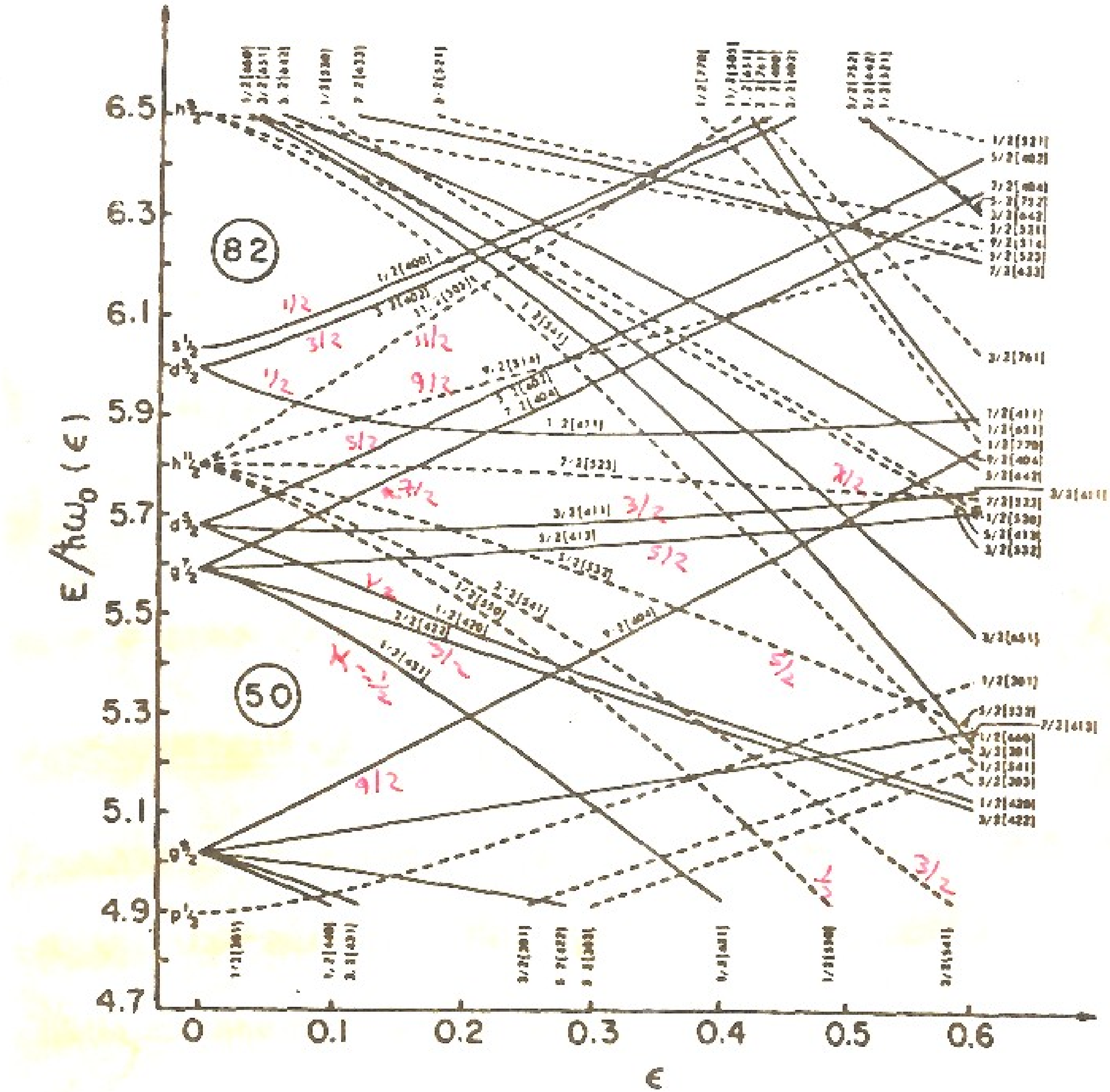
$\therefore$ Projections, $l_z \rightarrow \Lambda$, $s_z \rightarrow \Sigma$

and $K^{\pi} \equiv \Lambda + \Sigma$ are constants

Nilsson quantum numbers $K^{\pi}\,[\,N n_z \Lambda\,]$

Independent of Λ ($l.s.$, l^2 terms small) $\rightarrow$ Independent of K

Opposite to small def. limit.



Nilsson diagram for the $Z = 50$–82 regions.

K values in ink

Appendix E

Misc. Coriolis slides

Mixing

Coriolis: $\Delta K = \pm 1$

Matrix elements of $J_{\pm}$, $j_{\pm}$

Both are well known raising/lowering operators for J_z, j_z: *i.e.*, for K

$$\left\langle \Psi_K \,\middle|\, J_{\pm} \,\middle|\, \Psi_{K+1} \right\rangle = \sqrt{(J \pm K)(J + K + 1)}$$

$$\left\langle \Psi_K \,\middle|\, j_{\mp} \,\middle|\, \Psi_{K+1} \right\rangle = \sqrt{(j \pm K)(j + K + 1)}$$

Note: j is ***not*** a good quantum # –– deformed field ***mixes*** j values. So < $j_{\pm}$> depends on detailed structure of Nilsson wave function

$$\Psi_{\text{Nils}} \equiv \sum_j C_j \varphi_j \qquad \varphi_j = \text{shell model } \varphi$$

$$\left\langle \Psi_{\text{Nils}}(K) \middle| j_- \middle| \Psi_{\text{Nils}}(K+1) \right\rangle = \sum_j C_j^K C_j^{K+1} \sqrt{(j-K)(j+K+1)}$$

Note: for unique parity orbit $j \approx$ good quantum # = N + ½

$$\therefore \left\langle \Psi(K+1) \middle| j_- \middle| \Psi_{\text{Nils}}(K) \right\rangle \square \ (N+1) \qquad e.g. = 7 \text{ for } i_{13/2}$$

Off-Diagonal Effects

of Coriolis force

Mixing

$\Delta K = 1$

Effects on E's, Ψ's

How Big is Coriolis Mixing?

$\frac{\hbar^2}{2\mathcal{J}}$: Typ $E(2_1^+) \sim 100\,\text{keV} \Rightarrow \boxed{\frac{\hbar^2}{2\mathcal{J}} \sim 16\,\text{keV}}$

For states with J within a few units of K

$$\sqrt{(J-K)(J+K+1)} \sim \sqrt{(2-3)(2-8)} \approx \boxed{3-4}$$

Pairing factor: $\approx$ $\boxed{0.7}$

Attenuation: $\boxed{0.7}$

$\boxed{\langle \, | j_- | \, \rangle}$: Varies a lot

$$\langle \, | j_- | \, \rangle = \sum_j c_j^K \, c_j^{(K+1)} \sqrt{(j-K)(j+K+1)}$$

<u>3 cases:</u>

1. $\boxed{\text{Unique Parity Orbits}}$ $C_{j\,max} \sim 1$. One significant term in Σ

$$\langle \, | j_- | \, \rangle \sim \sqrt{(4)(9)} \sim \boxed{6}$$

2. $\boxed{\text{Allowed:}}$ $K_1 [N_1 n_{z_1} \Lambda_1]$ $K_2 [N_2 n_{z_2} \Lambda_2]$

If : $\boxed{\Delta n_z = -\Delta \Lambda_1 = \pm 1}$

e.g. $\frac{1}{2}[521]$ $\frac{3}{2}[512]$ $\langle \, | j_- | \, \rangle \sim \boxed{2-3}$
$\frac{5}{2}[512]$ $\frac{7}{2}[503]$

3. $\boxed{\text{Other cases}}$ $\langle \, | j_- | \, \rangle$ $\boxed{<1}$

$$V_{cor} \sim 16\text{ keV} \times (\sim 3-4) \times (\sim 0.7) \times (\sim 0.7) \times \langle \, | j_- | \, \rangle$$

$$\boxed{\approx 25\text{ keV x } \langle \, | j_- | \, \rangle}$$

Coriolis force mixes

Wave functions of Nilsson orbits differing

by $\Delta K = 1$

e.g. $\frac{5}{2}^-[512]$

$\frac{7}{2}^-[503]$ $K^\pi[N n_z \Lambda]$

Useful guideline:

largest Coriolis matrix elements

occur if $\Delta n_z = -\Delta\Lambda = \pm 1$

- (as in example above)

- as in *all* cases of unique parity

largest $\langle |V_{cor}| \rangle$

$\frac{1}{2}[660]$ $\frac{3}{2}[651]$ $\frac{5}{2}[642]$ $\frac{7}{2}[633]$ $\frac{9}{2}[624]$

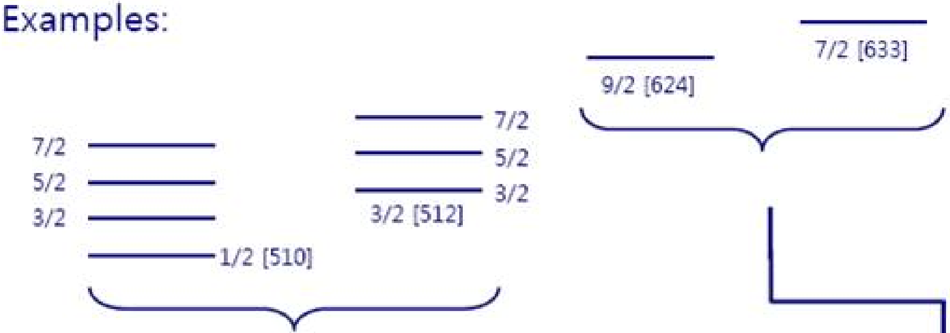
$\frac{11}{2}[615]$ $\frac{13}{2}[606]$

- But not as in $\frac{5}{2}[512]$ – $\frac{7}{2}[514]$

($V_{cor} \neq 0$ but is smaller)

Off-Diagonal Coriolis Mixing Effects

Examples:



$\langle \, | j_- | \, \rangle \sim 0.9$

V_{cor} ranges from ~ 16 keV for $J = \dfrac{3}{2}$

to ~ 30 keV for $J = \dfrac{7}{2}$

$\langle \, | j_- | \, \rangle \sim 6$

$V_{cor}\,(J = \dfrac{13}{2}) \sim 300$ keV

$\boxed{\text{Lower deformation}} \longrightarrow$ higher $E\,(2_1^+)$, higher $\dfrac{\hbar^2}{2\mathscr{I}}$

$\Longrightarrow$ $\boxed{\text{Larger Coriolis}}$

Note: $K = ½$ mixes by $\Delta K = 1$ with $K = ½$ through symmetrization of wave functions

e.g. ^{183}W: ½ [510] – ½ [521]

Nilsson code prints out: $\langle \varphi_i | j_- | \varphi_f \rangle$ values

So, Coriolis matrix element (interaction)

is given by

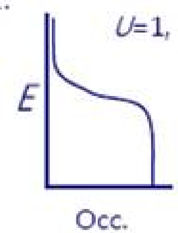
$$\underbrace{\langle \varphi_K | V_{cor} | \varphi_{K+1} \rangle}_{\text{only 1 term is non-vanishing depending on whether } K_f = K_i \pm 1} = -\frac{\hbar^2}{2\mathscr{I}}\left[J_+ j_- + J_- j_+ \right]$$

$$= -\frac{\hbar^2}{2\mathscr{I}} \sqrt{(J-K)(J+K+1)} \ \langle \varphi_K | j_- | \varphi_{K+1} \rangle$$

Usually, modify this in two ways:

1. Pairing $\quad \langle\ |\ |\ \rangle_{\text{pairing}} \rightarrow \langle\ |\ |\ \rangle_{\text{no pairing}} \times (U_1 U_2 + V_1 V_2)$
2. Extra attenuation factor ~ 0.7

1.

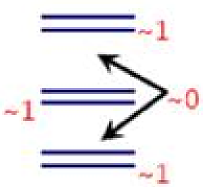


U=1, V=0

U= V=0.707

V=1, U=0

Nilsson Diagram



P a i r i n g

Estimate $\frac{\hbar^2}{2\mathscr{I}}$ from e-e neighbor

2^+ ——— $E = \frac{\hbar^2}{2\mathscr{I}}[2(3) - 0(1)] = \frac{6\hbar^2}{2\mathscr{I}}$

0^+ ——— $E = 0$

$$\frac{\hbar^2}{2\mathscr{I}} = E(2_1^+)/6$$

$$\langle\ | V_{cor} |\ \rangle \sim \frac{E(2_1^+)}{6} \sqrt{(J-K)(J+K+1)} \langle \varphi_K | j_- | \varphi_{K+1} \rangle \ (U_1 V_2 + U_2 V_1)\, 0.7$$