Script/Key Nuclear-Reaction Experiments – First Discoveries and Consequences

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Part I
Preface
Chapter 1

Introduction

Key experiments are those, which open up entirely new insights into unknown “territories” and start new fields of more detailed investigations in these areas. One indicator of key experiments can be Nobel prize awards to the principal investigators (examples are Robert Hofstadter (NP 1961), Jerome Isaac Friedman, Henry Way Kendall, and Richard Edward Taylor (NP 1990), for the study of the external and internal structure of the proton by electron scattering).

In nuclear physics, a field of science, which – by definition – has existed only for about 100 years these key developments are: radioactivity and active investigations of nuclei, either of their structure or their interactions in nuclear reactions. Both are intimately connected with the continuing progress in the development of particle accelerators and in part nuclear reactors.

1.1 Rutherford and Evidence for the Nuclear Atom

The famous Rutherford scattering experiment started the field of nuclear reactions around 1911 in Manchester. The behavior of α particles elastically scattered from gold and other nuclei suggested a very small (from the point of view of that time) and compact nucleus (i.e. containing most of the atom’s mass and all of the positive charge Ze compensating that of the atomic electrons). Energetic particles from radioactive sources were used as projectiles (α’s from heavy elements such as “radium emanation” ($^{222}_{86}$Rn) with sufficiently high energies and intensities). Even then scattering experiments were tedious: A MBq (in 4π solid angle) source corresponds to an incident “beam” current into a solid angle, small enough to define a reasonable scattering geometry, of only $\approx 10^{-6}$ nA. Single scintillation events had to be counted by observing them on a ZnS screen in the dark.

1.2 The First True Nuclear Reaction

Around 1917 Rutherford, using techniques similar to those of the famous scattering experiment, recognized that a different type of particle emerged
from the interaction of \( \alpha \)'s from radioactive sources with gas molecules. It had longer range in matter than the \( \alpha \)'s and proved to be the nucleus of the hydrogen atom. Rutherford therefore had performed the first true nuclear reaction with a rearrangement of the particles involved. Although in gas discharges the existence of negatively charged constituents (the atomic electrons) and positive ions had been seen, only Rutherford identified the particle which emerged from the

\[
^{14}\text{N} + \alpha \rightarrow ^{17}\text{O} + p
\]  

reaction as the very small nucleus of H and coined the term “proton”. Thus, he solved one part of the riddle of the structure and composition of nuclei; the other had to wait until the discovery of the neutron.

The nuclear charge number is identical to the element number of the periodic table and the \( Z \) dependence of the Rutherford cross section confirmed the periodic system of the elements. The explanation of the existence of isotopes and the correct placement of them in the chart of nuclides (\( Z \) vs. \( N \)) required the discovery of the neutron in 1932. Already Rutherford could – by comparing the measured scattering angular distribution of \( \alpha \) particles on gold with his ansatz of a point-Coulomb interaction – conclude that the nucleus is an object smaller than the scattering distances (order of magnitude: \( 1 \text{ fm} = 1 \cdot 10^{-15} \text{ m} \)). The very fact that scattering at backward angles occurred, showed that the scattering center had to be heavier than the \( \alpha \) (this is pure kinematics). The electron cloud relative to this is very large (order-of-magnitude radius: \( 1 \text{ Å} = 1 \cdot 10^{-10} \text{ m} \)) and carries the charge \(-Ze\) such that the atom is exactly neutral.

After the invention of accelerators the use of \( \alpha \) particles of much higher energies with penetration into the target nucleus was possible and the extension (the radius) of nuclei could be obtained by the onset of deviations from the point-Coulomb scattering. A key rôle is played here by the \textit{charge form factor} and its Fourier transform, the \textit{charge-density distribution}. It expresses how strongly the Coulomb potential of an extended (often simply assumed to be homogeneous) charge distribution in the nuclear interior deviates from that of a point charge or what the influence of the (hadronic) nuclear interaction on the observables is, see Fig. 4.3.

Using charged leptons as probes, which have no measurable extension and do not feel the strong interaction, charge (and current) distributions in nuclei and nucleons have been determined. At higher momentum transfer (i.e. at high energies and large scattering angles via \textit{inelastic or quasi-elastic scattering}) excited states of the nucleon and later-on, (via \textit{deep-inelastic scattering}), substructures of the nucleons (\textit{partons}) were discovered that had all the properties of quarks: \( 1/3 \) charges, spin \( 1/2 \bar{\hbar} \), color charge and confinement, characteristics of truly elementary particles (point shape, no internal structure), and they proved to be sources of the strong, electromagnetic, and weak interactions, also by probing them with neutrinos.
1.3 The Role of Accelerators

It is evident that the use of radioactive sources imposed severe restrictions: fixed or very limited energy range and extremely low intensities. It is clear that the field of nuclear reactions could only progress with the invention of particle accelerators. The first accelerator prototype important for nuclear physics was the linear accelerator ("LINAC") developed and published in 1929 by Ralf Wideröe at the Aachen Institute of Technology, also laying the ground for the betatron, which was realized by Kerst and Serber in 1940, and the cyclotron by Lawrence in 1931. Wideröe's ideas also included the synchrotron and storage-ring schemes. The first nuclear reaction initiated with accelerated beams was the reaction

\[ p + ^7\text{Li} \to 2\alpha \]  

(1.2)

by Cockroft and Walton in 1932 at the Cavendish Laboratory at Cambridge using a DC high voltage across several accelerating gaps and produced by the Delon/Greinacher voltage multiplication scheme. This and the ensuing developments in nuclear and particle physics up to the present energies of up to 14 TeV (at the Large Hadron collider LHC at CERN/Geneva) are intimately connected with the achievements in accelerator physics and technology. Likewise the development of detector technologies – from the first scintillators, later equipped with photomultipliers, to the cloud and the bubble chambers, the ionization chamber, Geiger-Müller counter, multiwire ionization chambers, and the large field of solid-state detectors – was essential. Not unjustifiably accelerators have been called “tools of our culture” or “Engines of Discovery” (see e.g. the book by Sessler and Wilson \[ ? \]). Their impact reaches now into social applications such as tumor diagnosis and therapy, materials identification and modification, age and provenience analyses in archaeology, geology, arts, environmental science etc.

1.4 The Neutron and the Correct Composition of Nuclei

With the detection of the neutron by Chadwick (1932) another branch of nuclear physics and especially nuclear reactions opened up that only partly depends on accelerators. Not only was the discovery of the neutron the keystone to the fundamental structure of nuclei removing all kinds of inconsistencies about e.g. nuclear isotopes, but immediately it incited Heisenberg to formulate the idea of charge independence of the nuclear interaction and the fundamental symmetry of isospin.

The neutrality of the neutron facilitates the description of nuclear reactions. On the other hand, production of neutrons for nuclear reactions as well as the detection methods are more complicated (see Section ??). Normally, except when neutrons from nuclear reactions are used, the choice or selection of specific neutron energies requires additional methods such as moderation by elastic collisions with light nuclei and/or chopper and time-of-flight facilities.
Much of neutron work relies on neutrons from fission in reactors (an example is the high-flux 660 MW research reactor with a thermal flux of $> 1 \cdot 10^{15} \text{s}^{-1}\text{cm}^{-2}$, at the Institut Laue-Langevin (ILL Grenoble)) or on spallation neutron sources where intense proton beams in the GeV and mA range incident on (liquid) metal targets release many (up to 30) neutrons per proton with high energies (a typical research center is the LANSCE facility with a proton LINAC, originally designed as meson factory at Los Alamos, New Mexico, another the spallation neutron source (SNS) at Oak Ridge, Tennessee, with 1.4 MW beam power and $4.8 \cdot 10^{16}$ neutrons/s.)

The neutron has fundamental properties in its own right that have been studied:

- $\beta$ decay
- The internal (quark + gluon) structure and charge and magnetic-moment distributions. They have been studied e.g. by elastic and inelastic electron scattering where deuterons and especially $^3\text{He}$ served as neutron targets. Polarized $^3\text{He}$ is an almost pure polarized neutron target. The charge and magnetic-moment distributions inside the neutron are proof of its inner structure.
- The possible electric dipole moment and thus time-reversal and parity violations were studied where the absence of the Coulomb force is experimentally advantageous.
- The wave nature of neutrons of low energies was studied in reflection, diffraction, and interference experiments.
- Especially ultracold neutrons offer many interesting properties and applications, e.g. its interaction with the gravitational field or that of its magnetic moment with magnetic fields.

### 1.5 Nuclear Spectroscopy

We define nuclear spectroscopy as the science of learning all about the properties of the thousands of nuclides, each with individual and also collective properties. Aside from early studies of radioactive decays, nuclear reactions have been the tool to investigate the action of nuclear forces (in the sense of an interplay of the strong interaction proper, the electromagnetic, and the weak force). In high-density situations, e.g. in neutron stars, even the gravitational force enters the stage via the density dependence of the nuclear interactions. The aim of modern nuclear spectroscopy is now moving away from stable nuclei, from deformed highly excited nuclei with high angular momenta on to the investigation of nuclei in the regions near the limits of existing nuclei with either high neutron excess, high neutron deficiency, or the region of new elements, the superheavy nuclei. They can be characterized by their isospin $T = (N - Z)/A$. 
Chapter 2

Rutherford Scattering and the Atomic Nucleus

We begin with a universal definition of the fundamental observable of nuclear reactions, the (differential) cross section that can be applied in classical as well as quantum-mechanical descriptions.

**Definition of Cross Section 2.1** The (differential) cross section is the number of particles of a given type from a reaction, which, per target atom and unit time, are scattered into the solid-angle element $d\Omega$ (formed by the angular interval $\theta...\theta + d\theta$ and $\phi...\phi + d\phi$), divided by the incident particle flux $j$ (a current density!).

With no azimuthal dependence this definition yields the classical formula for the cross section. With the number of particles $j d\sigma = j \cdot 2\pi bdb$ one obtains

$$\left( \frac{d\sigma}{d\Omega} \right)_{\text{class}} = \frac{2\pi bdb}{2\pi \sin \theta d\theta} = \frac{b}{\sin \theta} \cdot \left| \frac{db}{d\theta} \right|.$$  

(2.1)

$\theta(b)$, which contains the dynamics of the interaction (the dynamics) is called the *deflection function*. Its knowledge determines the scattering completely.

The measurements by Geiger and Marsden and the interpretation of the experiment by E. Rutherford [RUT11, GEI13] constitute a milestone in our understanding of the structure of nature and especially the true beginning of “nuclear physics”. Their apparatus carries all the features of modern scattering experiments, as can be seen in Fig. 2.1.

2.1 Rutherford Scattering Cross Section

For the derivation of the classical Rutherford scattering cross section we assume:

- The projectile and the scattering center (target) are point particles (with Gauss’s law it can be proved that this is also fulfilled for extended particles as long as the charge distribution is not touched upon)

- The target nucleus is infinitely heavy (i.e. the laboratory system coincides with the c.m. system)
The interaction is the purely electrostatic point Coulomb force

\[ F_C = \pm \frac{1}{4\pi \varepsilon_0} \cdot \frac{Z_1 Z_2 e^2}{r^2} = \frac{C}{r^2} \]  

with the Coulomb potential \( V_C = \pm C/r \).

"Classical" means that particles and trajectories are localized and no wave properties enter the description.

The classical scattering situation is shown in Fig. 2.2. The deflection function is most simply determined by applying angular-momentum conservation and the equation of motion in one coordinate (y):

\[ L = m v_\infty b = m r^2 \dot{\phi} = m v_{min} d \]  

and from this

\[ dt = r^2 d\phi / v_\infty b \]  

\[ m \Delta v_y = \int F_y dt \]

\[ v_\infty \sin \theta = \frac{C}{m v_\infty b} \int_0^\infty \phi \sin \phi dt \]

\[ = \frac{C}{m v_\infty b} \int_0^{\pi - \theta} \sin \phi d\phi = \frac{C}{m v_\infty b} (1 + \cos \theta) \]  

After transformation to half the scattering angle the deflection function is

\[ \cot(\theta/2) = m v_\infty^2 b/C = v_\infty L/C \]  

and

\[ b = \frac{C}{2E_\infty} \cdot \cot \left( \frac{\theta}{2} \right) \]
2.1. RUTHERFORD SCATTERING CROSS SECTION

Figure 2.2: Classical Rutherford scattering.

and

$$\frac{db}{d\theta} = \frac{C}{2mv_\infty^2} \cdot \frac{1}{\sin^2(\theta/2)} = \frac{C}{4E_\infty} \cdot \frac{1}{\sin^2(\theta/2)}$$  \hspace{1cm} (2.8)

and thus for the Rutherford cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{(4\pi\epsilon_0)^2} \left( \frac{Z_1Z_2e^2}{4E_\infty} \right)^2 \cdot \frac{1}{\sin^4(\theta/2)}.\hspace{1cm} (2.9)$$

Numerically:

$$\frac{d\sigma}{d\Omega} = 1.296 \left( \frac{Z_1Z_2}{E_\infty(MeV)} \right)^2 \cdot \frac{1}{\sin^4(\theta/2)} \left[ \frac{mb}{sr} \right]. \hspace{1cm} (2.10)$$

2.1.1 Minimal Scattering Distance d

For this quantity one needs additionally the energy-conservation law:

$$\frac{mv_\infty^2}{2} = \frac{mv_{\min}^2}{2} + \frac{C}{d}.$$  \hspace{1cm} (2.11)

The absolutely smallest distance $d_0$ is obtained in central collisions with:

$$E_\infty = \frac{mv_\infty^2}{2} = \frac{C}{d_0}.$$  \hspace{1cm} (2.12)
From this and the angular-momentum conservation Eq. 2.3 the relation

\[ b^2 = d(d - d_0) \] (2.13)

is obtained with the solution:

\[ d = \frac{C}{2E_{\infty}} \left( 1 + \sqrt{1 + b^2 \frac{4E_{\infty}^2}{C^2}} \right) \]

\[ = \frac{d_0}{2} \left( 1 + \frac{1}{\sin \theta/2} \right). \] (2.14)

The classical scattering distance in relation to the minimum distance \( d_0 \) as function of the scattering angle is shown in Fig. 2.3.

![Figure 2.3: Minimal scattering distance as function of the scattering angle.](image)

**2.1.2 Trajectories in the Point-Charge Coulomb Field**

For the motion in a central-force field with a force \( \propto r^{-2} \) classical mechanics shows that the trajectories for scattering, i.e. positive total energy, are hyperbolae, which can be derived using angular momentum and energy conservation (with the Coulomb potential):

\[ L = mr^2 \dot{\phi} = \text{const} \]

\[ E = \frac{mr^2}{2} + \frac{L^2}{2mr^2} + \frac{C}{r}. \] (2.15) (2.16)

In these equations \( dt \) can be eliminated. The integration of

\[ d\phi = -\frac{L}{mr^2} \left[ \frac{2}{m} \left( \frac{E - C}{r} - \frac{L^2}{2mr^2} \right) \right]^{-1/2} dr \]

(2.17)
results in
\[ r = \frac{L^2}{mC} \cdot \frac{1}{1 - \epsilon \cos \phi}. \] (2.18)
with \( b = L/\sqrt{2mE} \). With \( k = L^2/mC \) and \( \epsilon = \sqrt{1 + \frac{4Em^2}{c^2}} \) (the eccentricity) the standard form of conic sections is obtained
\[ \frac{1}{r} = \frac{1}{k}(1 - \epsilon \cos \phi). \] (2.19)
There is now a connection between impact parameter \( b \), scattering angle \( \theta \), and (quantized) orbital angular momentum \( L = \ell \hbar \)
\[ b = \frac{1}{2} \frac{d_0}{\cot \frac{\theta}{2}} = \frac{\ell \hbar}{p_\infty}. \] (2.20)

2.1.3 Quantum-Mechanical Derivation of Rutherford’s Formula

The point-Rutherford cross section can be derived quantum-mechanically with identical results. This can be done in two ways. One is to solve the corresponding Schrödinger equation exactly, resulting in the regular and irregular Coulomb functions \( F_\ell \) and \( G_\ell \) as solutions. The other is to use the First Born approximation together with Fermi’s Golden Rule.

Schrödinger Equation

The point-Rutherford cross section may be derived quantum-mechanically by solving the Schrödinger equation with the point (or extended) Coulomb potential as input. It has the form of a hypergeometric differential equation.
\[ -\frac{\hbar^2}{2\mu} u''(r) + \left( \frac{C}{r} - \frac{\hbar^2 \epsilon}{2\mu} \left( \frac{\ell(\ell+1)}{r^2} - \frac{\hbar^2 k^2}{2\mu} \right) \right) u_\ell = 0. \] (2.21)
This equation may be written in its “normal” form with the Sommerfeld parameter \( \eta_S \) and \( \rho = kr \):
\[ \frac{d^2 u_\ell(\rho)}{d\rho^2} + \left( 1 - \frac{\ell(\ell+1)}{\rho^2} - 2 \frac{\eta_S}{\rho} \right) u_\ell(\rho) = 0. \] (2.22)
It has the asymptotic solutions of the regular and irregular Coulomb Functions with the Coulomb phases \( \sigma_\ell = \arg \Gamma(\ell + 1 + i\eta_S) \):
\[ F_\ell \rightarrow \sin(kr - \ell \pi/2 - \eta_S \ln 2kr + \sigma_\ell), \] (2.23)
\[ G_\ell \rightarrow \cos(kr - \ell \pi/2 - \eta_S \ln 2kr + \sigma_\ell). \] (2.24)

With the usual partial-wave expansion with incident plane waves the Coulomb scattering amplitude of the outgoing wave results:
\[ \Psi_S \rightarrow \frac{1}{r} e^{i(kr - \eta_S \ln 2kr)} f_c(\theta), \] (2.25)
\[ f_c(\theta) = -\eta_S \frac{e^{i\sigma_0} \cdot e^{i\eta S \ln \sin^2 \theta/2}}{2k^2 \sin^2 \theta/2}. \] (2.26)
The amplitude squared \( f_C \cdot f_C^* \) provides the Rutherford cross section, which is identical to the classically derived equation.
1st Born Approximation

Starting points for appropriate descriptions are

- Fermi’s *Golden Rule* of perturbation theory
- The first Born approximation

For a “sufficiently weak” perturbation Fermi’s *Golden Rule* gives the transition probability per unit time $W$:

$$
W = \frac{2\pi}{\hbar} |\langle \Psi_{\text{out}} | H_{\text{int}} | \Psi_{\text{in}} \rangle|^2 \rho(E)
$$

$$
= \frac{V m p d\Omega}{4\pi^2 \hbar^3} \cdot |H_{if}|^2.
$$

(2.27)

The density of final states $\rho(E) = dn/dE$, which enters the calculation can be obtained from the ratio of the actual to the minimally allowed phase-space volumes:

$$
\frac{dn}{dE} = \frac{V 4\pi p^2 dp d\Omega}{(2\pi \hbar)^3 dE},
$$

(2.28)

$E = p^2/2m$ and $dp/dE = m/p = E/c^2 p$. Thus

$$
\rho(E) = \frac{dn}{dE} = V \frac{pm d\Omega}{(2\pi \hbar)^3}
$$

$$
= V \frac{pE d\Omega}{(2\pi \hbar)^3 c^2}.
$$

(2.29)

$W$ becomes the cross section according to the definition 3.1 on page 11 with the incident particle-current density $j = v/V = p/mV$:

$$
d\sigma = \frac{W}{j} = \frac{W}{(p/mV)} = \frac{V^2 m^2 d\Omega}{4\pi^2 \hbar^4} \cdot |H_{if}|^2.
$$

(2.30)

The 1st Born approximation consists in using only the first term of the Born series with plane waves in the entrance and exit channels:

$$
\Phi_{\text{in}} = \frac{1}{\sqrt{V}} e^{i\vec{k}_i \vec{r}} \quad \text{and} \quad \Phi_{\text{out}} = \frac{1}{\sqrt{V}} e^{i\vec{k}_f \vec{r}}.
$$

(2.31)

If $H_{\text{int}} = U(r)$ signifies a small time-independent perturbation then, with $\vec{K} = \vec{k}_f - \vec{k}_i$

$$
|H_{if}| = \left| \frac{1}{V} \int e^{i\vec{K} \vec{r}} U(r) d\tau \right|
$$

(2.32)

and

$$
\frac{d\sigma}{d\Omega} = \left( \frac{m}{2\pi \hbar^2} \right)^2 \left| \int e^{i\vec{K} \vec{r}} U(r) d\tau \right|^2 = |f(\theta)|^2.
$$

(2.33)

Inserting the Coulomb potential $U(r) = C/r$ the classically calculated formula for the Rutherford scattering cross section is obtained. The cross section
is (with the constant $Z_1 Z_2 e^2 / 16$ and the substitution $u = iKr \cos \theta$ and $du = -\sin \theta \, d\theta (iKr)$)

$$\frac{d\sigma}{d\Omega} = \text{const} \cdot \left| \int e^{iKr} \cdot \frac{1}{r} \, d\tau \right|^2$$

$$= \text{const} \cdot \left| \int \int \frac{1}{r} e^{iKr \cos \theta} 2\pi \sin \theta \, d\theta \, dr \right|^2$$

$$= \text{const} \cdot 2\pi \left| \int \int \frac{r}{iKr} \, e^u \, du \, dr \right|^2$$

$$= \text{const} \cdot \left( \frac{2\pi}{iK} \right)^2 \left| \int \left( e^{iKr \cos \pi} - e^{iKr \cos 0} \right) \, dr \right|^2$$

$$= \text{const} \cdot \left( \frac{2\pi}{iK} \right)^2 \left| \int \left( e^{-iKr} - e^{iKr} \right) \, dr \right|^2$$

$$= \text{const} \cdot \left( \frac{2\pi \cdot 2\pi}{iK} \right)^2 \left| \int_0^\infty \sin Kr \, dr \right|^2. \quad (2.34)$$

The integral is undefined. This is circumvented by a screening ansatz after Bohr, which corresponds to the real situation of the screening of the point Coulomb potential by the electrons of the atomic shell, with the screening constant $\alpha$. With

$$\int_0^\infty e^{-\alpha r} \sin K r \, dr = \frac{K}{K^2 + \alpha^2} \quad (2.35)$$

one obtains

$$\left( \frac{d\sigma}{d\Omega} \right)_{R,s} = \left[ \frac{2\mu Z_1 Z_2 e^2}{\hbar^2 (\alpha^2 + 4k^2 \sin^2 (\theta/2))} \right]^2 \quad (2.36)$$

with the momentum transfer $K = 2k \sin (\theta/2)$ for elastic scattering. This cross section is finite for $\theta \to 0^\circ$. By letting the screening constant go to zero a cross section results, which is identical with that from the classical derivation:

$$\left( \frac{d\sigma}{d\Omega} \right)_R = \lim_{\alpha \to 0} \left( \frac{d\sigma}{d\Omega} \right)_{R,s}$$

$$= \left( \frac{Z_1 Z_2 e^2}{4E_{\text{kin}}} \right)^2 \cdot \frac{1}{\sin^4 (\theta/2)}. \quad (2.37)$$

However, for all applications where there is interference the Rutherford amplitude has to be used including its (logarithmic) phase. Typical cases are that of identical particles (see Section 14) or of interference with nuclear (hadronic) amplitudes (see e.g. Section ??). Normally one has to assume a fundamentally quantum-mechanical description that only in special cases, i.e. when the relevant de Broglie wavelengths are small, may be approximated by classical methods. For this decision the Sommerfeld Criterion has been formulated.

$$\lambda_{\text{de Broglie}} = \hbar / p \ll d. \quad (2.38)$$

When choosing for a typical object dimension half the distance of the trajectory turning point $d_0$ for a central collision the Sommerfeld criterion for
classical scattering is obtained

\[ \eta_S = \frac{Z_1 Z_2 e^2}{\hbar v} = Z_1 Z_2 \frac{e^2}{\hbar c} \cdot \frac{c}{v} = Z_1 Z_2 \cdot \frac{\alpha}{\beta} \gg 1 \]  

(2.39)

or numerically (for a heavy target)

\[ \eta_S \approx 0.16 \cdot Z_1 Z_2 \sqrt{\frac{A_{\text{proj}}}{E_{\text{lab}}(\text{MeV})}} \gg 1. \]  

(2.40)

2.1.4 Result of the Experiment

The results of the Rutherford-Geiger-Marsden experiment are shown in Fig. 2.4. The figure exhibits the strong angle dependence of this cross section together with the original data of Ref. [GEI13], adjusted to the theoretical curve shown.

Figure 2.4: The curve shows the angular dependence of the theoretical Rutherford cross section \( \propto \sin^{-4}(\theta/2) \). The points are the original data (that consisted of tabulated numbers of counts with no error bars, and not transformed into cross-section values) of Ref. [GEI13], adjusted to the theoretical curve, giving a nearly perfect fit (Nowadays data with at least an error estimate or, better, error bars are mandatory).

2.1.5 Consequences of the Rutherford Experiments and their Historic Significance

Rutherford and his collaborators Geiger and Marsden (later also Chadwick) used \( \alpha \) particles from radiactive sources as projectiles. Their energies were
so small that for all scattering angles the minimum scattering distances \( d \) were large compared with the sum of the two nuclear radii of projectiles and targets. The complete agreement between the results of the measurements and the (point-)Rutherford scattering cross section formula shows this in accordance with Gauss’s law of electrostatics: a finite charge distribution in the external space beyond the charges cannot be distinguished from a point charge with an \( r^{-1} \) potential. In addition, the mere occurrence of backward-angle scattering events proves uniquely by simple kinematics that the target nuclei were heavier than the projectiles. Thus the existence of the atomic nucleus as a compact (i.e. very small and heavy object) was established (and Thomson’s idea of a “plum-pudding” of negative charges from distributed electrons, in which the positive charges of ions were suspended, was refuted).

Later, the energy dependence as well as the dependence on charge numbers could be fully corroborated leading to a confirmation and a few corrections to the periodic table of the elements.
CHAPTER 2. RUTHERFORD SCATTERING AND THE ATOMIC NUCLEUS
Bibliography


Chapter 3

The First True Nuclear Reaction and the Discovery of the Proton

The first “true” nuclear reaction (i.e. one with transmutation into different particles) was discovered by Rutherford in 1919 (after earlier work together with Ernest Marsden, in which particles with larger range, $^1$H nuclei, than that of the $\alpha$’s in scattering from different targets were observed:

$$\alpha + ^{14}\text{N} \rightarrow ^{17}\text{O} + p$$ (3.1)

using 6 MeV $\alpha$’s from a radioactive source. Fig. 3.2 shows such an event in a cloud chamber. It also shows an event of “Rutherford” scattering of the recoil $^{17}\text{O}$ nucleus on an $^{14}\text{N}$ nucleus.

The cloud chamber, which is still unsurpassed as an instrument for visualizing such events but also cosmic rays etc. was invented by Charles Wilson following 1911, but developed for practical use by Blackett only since 1921, was not yet used by Rutherford. Consequently the $^1$H nuclei were identified as part of all nuclei and Rutherford coined the term “proton”. However, the still remaining puzzles about the true structure of nuclei were only resolved after the neutron was discovered by James Chadwick in 1932 (after Rutherford had already speculated about neutrons in nuclei and others had mistakenly interpreted the neutron radiation from the reaction $\alpha + ^9\text{Be} \rightarrow ^{12}\text{C} + n$ (with $\alpha$’s from a polonium source) to be an energetic $\gamma$ radiation).
Figure 3.1: Apparatus used by Rutherford from 1917 to 1920 to bombard $^{14}\text{N}$ with $\alpha$ particles. The emitted particle radiation of longer range was identified as consisting of $Z=1$, $A=1$ particles, forming the nucleus of the hydrogen atom, and for which Rutherford coined the word “proton” in 1919.

Figure 3.2: Cloud chamber photograph by Blackett of the first nuclear reaction $\alpha + ^{14}\text{N} \rightarrow ^{17}\text{O} + p$ observed by Rutherford in 1919 [RUT19, BLA25].
Bibliography

Chapter 4

Extended Matter and Charge Distributions of Nuclei

Naturally the study of the finite size of nuclei requires higher-energy projectiles, as has been indicated by classical arguments in Chapter 2. These can be hadronic particles such as $\alpha$’s with medium energies (around 50 MeV would be sufficient due to the small de Broglie wavelength) or e.g. electrons needing more like hundreds of MeV. Because interference effects between strong and Coulomb interactions occur and electrons are relativistic a classical description is impossible.

The extension of the derivation of the Rutherford cross section to an extended (especially a homogeneous and spherically-symmetric) charge distribution is simple and leads to the fundamental concept of the form factor.

We start with the Coulomb potential of such an extended homogeneous spherical charge distribution (Fig. 4.1). It is calculated with Gauss’s theorem of electrostatics:

$$V(r) = \begin{cases} \frac{ze^2}{4\pi\epsilon_0 r} & \text{for } r > R \\ \frac{ze^2}{4\pi\epsilon_0 2R} \left(3 - \frac{r^2}{R^2}\right) & \text{for } r \leq R \end{cases}$$

(4.1)

In the exterior space the potential is identical with that of a point charge, continues at $r = R$ to a parabolic shape in the interior of the distribution. It is therefore to be expected that in the scattering with sufficiently high energy the scattering cross section would strongly deviate from the Rutherford cross section as soon as the nuclear surface is touched. In addition, the onset of the short-range strong interaction will influence the scattering, especially by absorption. For the calculation of the cross section an integral over the contributions from all charge elements $dq = Ze\rho(\vec{r})d\tau$ to the potential $U(\vec{r}) = -Z\frac{Ze^2}{R}e^{-\alpha R}\rho(\vec{r})d\tau$ has to be performed.

$$U(\vec{r'}) = -Z_1 Z_2 e^2 \int \rho(\vec{r'}) \frac{e^{-\alpha R}}{R} d\tau. \quad (4.2)$$

By inserting this into the Born approximation Eq. 2.33 (with $d\vec{R} = d\vec{r}'$
and \( \vec{R} = \vec{r}' - \vec{r} \) one obtains:

\[
\frac{d\sigma}{d\Omega} = \left( \frac{Z_1 Z_2 e^2 m}{2\pi \hbar^2} \right)^2 \left[ \int \rho(\vec{r}) e^{i\vec{K}\vec{r}} d\tau \cdot \int e^{-\alpha R} e^{i\vec{K}\vec{R}} d\vec{R} \right]^2
\]

\[
= \left[ F(K^2) \frac{K}{K^2 + \alpha^2} \right]^2. \tag{4.3}
\]

The cross section factorizes into two parts, one of which (after a transition to the limit \( \alpha \to 0 \)) results again in the point cross section, the other in the form factor:

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{point nucleus}} \cdot \left| F(K^2) \right|^2. \tag{4.4}
\]

This separation is characteristic for the interaction between extended objects and signifies a separation between the interaction (e.g. the Coulomb interaction) and the structure of the interacting particles.

For rotationally-symmetric problems the form factor has a simplified interpretation:

\[
F(K) = \int \rho(r) \exp(iKR) 2\pi r^2 dr \sin \theta d\theta. \tag{4.5}
\]

On substitution \( u = iK r \cos \theta \) and \( du = -iKr \sin \theta d\theta \) this becomes

\[
F(K) = 2\pi \int \rho(r) e^u r^2 dr \frac{du}{-iKr} = \int \rho(r) 4\pi r^2 dr \cdot \left( \frac{\sin(Kr)}{Kr} \right)_{\text{purely real}}. \tag{4.6}
\]

Thus the form factor is a folding integral of the density with the sampling function (in parentheses). This function is oscillatory and its oscillation
“wavelength” $1/K$ (which depends on the energy of the transferred radiation) has to be adjusted to the rate of change of the density. If the oscillation is too frequent the integral results in $\approx 0$ revealing no information on $\rho$. If it is too slow the sampling function is $\approx$ constant, and the integral results in just the total charge $Ze$. Fig. 4.2 illustrates this for different momentum transfers on a given nuclear density distribution. Experimentally the form factor is obtained as the ratio

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{experimental}} / \left(\frac{d\sigma}{d\Omega}\right)_{\text{point, theor.}}. \quad (4.7)$$

The charge distribution (or more generally: the density distribution e.g. of the hadronic matter) is obtained by Fourier inversion of the form factor $F$:

$$\rho_c(\vec{r}) = \frac{1}{(2\pi)^3} \int_{0 \to \infty} F_c(\vec{K}^2) e^{(-i\vec{K}\vec{r})} d\vec{K}. \quad (4.8)$$

This means that (in principle) for a complete knowledge of $\rho(\vec{r})$ $F$ must be known for all values of the momentum transfer. Since $\rho(\vec{r})$ for small $\vec{r}$ is governed by the high-momentum transfer components of $\vec{K}$ this cannot be achieved in practice. For this reason the following approximations may be used:

Figure 4.2: Sampling functions for different momentum transfers show that in order to sample details of a given structure (e.g. the shape around the radius of a nuclear density (charge or mass) distribution) the momentum transfer (given by the incident energy and the scattering angle) has to have an appropriate intermediate value. In the example shown the value of $K = 0.5 \text{ fm}^{-1}$ is suitable for sampling the region around the nuclear radius of 5.0 fm. The vertical dotted lines indicate a 10 to 90% sampling region.
• Model assumptions are made for the form of the distribution: e.g. homogeneously charged sphere, exponential, Yukawa, or Woods-Saxon behavior.

• The model-independent method of the expansion of \(e^{i\mathbf{K} \cdot \mathbf{r}}\) into moments.

### 4.0.6 Ansatz for Models

It is useful to get an impression of the Fourier transformation of different model density-distributions as shown in Fig. 4.3: It is a general observation that “sharp-edged” distributions lead to oscillating form factors (and therefore cross sections), and smooth distributions to smooth form factors. In agreement with our ansatz a \(\delta\) distribution (characteristic for a point charge or mass) corresponds to a constant form factor (This is called “scale invariance”).

### 4.0.7 Expansion into Moments

With the power-series expansion of \(e^{i\mathbf{K} \cdot \mathbf{r}}\) the form factor becomes

\[
F(\mathbf{K}^2) \propto \int \rho(\mathbf{r}) \left[ 1 + i\mathbf{K} \cdot \mathbf{r} - \frac{(\mathbf{K} \cdot \mathbf{r})^2}{2!} + ... \right] d\tau \tag{4.9}
\]

By assuming a spherically symmetric distribution (with pure \(r\) dependence only) and with a normalization such that for a point object the constant form factor is 1, we have:

\[
F(\mathbf{K}^2) = 1 - \text{const} \cdot K^2 \int_{0\to\infty} r^2 \rho(r) d\tau \pm ...	ag{4.10}
\]

The second term contains the average square radius \(\langle r^2 \rangle = r_{\text{rms}}^2\). For small values of \(K^2\langle r^2 \rangle\) one gets in a model-independent way (i.e. for arbitrary form factors):

\[
F(\mathbf{K}^2) \approx 1 - \frac{1}{6} K^2 \langle r^2 \rangle\tag{4.11}
\]

Of course this approximation is becoming worse with smaller \(r\) (because one needs higher moments), i.e. if one wants to resolve finer structures.

### 4.1 Hadron Scattering Experiments

After accelerators were available charge and matter density distributions of nuclei and their radii could be investigated by probing the distributions with hadronic projectiles. With light as well as with heavy ions, but also with neutrons as projectiles it is evident that they are extended and possess structure. The consequence is that detailed statements about the density distributions are difficult to make and may need the deconvolution of the contributions from projectile and target nuclei. However, statements about nuclear radii are possible, even with quite simple semi-classical assumptions such as absorption between nuclei settling in sharply at a well-defined distance and pue
Coulomb scattering beyond that distance. Systematic $\alpha$ scattering studies on many nuclei (where we already have strong absorption at the nuclear surfaces) revealed good $A^{1/3}$ systematics for the nuclear radii. A dependence of $\sigma_{\alpha,\alpha}$ on $R = R_0(A^{1/3} + 4^{1/3})$ was fitted to the data, assuming a sharp-cutoff model for the cross sections and taking into account the finite radii of both nuclei. It yielded a radius constant of

$$R_0 = 1.414 \text{ fm}$$

(4.12)

However, when considering the range of the nuclear force for both nuclei of about 1.4 fm a radius constant of $\approx 1.2$ fm resulted.

### 4.1.1 Nuclear Radii from Higher-Energy $\alpha$-Particle Scattering

Already without detailed knowledge of the density distribution and of the potential some quite precise statements about nuclear radii by scattering of charged projectiles from nuclei were possible. One condition for this is, however, that the potential, which is responsible for the deviations from the point cross section is of short range, i.e. the charge distribution has a relatively sharp edge.

Most impressively these deviations from the point cross section appear with diminishing distances between projectile and target in a suitable plot. Because the Rutherford cross section itself is strongly energy and angle dependent one may choose to plot the ratio

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}}}{\left(\frac{d\sigma}{d\Omega}\right)_{\text{point, theor.}}}$$

as function of the minimum scattering (the *apsidal*) distance $d$. Thus data at very different energies and angles can be directly compared (see Fig. 4.8 in Section 4.1). If, in addition, one wants to check on the assumption of the systematics of nuclear radii to follow $r = r_0 A^{1/3}$ a universal plot for all possible scattering partners by plotting the above ratio against $d/(A^{1/3} + A_2^{1/3})$ is useful. The experimental results show the extension of the charge distribution and the rather sudden onset of (hadronic) absorption (provided the interaction has a strong absorption term, which is typical for $A \geq 4$).

Around 1954 $\alpha$-particle beams from cyclotrons with energies much higher than those from radioactive sources became available, typically from about 20 MeV to 40 MeV. In the classical Rutherford picture these energies were high enough that the colliding nuclei could be brought into contact in order to “feel” the (hadronic) nuclear interaction in addition to the Coulomb field. According to this classical model the point of “grazing” depends on the energy and the scattering angle, see Eq. 2.14.

The key experiments were performed at the Brookhaven cyclotron with 40 MeV beams on heavy nuclei. Using heavy targets has the significant advantages

- The Coulomb interaction is quite strong.
• Quantum-mechanical interference or diffraction effects are small.
• The onset of strong absorption by the nuclear force is quite abrupt.

The following figures are from the key paper by Ref. [WEG55].

The point of deviation from the Rutherford cross section is clearly visible and corresponds to

\[ d_{\text{min}} \approx 1.7 \text{ fm} \]  

(4.14)

A best description of the data was achieved with the assumption that nuclear radii (including that of the \( \alpha \) particle) follow a

\[ r = r_0 A^{1/3} \]  

(4.15)

law. With the additional assumption of a range of the nuclear force of \( \approx 1.4 \text{ fm} \) a “strong” radius constant of

\[ r_0 \approx 1.45 \text{ fm} \]  

(4.16)

resulted.

### 4.1.2 Heavy-Ion Scattering

As in \( \alpha \)-particle scattering the strong absorption properties of the nuclear interaction heavy-ion scattering experiments have been very useful to gain insights into nuclear radii and other surface properties. A great number of different pairs of collision partners yielded very good systematics as shown in Fig. 4.8. It becomes evident especially by plotting the relative cross sections against the distance parameter \( d \), for which an assumed \( A^{1/3} \) dependence of the radii of both collision partners was applied

\[ d = D_0 (A_1^{1/3} + A_2^{1/3})^{-1} \]  

(4.17)

with \( D_0 \) the distance of closest approach, as calculated from energies and scattering angles. A well-defined sharp distance parameter of \( d_0 = 1.49 \text{ fm} \) for the onset of absorption results. This corresponds to a universal radius parameter of \( r_0 = 1.1 \text{ fm} \) if the range of the nuclear force is set to 1.5 fm. The simple model applied was to assume

• Pure point-Rutherford scattering outside the range of nuclear forces,
• Ratio of elastic to Rutherford cross section

\[
\frac{d\sigma}{d\sigma_R} = 1 + P_{\text{abs}}(D)
\]

(4.18)

and

\[
P_{\text{abs}}(D) = \begin{cases} 
0 & \text{for } D \geq D_0, \\
1 - \exp \left( \frac{D - D_0}{\Delta} \right) & \text{for } D < D_0,
\end{cases}
\]

(4.19)

with \( P_{\text{abs}}(D) \) the probability of absorption out of the elastic channel, \( D_0 \) the interaction distance, and \( \Delta \) the “thickness” of the transition region.

The latter depends on the \( A \) of the nuclei involved and could be determined with good accuracy to be e.g. \( \Delta \approx 0.33 \text{ fm} \) for scattering of nuclei near \( ^{40}\text{Ca} \) from \( ^{208}\text{Pb} \).
4.2 Elastic Electron Scattering – Hofstadter’s Experiments

Since all electrons (and all leptons) are considered to be point-particles they are – as long as not the hadronic interaction region proper shall be probed – the ideal projectiles. They "see" the electromagnetic (and weak) structure of the nuclei. Of course, the treatment must be relativistic. Instead of the Rutherford- (point-Coulomb) approach one has to use the proper theory.

Besides the relativistic treatment differences to the (classical) Rutherford cross section come about by the lepton spin. The derivation of the correct scattering cross section relies on the methods of Quantum Electrodynamics (QED) and techniques such as the Feynman diagrams. Here only the results will be presented. The electromagnetic interaction between the electron and a hadron is mediated by the exchange of virtual photons, which is accompanied by a transfer of energy and momentum. The wavelength of these photons derives directly from the momentum transfer \( \bar{hK} = 2(\bar{h}\nu/c)\sin(\theta/2) \) to be

\[
\lambda_{\text{de Broglie}} = \frac{\hbar}{\bar{h}K} = \frac{1}{K}.
\]

(4.20)

The argument of diffraction limitation may also be formulated in the complementary time picture; it may be said that at long wavelengths, due to the uncertainty relation, one needs long measurement times, in which the projectile sees only a time-averaged picture of the object considered while small wavelengths allow measurement times equivalent to snapshots of the object or its substructures (partons).

Principally in lepton scattering at higher energies three distinct regions of momentum transfer can be distinguished:

- Elastic scattering at small momentum transfer is suitable to probe the shape of the hadrons. The resulting two form factors produce again the charges and current (magnetic moment) distributions and the radii of the hadrons by Fourier inversion.

- (Weakly) inelastic scattering at higher momentum transfer leads to excitations of the hadrons (e.g. Delta- or Roper excitations (resonances) of the nucleons). The form factors are quite similar to those from the elastic scattering, which means that we have some excited state of the same nucleons.

- Deep-inelastic scattering is the suitable method to see partons inside the hadrons. In this way in electron and muon scattering the quarks bound in nucleons and their properties (spin, momentum fraction) and also the existence of sea quarks (s quark/anti-quark pairs were identified). Especially the pointlike character of these constituents was shown by the near constancy of the form factors (here called: structure functions with the momentum transfer (Bjorken scaling)).

Here only elastic scattering will be discussed in detail. In QED theory for
the differential cross section the Rosenbluth formula was deduced:

\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{point}} \cdot \left( \frac{F_E^2 + bF_M^2}{1 + b} + 2bF_M^2 \tan^2 \frac{\theta}{2} \right). \tag{4.21}
\]

The point cross section \((d\sigma/d\Omega)_{\text{point}}\) is a generalized Rutherford cross section and is calculable with the methods of QED (e.g., using Feynman diagrams). The most general form of this cross section (the Dirac scattering cross section) contains as main part the electrostatic scattering, a contribution from the magnetic (spin-dependent) interaction, which depends on the momentum transfer, and a correction for the nuclear recoil:

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{Dirac}} = \frac{\alpha^2}{4\rho_0^6 \sin^4(\theta/2)} \left[ 1 + \frac{2p_0}{M} \sin^2 \frac{\theta}{2} \right] \left( \cos^2 \frac{\theta}{2} + \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right) \ldots \tag{4.22}
\]

For small energies or momentum transfers the cross section simplifies to:

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} = \frac{[2e^2/(E/c)]^2}{q^4} \cdot \frac{E'}{E} \cos^2 \frac{\theta}{2}. \tag{4.23}
\]

The symbols used here mean: \(q\) = four-momentum transfer, \(b = -q^2/(4m^2c^2)\), \(E'\), and \(E\) the energies of the outgoing and incoming electrons. \(F_E\) and \(F_M\) are the electric and magnetic form factors of the nucleons. Experimentally they are obtained from the measured data by least-squares fitting of the parameters of the theory, graphically through the Rosenbluth plot, i.e., by plotting \((d\sigma/d\Omega)_{\text{exp}}/(d\sigma/d\Omega)_{\text{point}}\) against \(\tan^2(\theta/2)\).

In analogy to the Rutherford cross section here the form factors (or structure functions) are Fourier transforms of the charge and current-density distributions (or: distributions of the (anomalous) magnetic moments). Like there, these distributions result from Fourier inversion of the form factors, and at the same time quantitative values of the shape and size of the nucleons are obtained.

The measured form factors as functions of \(q^2\) are normalized such that for \(q \rightarrow 0\) they become the static values of the electric charge and magnetic moments. Except for the electric form factor of the neutron all others are well described by the dipole ansatz corresponding to a density distribution of an exponential function.

An early model for the charge-density distribution was – besides the homogeneously charged sphere with only one parameter, its radius – a modified Woods-Saxon distribution with three parameters, because, besides the radius parameter \(r_0\) and the surface thickness \(a\), also the central density \(\rho_0\) must be adjustable because it varies especially in light nuclei:

\[
\rho_c(r) = \frac{\rho_0}{1 + e^{r/r_1/2}}. \tag{4.24}
\]

The surface thickness \(t = 4 \ln 3 \cdot a\) signifies the 10 to 90% thickness range centered around \(r_{1/2}\). From this parametrization an electromagnetic radius constant of \(r_{1/2} = 1.07\) fm, a surface-thickness parameter of \(a = 0.545\) fm,
and a central density of \( \rho_N = 0.17 \text{nucleons/fm}^3 \) or \( 1.4 \cdot 10^{14} \text{g/cm}^3 \) for nuclei with \( A > 30 \) have been derived. The description of “modern” density distributions is not so simple because the nuclei have individual and more complex structure even if the essential features such as the three parameters do not vary too much. The detailed structure information is obtained from model-independent approaches such as Fourier-Bessel expansions. Radii are given as rms radii or converted into the equivalent radii \( R_0 \). \( R_0 \) is the radius of a homogeneously charged sphere of equal charge using the relation

\[
  r_{\text{rms}} = \sqrt{\frac{3}{5}R_0}. \tag{4.25}
\]

The definition of the (model-independent) Coulomb rms radius is

\[
  r_{\text{rms}} = \langle r^2 \rangle^{1/2} = \left( \frac{1}{Ze} \int_0^\infty r^2 \rho_C(r) 4\pi r^2 dr \right)^{1/2}. \tag{4.26}
\]

The results of elastic electron scattering on the proton show that the proton – different from heavier nuclei – has no sharp surface. Its density distribution has been described appropriately by an exponential with an rms radius of

\[
  r_{\text{rms}} \approx 0.888 \text{ fm} \tag{4.27}
\]

### 4.3 Complementary Methods

The scattering methods of determining density distributions and radii of nuclei are well complemented by methods, which rely on the influence of the extended nuclear charge distribution on atomic levels. Laser-spectroscopy methods as well as exotic-atom methods are sensitive enough to compete with them. As a model the interaction of an extended (homogeneous, spherical) charge distribution with atomic electrons shows the salient features of the modifications from a point charge. Fig. 4.11 depicts the charge-density distribution together with the potential as functions of \( r \). The Coulomb-energy difference \( \Delta E_C \) is determined by the integral over the potential difference for the two cases

\[
  \Delta E_C = \int_0^R e \rho_e(r) \Delta \Phi_N(r) 4\pi r^2 dr \tag{4.28}
\]

\[
  \equiv 4\pi e \rho_e(r) \int_0^R \left[ \frac{C}{2R} \left( 3 - \frac{r^2}{R^2} \right) - \frac{C}{R} \right] r^2 dr \tag{4.29}
\]

\[
  \approx -|\psi_e(0)|^2 \cdot \frac{Ze^2}{4\pi\epsilon_0} \cdot \frac{4\pi R^2}{10} \tag{4.30}
\]

For the 1S ground state of the hydrogen atom the electron’s radial wave function is

\[
  \psi_{1s} \propto \left( \frac{Z}{a_0} \right)^{3/2} \tag{4.31}
\]
leading to
\[ \Delta E_C \approx \frac{2}{5} \cdot \frac{Z^4}{4\pi\epsilon_0} \cdot e^2 \cdot \frac{R^2}{a_0^3}, \tag{4.32} \]
assuming a "contact interaction" at the nuclear center.

### 4.3.1 High Precision Laser Spectroscopy

Although the effects of nuclear size and shape on the energies and transitions of atomic electrons are very small the very high precisions reached in laser-spectroscopy measurements allows to reach results comparable to other methods (such as muonic atoms). The most intense efforts have been spent on the spectroscopy of the hydrogen atom, especially on the measurement of the famous Lamb shift, i.e. the energy separation between the 2S and 2P states, which are predicted to be degenerate in Dirac theory but split by different effects of QED (among them vacuum fluctuations and polarization of the vacuum). This is why the measurement of the Lamb shift to very high precision is essential.

The interpretation of the measurements requires the evaluation of the influence of nuclear effects, especially the size (radius and shape) of the nucleus. The present state is that the precision of the measurements (in atomic spectroscopy as well as in muonic atoms) is now so high that the e.g. the proton radius has become the final limitation to higher precision of the Rydberg constant and comparisons to QED. The results obtained for the proton radius agree within the errors with those from medium-energy electron scattering. However, a very recent, very puzzling disagreement (by 5 standard deviations!) with results from muonic atoms is unresolved, see next subsection.

### 4.3.2 Muonic Atoms

The 1S (n = 1) Bohr radius of a negative particle with mass m \( a_0 \) is \( \propto m^{-1} \) thus
\[ \frac{a_0(\mu^-)}{a_0(e)} = \frac{m_e}{m_\mu} = 4.75 \cdot 10^{-3} \tag{4.33} \]
and
\[ \frac{\Delta E_C(\mu^-)}{\Delta E_C(e)} = \frac{|\psi_\mu(0)|^2}{|\psi_e(0)|^2} \left[ \frac{a_0(e)}{a_0(\mu)} \right]^3 \tag{4.34} \]
\[ = \left( \frac{m_\mu}{m_e} \right)^3 = 210.5^3 = 9 \cdot 10^6 \tag{4.35} \]
testifying to a large increase in sensitivity of the position of energy levels and transitions between them for muonic atoms as compared to electronic ones. However, because the level energies are larger by a factor \( m_\mu/m_e = 210.5 \), the transitions are in the X-ray region.

The first experiments with muons (the term "\( \mu^- \)-mesons is erroneous because muons are leptons, mesons are hadrons underlying the strong interaction) could be performed in the 1950s after cyclotrons with sufficient energy
became available to produce pions ($\pi$ mesons), which in turn decay weakly via

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu + 34 \text{ MeV}$$ (4.36)

(lifetime $\tau = 2.603 \cdot 10^{-8}$ s, corresponding to a flight path of $c\tau = 7.8 \text{ m}$ for highly relativistic pions). By magnetic deflection and time-of-flight techniques the muons are separated from electrons and formed into a muon beam in a *muon channel*. Their lifetime is $\tau = 2.197 \cdot 10^{-6}$ s and $c\tau = 658.65 \text{ m}$. Besides the properties of the muons their interactions with nuclei were studied. Negative muons, after slowing down by energy-loss processes in matter, may be captured by the nuclear Coulomb field and forced into outer Bohr orbits of the muonic atoms formed, from where they cascade down into the ground state, thereby emitting characteristic X-rays. References for the earliest such investigations are Refs. [FIT53, COO53, WHE53]. In Ref. [FIT53] muonic X-rays on a number of nuclei were measured, of which two examples are shown in Figs. 4.14 and 4.15. The Figs. 4.12 and 4.13 show the apparatus used for these experiments. From the shift between the 2p-1s transition energies of point and extended-charge distributions the nuclear-radius systematics of $r = r_0 A^{1/3}$ was established with a best value of

$$r_0 = 1.17 \ldots 1.22 \text{ fm}$$ (4.37)

and a muon mass of $m_\mu = 210m_e$. More modern methods used Ge(Li), Si(Li), HPGe, crystal spectrometers, and LAAPD photodiode detectors.

Nuclear radii from muonic atoms are often more precise than those from lepton scattering but they are in a way complementary in relation to the radius region probed they measure different moments). Thus, the results of both methods can be combined (Fig. 4.16). The distributions are quite well reproduced by “mean-field” calculations, see e.g. [FRO87, DEC68]. The salient results of these investigations are:

- From the distributions a central density is derived, which for heavier nuclei is constant in first approximation. This and the systematics of radii are characteristic for nuclear forces; their properties are: short range, saturation and incompressibility of nuclear matter, and suggest the analogy to the behavior of liquids, which led to the development of collective nuclear models (liquid-drop models, models of nuclear rotation and vibration).

- The radii of spherical nuclei follow more or less a simple law $r = R_0 A^{1/3}$. For the radius parameter $R_0 = 1.24 \text{ fm}$ is a good value. From Coulomb-energy differences of mirror nuclei a value of $R_0 = (1.22 \pm 0.05) \text{ fm}$ has been derived.

- The surface thickness of all nuclei is nearly constant with a 10-90% value of $t = 2.31 \text{ fm}$ corresponding to $a = t/4\ln 3 = 0.53 \text{ fm}$. This is explained by the range of the nuclear forces independent of the nuclear mass number $A$. 

• The nucleons have no nuclear surface. The charge and current as well as the matter densities of the proton follow essentially an exponential distribution. For the neutron the charge distribution is more complicated because volumes of negative and positive charges must compensate each other to zero notwithstanding some complicated internal charge distribution that originates from its internal quark-gluon structure.

• The rms radii for the current distributions of protons and neutrons and the charge distribution of the protons are $0.88768(69)$ fm (accepted CODATA value) in agreement between atomic spectroscopy and electron-scattering results. Recently, with increased experimental precision an unresolved discrepancy between values from lepton scattering and muonic-atom work has been published [COD08, POH10], see Fig. 4.17. The rms charge radius of the neutron is $-0.1161 \pm 0.0022$ fm [EID04], which – with total charge zero – means that there must be positive and negative charges distributed differently over the nuclear volume.

• Thus, nucleons are not “elementary”, but have complicated internal structures.

### 4.3.3 Matter-Density Distributions and Radii

The matter density – apart from and independent of the charge or current distributions – can be investigated only by additional hadronic scattering experiments because neutrons and protons in principle need not have the same distributions in nuclei.

### 4.3.4 Hadronic Radii from Neutron Scattering

The total cross sections of 14 MeV neutron scattering under simple assumptions have been shown to also follow a $A^{1/3}$ law, see e.g. Ref. [SAT90], p. 32, cited from Ref. [?]. The assumptions were that the sharp-edged range of the nuclear force was 1.2 fm and the total cross section $\sigma_{\text{tot}}$ follows $2\pi(R + \lambda)^2$ with R the nuclear (hadronic) radius, i.e. the nuclei are considered to be black (totally absorbent) to these neutrons, which is not exactly fulfilled, as the structures in this dependence show. These can be explained with the optical model, see below. The radius constant extracted from this systematics is

$$R_{\text{hadr}} = 1.4 \text{ fm}. \quad (4.38)$$

In addition, there have been attempts to extract the neutron radius of $^{208}\text{Pb}$ from parity-violating electron scattering [?].

### 4.3.5 Special Cases – Neutron Skin

Especially the question of a neutron skin in nuclei with neutron excess is interesting and only recently such a thin skin was consistently shown to exist, see e.g. [?] and references therein. Among the hadronic probes used have been protons, $\alpha$’s, heavy ions, antiprotons, and, recently, also pions.
4.4. THE SIZE AND SHAPE SYSTEMATICS OF NUCLEI

...the extraction of rms radii requires some model assumptions concerning the reaction mechanism and the interplay of hadronic and Coulomb interactions. The pion results are derived from two sources: pionic atoms (in analogy to the derivation of the electromagnetic radii from muonic atoms) and total reaction cross sections of $\pi^+$ [?]. The neutron skin is related to the symmetry energy, which plays a role in the in the mass formula of Bethe and Weizsäcker for the binding energies of nuclei, especially for “asymmetric” nuclei with strong neutron excess, but also for astrophysics and nuclear-matter calculations. The radius of neutron stars is closely related to the symmetry energy value in high-density nuclear matter, see e.g. Ref. [?].

Usually the quantity

$$\delta R_{np} = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}. \quad (4.39)$$

The experimental values deduced from different experiments are on the order of $\delta R_{np} \approx 0.2$ fm.

4.4 The Size and Shape Systematics of Nuclei

Whereas the charge and magnetic distributions are best obtained with charged projectiles, for which the interaction is exactly known (e.g. electrons, which do not interact via the strong force), for neutrons one needs nuclear scattering models (e.g. the optical model, see Section 8.1). The assumption that neutron and proton radii of heavier nuclei are about equal has proved too simple with the evidence of neutron-halo and neutron-skin nuclei, see Chapter 5.
Figure 4.3: Squares of the Fourier transforms – basically the form factors determining the shapes of the cross sections – of different charge-density distributions.
FIG. 3. Angular dependence of the elastic scattering cross section for alpha particles scattered from Ta, Pb, Th, and Au.

Figure 4.4: TEXT.................
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Figure 4.5: TEXT...........................

Figure 4.6: TEXT.......................
Figure 4.7: By plotting the ratio of the experimental differential cross sections to the calculated (point) Rutherford cross sections as functions of the classical minimum distances between the centers of the colliding nuclei, a universal behavior becomes apparent.
Figure 4.8: Plot of the scattering cross sections (relative to the Rutherford cross section) of many different HI pairings vs. the distance parameter d in fm. The data used for the fit are in Ref. [?], see also [?].

Figure 4.9: Connection between electron scattering cross sections and density distributions on pointlike and extended nuclei.
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Figure 4.10: Differential cross section for 500 MeV electrons fitted by an exponential form factor.
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Figure 4.11: Charge density distribution and Coulomb potential of a point charge compared to an extended homogeneous charge distribution and its potential.

Figure 4.12: Experimental setup for the production of a beam of muons and slowing-down and capturing the muons into Bohr orbits. After [FIT53].
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Figure 4.13: NaI scintillator detector setup for the measurement of muonic X-rays, emitted in 2p-1s transitions to the ground states of different muonic atoms. After [FIT53].

Figure 4.14: Muonic X-ray spectrum of Pb obtained with a NaI scintillation detector and showing the large energy shift between a point charge and the actual extended-charge distributions. After [FIT53].
Figure 4.15: Muonic X-ray spectrum of Ti obtained with a NaJ scintillation detector and showing the large energy shift between a point charge and the actual extended-charge distributions. After [FIT53].

Figure 4.16: Charge density distributions of different doubly closed-shell nuclei with electron-scattering and muonic-atom data combined. After [FRO87].
4.4. THE SIZE AND SHAPE SYSTEMATICS OF NUCLEI

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Figure 4.17: Unresolved discrepancies between determinations of the proton’s rms radius by different methods. The accepted CODATA-06 value is \( r_{\text{rms}}(p) = 0.88768(69) \text{ fm} \) whereas the new muonic-atom value is \( r_{\text{rms}}(p) = 0.84184(67) \text{ fm} \). After [COD08, POH10].

Figure 4.18: Stanford facility.
Figure 4.19: Stanford spectrometer.
Bibliography


Chapter 5

Halo Nuclei and Farewell to Simple Radius Systematics

At the “rims” of the valley of stability (the neutron or proton driplines) there are a number of nuclei that have much larger radii than expected from the systematics. $^{11}$B has about the same radius as $^{208}$Pb.

First experimental evidence of halos in 1985 were deviations of reaction cross sections $\sigma = 4\pi r^2$ from the systematics expected as described in Chapter 4 in light nuclear isotopes far from the valley of stability, such as $^{11}$Li [TAN85], see also Refs. [OZA01, KRI12]. With increased interaction (absorption) radii between nuclei in heavy-ion reactions also narrower momentum distributions of breakup fragments in such reactions have been observed. [DOB06].

Also the deuteron has an extreme rms radius of about 3.4 fm. In all cases the nuclei seem to have a halo of weakly bound neutrons (or protons), which surrounds a more strongly bound core. Different cores are possible, i.e. besides the strongly bound $\alpha$ making $^5$He and $^6$He one- and two-neutron halo nuclei also $^4$Be forms some type of core. Generally indications of halo structures are – among others – the exceptionally large cross sections in heavy-ion reactions, narrower momentum distributions of the nucleons in the nuclei, and larger radii, as compared to the $A^{1/3}$ law. Fig. 5.1 shows the low-mass portion of the chart of nuclides where halo nuclei have been found. The scientific interest in halo nuclei is manifold. They were among the first where the driplines have been reached. The results show that the shell structures established for the valley of stability can be extended to “exotic” nuclei, but with modifications of the closed shells, i.e. with new magic numbers emerging. The low mass numbers invite application of microscopic theories such as Faddeev-(Yakubowsky), no-core shell models, Green’s function Monte Carlo (GFMC), and other approaches to test nuclear forces, e.g. three-body forces, or effective-field (EFT) approaches. Impressive results have been obtained by such “ab initio” calculations, see e.g. Ref.[PIE01, DEA07]. A special role is played by the so-called Borromean nuclei, i.e. those that consist of a core plus two weakly (un)bound neutrons at large radii, and for which any of the two-particle subsystems are unbound (Example: $^4$He + n + n). They can be treated by well-established three-body methods. Their name is derived from the three intertwined Borromean rings that fall apart when one ring is
Figure 5.1: Halo nuclei at the driplines of the chart of nuclides.

Figure 5.2: Coat of arms and symbol of the Renaissance Borromean family (and other north Italian families like the Sforzas) at their castle on the Borromean island Isola Bella in the Lago Maggiore, Italy.

removed and hold together only when united, see Fig. 5.2. Many nucleosyn-
thesis processes pass through nuclei that are neutron rich or neutron poor and are not well known. Thus, for nuclear astrophysics, a better understanding of all these reactions and their reaction rates is essential.

Since we deal with unstable (radioactive) nuclei the “radioactive-ion beams (RIB)” facilities, which are being developed are especially suited for their investigation. These facilities collect, focus and accelerate nuclear reaction products in order to use them as projectiles in reactions. The Figs. 5.3 and 5.4 show the properties typical for halo nuclei:
• They have radii, which are larger than predicted from the usual $A^{1/3}$ systematics.

• Their density distributions reach further out than usual.

• In agreement with this they show narrower momentum distributions of the breakup fragments of the halo nuclei (one example: $^{19}\text{C} \rightarrow ^{18}\text{C} + \text{n}$, compared to $^{17}\text{C} \rightarrow ^{16}\text{C} + \text{n}$).

The latest discovery of a halo nucleus is that of $^{22}\text{C}$ [TAN10] which showed an increased reaction cross section and an rms radius of $r_{\text{rms}} = 5.4 \pm 0.9 \text{ fm}$, both larger than expected from the usual systematics.

Figure 5.3: Fragment-momentum distribution and density distributions in halo nuclei.

Figure 5.4: Radii of halo nuclei.
Bibliography


Chapter 6

The Particle Zoo

6.1 The Pion

Yukawa in 1935 postulated an exchange particle with a mass of \( \approx 130 \text{ MeV}/c^2 \), commensurate with the range of the (hadronic) nuclear force of about 1.4 fm. For a time the muon, found in 1932 in cosmic radiation, was mistaken as this particle, but – being a heavy lepton – did not have the expected properties. Only in 1947 the pion was detected as a component of cosmic rays [LAT47a, LAT47b, LAT47c] in photoplates, see Fig. 6.1, decaying as

\[ \pi^\pm \rightarrow \mu^\pm + \nu_\mu. \]  

(6.1)

The sign could not be determined due to the lack of a strong magnetic field. The pion was identified with a mass of \( \approx 300 \text{ me} \) by its shorter lifetime (range) and higher ionization grain density than those of the muon. All muon tracks observed were of about equal lengths, corresponding to a pion decay at rest. Sometimes the pion decay would be accompanied by a number of strong

hadronic tracks of short range, i.e. \( \alpha \) particles etc, forming a “star”, which can be taken as evidence of an hadronic interaction, i.e. through the strong interaction between the pion and an emulsion nucleus, in contrast to the pion decay that is a weak-interaction process. Fig. 6.2 shows such an event. Soon

![Figure 6.1: Photoplate tracks of the decay of a cosmic pion into a muon and an invisible muonic (anti)neutrino, as seen through a microscope. The typical length of the muon track is \( \approx 0.61 \text{ mm} \).](image-url)
after this discovery – in 1948 – the artificial production of pions in accelerators of sufficient energy, e.g. the 184" synchrocyclotron at Berkeley was successful [GAR48, BUR48, JON49]. A carbon target positioned inside the magnetic field of the synchrocyclotron was bombarded with protons of energies starting near the production threshold just below 200 MeV. Positive and negative pions were deflected by the magnetic field in opposite directions and registered by two stacks of photoplates. These showed not only microscopic tracks of pions but also their decay into muons (with typical track lengths of 0.6 mm) and the $\beta$ decay of the muons. The properties of pions could now be studied as function of their energies, e.g. by scattering them from hydrogen. The interaction strength and the shape of the differential cross sections pointed to the strong interaction as the one keeping nuclei together (against the repulsive Coulomb force between the protons). The spin and parity of the pions were determined to be $J = 0^-$ (they are pseudoscalars), their isospin $T = 1$, meaning that they come as an isobaric triplet $\pi^\pm, \pi^0$.

Especially the scattering of pions from protons proved to be very fruitful. Resonances in excitation functions were indications of the formation of intermediate states, corresponding to excited states of the nucleons. The most
prominent is the excitation of the $\Delta$ resonance. Because of the high energies involved and thus the high number of decay channels these excited states are rather short-lived, i.e. the resonances are quite broad. The $\Delta$ has $J^\pi = 3/2^+$ and isospin 3/2 meaning that it comes as an iso-quartet. These quantum numbers require a spin flip whereas another such resonance, the Roper resonance has the same quantum numbers as the proton and can be interpreted as an excited proton without change of internal structure. In this way a large number of “particles” could be created, such as the baryons, which encompass the nucleons and the hyperons $\Delta, \Lambda, \Sigma, \Xi$, and $\Omega$, the latter four carrying the new quantum number of strangeness. In the constituent-quark model all baryons are made of three quarks $u$ and $d$, quark-antiquark pairs, allowing for $s$ and $\bar{s}$, quarks, and gluons. In similar experiments a number of mesons was discovered: The scalar mesons: $\rho, \omega, \phi$, and $K^*$ and the pseudoscalar mesons $\pi, \eta, \eta'$, and $K$. All mesons consist of quark-antiquark pairs. The multitude of particles is often called “The Particle Zoo”, into which only the assignment of quark combinations brought a perfect order.

6.2 Quasi-Elastic Electron Scattering – Excited Nucleons and the Particle Zoo

Spectra of the inelastic electron scattering from the proton at relatively high energies show besides the elastic peak a number of excited states (like in nuclear scattering spectra), corresponding to excited nucleon states (nucleon resonances),

6.3 Deep-Inelastic Lepton Scattering – Partons inside Hadrons

Once higher electron energies became available, finer structures of nucleons and nuclei could be explored. The key experiment was performed by Jerome Isaac Friedman, Henry Way Kendall, and Richard Edward Taylor (NP 1990), see Ref. [TAY67] around 1966 and yielded clear evidence of “parton” structures inside the nucleons that had all the properties of the quarks. The “scaling” behavior (as in the classical Rutherford scattering proves that the constituents apparently are pointlike and therefore truly “elementary”.

The scattering of neutrinos from nuclei required a number of special preparations. In order to form an “intense” beam of neutrinos relativistic kinematics had to be applied, which causes reaction products to be emitted into narrow forward cones. The neutrinos were produced in the decay of pions and muons that accompany the reactions of high-energy protons with suitable solid targets. The invention of the neutrino horn [?] appreciably increased the neutrino beam density.

The search for neutrino oscillations (transformation into other neutrino flavors as function of the travelling distances or flight time) has led to very large detectors filled with neutrino-sensitive liquids and scintillators and surrounded by very many photomultipliers (SUPER-KAMIOKANDE, SUD-
bury observatory, kamland, gran sasso etc.) looking for solar, cosmic, and reactor-produced neutrinos. Thus, neutrino oscillations have been discovered and the solar-neutrino puzzle has been solved.
6.3. DEEP-INELASTIC LEPTON SCATTERING – PARTONS INSIDE HADRONS

FIG. 1. $(d^2\sigma/d\Omega dE')/\sigma_{\text{Mott}}$, in GeV$^{-1}$, vs $q^2$ for $W = 2$, 3, and 3.5 GeV. The lines drawn through the data are meant to guide the eye. Also shown is the cross section for elastic $e-p$ scattering divided by $\sigma_{\text{Mott}}$, $(d\sigma/d\Omega)/\sigma_{\text{Mott}}$, calculated for $\theta = 10^\circ$, using the dipole form factor. The relatively slow variation with $q^2$ of the inelastic cross section compared with the elastic cross section is clearly shown.

Figure 6.3: Double differential cross section of deep-inelastic scattering of 500 MeV electrons [BLO69].
Figure 6.4: Big European Bubble Chamber at CERN.
Figure 6.5: Neutrino Scattering event in the Big European Bubble Chamber at CERN.
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Neutrino horn ???????????????
Chapter 7

Production of the Antiproton, the first Antiparticle from a Nuclear Reaction

The conservation laws (energy and momentum conservation, and charge conjugation) require that when bombarding a nuclear target with (high-energy) protons and with an antiproton in the exit channel the antiparticle must be one of a particle-antiparticle pair (i.e. $p + \bar{p}$). In addition the ejectile and recoil particles will appear, thus four particles are emitted. The proton lab. energy threshold for antiproton production on a fixed proton target is thus

$$E_{\text{thr}} = 7 \cdot m_p = 6.537 \text{ GeV}. \quad (7.1)$$

Such an energy could only be reached with synchrotron accelerators. In this case the Bevatron at Berkeley, which was designed for a maximum beam momentum of 6.3 GeV/c was used to create the first antiprotons in 1955 [CHA55]. Fig. 7.1 shows schematically the setup to produce antiprotons.

In the actual experiment – because of the insufficient energy for protons on protons – a copper target was used. Because of the binding of the protons in the heavier target the Fermi momentum of the nucleons could be exploited to add some relative energy to the projectile-target system with the effect of lowering the threshold energy for $\bar{p}$ production. With the approximate factor

$$1 - \frac{p_F}{M_p c} \quad (7.2)$$

the threshold momentum is lowered to a value of about 4.8 GeV/c.
Figure 7.1: Experimental setup for the first detection of antiprotons at the Berkeley Bevatron.
Bibliography

Chapter 8

Observation of Direct Interactions

Together with compound nuclear CN reactions direct interactions DI are two forms of reactions of composite nuclei that are best classified according to their time behavior. They mark the extremes in this classification. The DI are processes, which occur at time scales of the traversal times of projectiles past target nuclei (typical times are $10^{-22}$ s). On the other end are the CN reactions where projectiles are more or less completely absorbed by the target to form a compound nucleus living a long time during an equilibration process and decaying into open channels without memory of the formation process (typical times are $10^{-16}$ s).

8.1 Elastic Scattering and the Optical Model

Around 1950 it was noticed that the scattering data of intermediate-energy nucleons (in the beginning mainly neutrons) could not be described well in the framework of the compound-nucleus models. Angular distributions as well as excitation functions showed marked diffraction patterns. Very early proton data were measured by Burkig et al. [BUR51] using 18.6 MeV protons on several nuclei as shown in Fig. 8.1. Early neutron elastic scattering was reported by Bratenahl et al. [BRA50]. For the description of the data two limiting cases of models inspired by phenomena in optics, the opaque model and the transparent model [FER49]. These data were then fit with two different optical model theories [LEL52]. For the one of them that gave a better fit to the diffraction pattern the authors coined for the first time the term Optical Model. A picture of the first “Optical Model” fit is shown in Fig. 8.2.

As standard literature on the optical model only a few references will be given here: [HOD63, HOD67, MAR70] The radial Schrödinger equation for protons (spin $s = 1/2$) reads:

$$\frac{d^2}{dr^2} + k^2 - \frac{\ell(\ell + 1)}{r^2} + Vf(r) + iWg(r) - V_C(r) + (V_{s.o.} + iW_{s.o.})h(r) \cdot \left\{ \begin{array}{c} \ell \\ -(\ell + 1) \end{array} \right\} u_{ij}^{(+)}(kr) = 0. \tag{8.1}$$
Here: \( V, W \) are real and imaginary parts of the central potential, \( V_C \) the Coulomb potential, and \( V_{\text{o.o.}}, W_{\text{s.o.}} \) real and imaginary parts of the spin-orbit potential, especially important for die description of polarization observables. For neutrons the Coulomb term vanishes. The solution of this equation is part of the total scattering function

\[
\Psi = \frac{1}{kr} \sum_{\ell j \lambda} i^\ell \left[ \frac{4\pi (2\ell + 1)}{\ell 0 s \mu | jm \rangle (\ell \lambda s \nu | jm \rangle u_{\ell j}^{(\pm)}(kr) Y_{\ell}^{\lambda}(\theta, \phi) \chi^\mu e^{i\sigma_\ell}} \right]^{1/2}
\]

with

\[
u_{\ell j} \to r \to \infty \frac{1}{2i} \left[ e^{-i(kr-\eta \ln 2kr-\ell \pi/2)} - e^{2i\delta_{\ell j} e^{i(kr-\eta \ln 2kr-\ell \pi/2+2\sigma_\ell)}} \right].
\]

\( \delta_{\ell j} \) are the complex nuclear scattering phases, \( \sigma_\ell = \arg \Gamma(1 + \ell + i\eta) \) the Coulomb scattering phases, \( \eta_\ell = e^{2i\delta_{\ell j}} \) the “reflection coefficients”, \( \eta = Z_1 Z_2 e^2/\hbar^2 v \) the Coulomb (or Sommerfeld) parameter (see Eq. 2.39), and \( k = \sqrt{2\mu E_{\text{kin}}/\hbar^2} \) the entrance-channel wavenumber.

The potential form factor \( f(r) \) is defined in analogy to the shape of the usual nuclear density or potential distributions that are used in the classical shell model (Woods-Saxon form). The absorption occurs predominantly at the nuclear surface. Thus, as form for \( g(r) \) at low energies one chooses the derivative of the Woods-Saxon-form factor, and for the spin-orbit term \( h \) the Thomas form \( g(r)/r \). At higher energies more absorption in the nuclear volume is plausible, which is taken into account by a gradual transition from the surface absorption to volume absorption.

The best sets of parameters have been obtained by fits with \( \chi^2 \) minimization to a large number of data sets of cross sections as well as analyzing powers. The latter are important for fixing the \( (L \cdot S) \) potential and removing typical ambiguities in the potential parameters. A few of these parameter sets have become standards for the optical model. For nucleon scattering the parametrization most used is that of Greenlees and Becchetii [BEC69], especially because they provide a global set of parameters (i.e. valid over a large region of the periodic table). However, in special cases e.g. near doubly-magic nuclei this set is not as good as a single fit. It is interesting that the depth of the real potential corresponds closely to that of the shell model potential, similarly for the LS term. For light projectiles consisting of \( A \) nucleons (deuterons, \( \alpha \) particles etc.) one has potential depths that are \( A \)-fold the nucleon potential depths. For heavy-ion scattering there are quite different approaches, partly with very shallow potentials. Fig. 8.3 shows the form factors and the behavior of imaginary potentials with energy. It can only be mentioned in passing that steps to found the optical potentials on more microscopic grounds have been undertaken by creating folding potentials. In these the potential of one nucleon of the projectile with the target nucleus is folded with the nucleon density of the projectile nucleus and vice versa, or the same for both nuclei (double folding potentials).

### 8.2 Direct (Rearrangement) Reactions

The multitude of these reactions may be classified:
8.3. STRIPPING REACTIONS

- Reactions without change of the mass number
  - Elastic potential scattering (see above; description by the optical model).
  - Direct inelastic scattering \([p,p'\gamma], (\alpha, \alpha'),...\]. It leads preferentially to collective nuclear excitations (such as rotation, vibration etc.).
  - Quasi-elastic (charge-exchange) processes \((p,n), (n,p), (^3\text{He},t), (^{14}\text{N},^{14}\text{C}),...\). These lead e.g. to isobaric-analog states of the target nucleus.

- Reactions with change of the mass number
  - Pickup reactions [One-nucleon transfer: \((p,d), (d,^3\text{He}), (d,t),...,\], few or multi-nucleon transfer: \((p,\alpha), (d,^6\text{Li}),...\].
  - Stripping reactions [One-nucleon transfer: \((d,p), (d,n), (^3\text{He},d),...\] few- or multi-nucleon transfer: \((^6\text{Li},d), (\alpha,p), (^3\text{He},p),...\].
  - Knockout reactions \([p,\alpha), (p,p')...,\].
  - Direct breakup processes like knockout with few-particle exit channels \([p,pp), (\alpha, 2\alpha),...\].
  - Induced fission is a special case of a rearrangement reaction resulting in larger debris.
  - Processes of higher order (multi-step processes via excited intermediate states, coupled channels).

Here only the simplest case of the stripping reaction will be discussed. The many details of direct interaction processes are subjects of a large number of books, see e.g. Refs. [AUS70, SAT83, GLE63, GLE83]. Standard computer codes such as DWUCKn (distorted wave code) and CHUCKn (coupled channels distorted wave code) [KUNZ] are available. For induced nuclear fission see Section ??.

8.3 Stripping Reactions

Already a semi-classical ansatz provides a qualitative picture of the angular distributions of stripping reactions. It explains the expected behavior with the assumption of a rapid process that is localized at the nuclear rim and is non-equilibrated. The wave-number vector of the incoming deuterons is \(\vec{k}_d\), those of the transferred nucleon and of the outgoing nucleons are \(\vec{k}_n\) and \(\vec{k}_p\), respectively. They form a momentum diagram, from which the connection between a preferred scattering angle \(\theta\) and the transferred momentum and also the angular momentum can be deduced:

\[ p_n R = \hbar k_n R = \hbar \ell_n \]  \hspace{1cm} (8.4)

For small \(\theta\)

\[ \theta_0 \approx \frac{k_n}{k_d} = \frac{\ell_n}{k_d R} \]  \hspace{1cm} (8.5)
Because of the quantization of $\ell$ there are discrete values of $\theta$ increasing with $\ell$. This qualitative picture is not changed when calculating the angular distributions quantum-mechanically. As an example for the reaction $^{52}\text{Cr}(d,p)^{53}\text{Cr}$ the angles of the stripping maximum in calculated in different ways are

\begin{align*}
\theta_{\text{DWBA}} & \quad \theta_{\text{PWBA}} & \quad \theta_{\text{s.c.}} \\
\ell = 0 & \quad 0^0 & \quad 0^0 & \quad 0^0 \\
\ell = 1 & \quad 18^0 & \quad 13^0 & \quad 13^0 \\
\ell = 2 & \quad 34^0 & \quad 19^0 & \quad 26^0 \\
\ell = 3 & \quad 49^0 & \quad 30^0 & \quad 39^0 \\
\ell = 4 & \quad 64^0 & \quad 40^0 & \quad 52^0 \\
\end{align*} (8.6)

The measured angular distributions of the cross sections show – in addition to diffraction structures – marked stripping maxima, which often allow the determination of the angular momentum of the transferred nucleon. Historically this feature was (and still is) important for the assignment of the final-nuclear states of a reaction to orbitals in the one-particle shell model. The energy relations in stripping reactions are such that the transferred nucleon near magic shells is preferentially inserted into low-lying shell-model states. Because of the spin-orbit splitting the complete assignment requires that also the total angular momentum $j$ of the transferred nucleons is known. A good method is the measurement of the analyzing power of the stripping reaction, i.e. the use of polarized projectiles. In many cases the distinction between the two possibilities for $j$ can be made just from the sign of the analyzing power alone. For an intuitive description of this fact there exists again a simple semi-classical model (Newns). It is assumed that the interaction happens at the nuclear surface and that we have a relatively strong absorption in the nuclear matter. Thus the front side of the nucleus directed towards the projectile contributes more strongly to the reaction than the backside of the target nucleus. In the front part the orbital angular momentum vector points upward perpendicularly to the reaction plane whereas in the back half the orbital angular momentum points down. If the incident deuteron is polarized up or down perpendicularly to the scattering plane – under the assumption of the existence of a spin-orbit force – the transferred nucleon in the scattering to the left ends preferentially in a state with $j = \ell + 1/2$ for the up case, in the down case with $j = \ell - 1/2$. The measured analyzing power

\[ A_y = \frac{1}{P_d} \frac{N_{\text{up}} - N_{\text{down}}}{N_{\text{up}} + N_{\text{down}}}, \] (8.7)

will show opposite signs for the two cases. This behavior has been confirmed for many examples not only for stripping reactions. If one wants to know the degree, to which the transition considered is a single-particle transition the spectroscopic factor has to be determined. For that – at least approximately – quantitative theories are required (DWBA, CC-DWBA).

Because the single-particle strength is often strongly fractionated by the residual interaction, i.e. spread out over many states in a range of energies spectroscopic investigations on very many final nuclear states are necessary.
Often these states are close together and high detector resolution to get a complete picture is necessary. Especially useful tools for this purpose are magnetic spectrographs with high resolution at tandem Van-de-Graaff accelerators, also with polarized particle beams (for more details see Section ??).

8.4 The Born Approximation

Here one uses the first Born approximation, i.e. the first term of the Born series. Starting from Fermi's Golden Rule of perturbation theory, which predicts for the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{(2I_b + 1)(2I_B + 1)}{2\pi^2\hbar^2} \mu_I^\mu_f^\mu_i^\mu_f^k_i^k_f^|T_{if}|^2.$$  (8.8)

one has to make assumptions about the transition matrix element.

In the Plane Wave Born Approximation PWBA (also: Butler theory) for the incoming and outgoing waves plane waves are used. Since the radial wave functions are Bessel functions one finds a simple diffraction pattern for the cross section

$$\frac{d\sigma}{d\Omega} \propto [j_\ell(kR)]^2.$$  (8.9)

For illustration Fig. 8.8 shows the few lowest-order spherical Bessel functions squared. The angle dependence of the stripping maximum is contained only in the momentum relation

$$k^2 = k_{in}^2 + k_{out}^2 - 2k_{in}k_{out}\cos\theta.$$  

Only in a few simple cases the angular distribution near the maximum are satisfactorily described by PWBA. It also makes no statements about polarization observables and contains no information about nuclear structure.

Better results at least for forward angles are obtained with the Distorted Wave Born Approximation DWBA. It was formulated with a number of additional and far-reaching assumptions:

- In the entrance and exit channels distorted waves are used, i.e. the wave functions are the solutions obtained from fits of the optical model (OM) to the elastic-scattering data in each pertaining channel at the proper channel energy. E.g. for the description of the reaction A(d,p)B the OM wave functions from the fit to the data of the scattering A(d,d)A as well as of B(p,p)B are needed. Thus – still in the first Born approximation – the diffraction and absorption of the incoming and outgoing waves in the nuclear (and eventually the Coulomb) field, as well as the effect of the LS potential (see also Fig. ??) are taken into account.

- The nuclear initial and final states are shell-model states.

- The finite range of the nuclear forces is taken care of by a finite-range or even zero-range approximation.

- The T matrix is expanded into partial waves belonging to fixed angular-momentum transfer.
The transfer matrix is factorized into a nuclear-structure dependent and into a kinematical part.

Thus the cross section reads:

\[
\frac{d\sigma}{d\Omega} = \frac{\mu_a h_b (m_B)^4}{\pi h^4 m_A} \frac{2J_B + 1}{(2J_A + 1)(2s_a + 1)} \sum_{\ell sj} \left| A_{\ell sj} \right|^2 \sum_m \left| \beta_{\ell sj}^m \right|^2
\]

The experimental cross section is a product of a fit parameter, the spectroscopic factor \( S_{\ell j} \), and a theoretical cross section calculated in the framework of the DWBA with the assumption of single-particle states:

\[
\left( \frac{d\sigma}{d\Omega} \right)^{\ell j}_{\text{exp}} = S_{\ell j} \left( \frac{d\sigma}{d\Omega} \right)^{\ell j}_{\text{DWBA}}
\]

In a stripping process the spectroscopic factor is the square of the amplitude of a fragment of a single-particle state of the final nucleus. Because of this fractionization (which in reality is caused by the residual interaction of the many other nucleons) into many states with equal quantum numbers the strengths of all these states have to be summed up. If a complete collection from all these states is possible one obtains the total strength, which can also be calculated because the number of nucleons \( N \) in a subshell is known. Therefore sum rules can be applied, e.g. for single-particle stripping \( \sum S_{\ell j} = (2J + 1) \). Mathematically the spectroscopic factor is the overlap integral between the anti-symmetrized \( k \)-particle final-nuclear state \( \Psi_A(i) \), into which the nucleon is inserted, and the single-particle configuration of the anti-symmetrized \( (k-1) \)-particles target, the nuclear ground state (core), and the single-particle wave function of the transferred \( k \)-th particle \( \Psi(j) \). It thus gives the probability, with which a certain state is present in this configuration. When averaging over the strength distribution of all states that are fractions of one single-particle state (e.g. while assuming a Breit-Wigner distribution function) the position of the average energy provides the energy of the single-particle state, whereas the width of the distribution is a measure of its lifetime, the (spreading width) \( \Gamma \). It measures the decay of the single-particle state into the real nuclear states, split and spread out by the residual interaction, and thus its strength.
Figure 8.1: Angular distributions of 84 MeV neutrons scattered from different targets.

Fig. 5. Differential scattering cross sections in units of $10^{-28} \text{ cm}^2$ per steradian. The dashed and dotted curves show expected patterns from opaque nuclei with cross sections $\sigma_0$ and $(\sigma_0 - \sigma)$, respectively. (See Section VI-C.)
CHAPTER 8. OBSERVATION OF DIRECT INTERACTIONS

Figure 8.2: Fit with the first “Optical Model”.

Figure 8.3: Form factors of the optical model. Upper: Woods-Saxon form of the real part $f$. Center: Derivative Woods-Saxon form $g = f'$ of the imaginary part. Lower: Sliding-transition form of the surface-to-volume imaginary part as function of energy.
Figure 8.4: Global fit of the optical model to elastic scattering data of 14.5 MeV protons for a large nuclear mass range. The cross sections are normalized to the Rutherford cross sections (i.e. to 1 at 0°), the analyzing powers are 0 at 0°. The arrows indicate the systematic variation of characteristic diffraction maxima with the target mass.
Figure 8.5: Angular and energy dependence of the cross section of elastic proton scattering from $^{90}$Zr calculated with standard Greenlees-Becchetti parameters of the optical model. The interference structure of the angular distributions may be interpreted as “resonant” (single-particle) structures of the excitation function with widths typical for fast (i.e. direct) processes. They are also analogous to diffraction structures in classical optics.

Figure 8.6: Characteristic and systematic features of the stripping maximum as function of the transferred orbital angular momentum $\ell$. The arrows indicate the increase of the reaction angle of the stripping peak with $\ell$. 
8.4. THE BORN APPROXIMATION

Figure 8.7: Sensitivity (sign!) of the analyzing power of the stripping reaction to the total angular momentum $j$ of the transferred nucleon.

Figure 8.8: The behavior of the squares of the lowest-order spherical Bessel functions $j_\ell(kr)$ as functions of $x = kr$. 
Bibliography

Chapter 9

Resonances and Compound Reactions

9.1 Generalities

Resonances are a very general phenomenon in nature and therefore in all of physics. In classical physics they appear when a system capable of oscillations is excited with one or more of its eigenfrequencies, which – depending on the degree of damping – may lead to large oscillation amplitudes of the system. Nuclei are no exception. When tuning the system (changing the exciting frequency) these amplitudes pass through a resonance curve of Lorentz form. In particle physics most of the many known “particles” actually appear as resonances, i.e. as quantum states in the continuum, which decay with characteristic widths (or equivalently: lifetimes).

Resonances can be discussed in the energy picture where as function of energy excursions of Lorentz form (Breit-Wigner form) with a width $\Gamma$ appear, but also in the complementary time picture where they appear as quantum states in the continuum, i.e. as states, which decay with finite lifetime $\tau$. Between them there is the relation

$$\Gamma = \hbar/\tau$$

(9.1)

In nuclear physics resonances appear in the continuum (i.e., in scattering situations, at positive total energy) when the projectile energy in the c.m. system plus the Q value of the reaction just equals the excitation energy of a nuclear state.

The excitation functions of observables such as the cross section show characteristic excursions from the smooth background when varying the incident energy. The background may be due to a direct-reaction contribution from Coulomb or shape-elastic scattering or – in a region of high level density – may be the energy-averaged cross section of unresolved overlapping compound resonances; in this case resonant excursions would be due to doorway mechanisms. Likewise the scattering phases and scattering amplitudes change in characteristic ways over comparatively small energy intervals. Nuclei may be excited into collective modes such as rotations and/or vibrations.
of a part of the nucleons. At still higher energies new phenomena with high cross sections in charged-particle, neutron, $\gamma$, and $\pi$ induced reactions appear involving up to all nucleons of a nucleus, the Giant Resonances, see Section ??.

Fig. 9.1 shows schematically the phenomena in different energy regions.

![Figure 9.1: The excitation of single resonances, overlapping resonances (with and without Ericson fluctuations) and giant resonances as functions of the energy in the continuum region above the bound-state energy.](image)

### 9.2 Theoretical Shape of the Cross Sections

A model assumption for resonances is – in contrast to direct processes – that the system goes via an intermediate state from entrance into the exit channel. For this case perturbation theory gives the following form of the transition matrix element

$$
\langle \Psi_{\text{out}} | H_{\text{int}} | \Psi_{\text{in}} \rangle = \text{const} \frac{E - E_R}{E - E_R}.
$$

(Eq. 9.2)

$E_R$ is the energy of the nuclear eigenstate. However, since it is a state in the continuum it is not a stationary but one, which decays in time. Such states are best described by giving it a complex eigen-energy:

$$
\tilde{E_R} = E_R + i\Gamma/2.
$$

(Eq. 9.3)

The interpretation of the imaginary part is: the time development of a state has the form $e^{i\tilde{E}/\hbar}$, on the other hand the state decays with a lifetime $\tau$, whence

$$
\frac{1}{\tau} = Im(\tilde{E}) = \Gamma/2.
$$

(Eq. 9.4)

The resonance amplitude thus has the form:

$$
g(E) = \frac{F(E)}{E - E_R + i\Gamma/2}.
$$

(Eq. 9.5)
The meaning of \( F(E) \) has to be determined. In the sense of Bohr’s independence hypothesis formation and decay of a resonance are independent (i.e., decoupled). Therefore, one writes the amplitude as the product of the probability amplitude for its formation and its probability of decaying into the considered exit channel. In general, for one formation channel (the entrance channel \( c \)) there will be several exit channels \( c' \).

The width \( \Gamma \) of the Breit-Wigner function is inversely proportional to the formation probability \( P \) and is the integral over the cross section in the energy range of the resonance:

\[
P = \int \frac{\sigma_{aA} v_{aA}}{V} \cdot \frac{V p_{aA}^2 d\mu_{aA}}{2\pi^2 \hbar^2} = \int \frac{\sigma_{aA} k_{in}^2 dE}{2\pi^2 \hbar} \approx \frac{k_{in}^2 F(E_R)}{2\pi^2 \hbar} \int \frac{dE}{(E - E_R)^2 + \Gamma^2/4} = \frac{k_{in}^2 F(E_R)}{\pi \hbar \Gamma}
\]  

(9.6)

In equilibrium this is equal to the probability that the resonance re-decays into the entrance channel (purely elastic case). A measure for this is the partial width \( \Gamma_{aA} \) formed similarly as \( \Gamma \), thus:

\[
\Gamma_{aA} = \frac{k_{in}^2 F(E_R)}{\pi \hbar \Gamma},
\]  

(9.7)

\[
F(E_R) = \frac{\pi}{k_{aA}^2} \Gamma_{aA} \Gamma
\]  

(9.8)

By definition \( \Gamma \) is the sum of all partial widths over the open channels. Thus, the branching ratio for the decay into one definite channel \( bB \equiv c' \) is equal to \( \Gamma_{c'}/\Gamma \) and the Breit-Wigner cross section for the formation of the resonance via channel \( c \) and the decay via channel \( c' \) is

\[
\sigma(E) = \frac{\pi}{k_{in}^2} \cdot \frac{\Gamma_c \Gamma_{c'}}{(E - E_R)^2 + \Gamma^2/4}
\]  

(9.9)

This derivation is simplified and must be carried out – when there is interference with a direct background contribution and for the description of a differential cross section via a partial-wave expansion near a resonance – with complex scattering amplitudes. For elastic s-wave scattering it results in a resonant scattering amplitude of the form:

\[
A_{res} = \frac{i \Gamma_{aA}}{(E - E_R) + i \Gamma/2}
\]  

(9.10)

When a direct background is present, then, besides the pure resonance term and the pure direct (smooth) term, a typical interference term appears, which may be constructive or destructive. For \( \sigma \) we have then:

\[
\sigma_{tot} = |A_{res} + A_{pot}|^2 = \sigma_{res} + \sigma_{pot} + 2 \text{Re}(A_{res} A_{pot}^*)
\]  

(9.11)

where \( A_{pot} \) is the amplitude of the weakly energy-variable potential scattering.
9.3 Derivation of the Partial-Width Amplitude for Nuclei (s Waves only)

The connection between the resonant scattering wave function and the wave function of the eigenstate of the nucleus is made by the \textit{R-matrix theory}. Their basic features (for more details see [?]) are approximately:

- The two wave functions and their first derivatives are matched continuously at nuclear radius (edge of the potential or similarly).

- The condition for a resonance is equivalent with the wave-function amplitude in the nuclear interior taking on a maximum value. This happens exactly if the matching at the nuclear radius occurs with a wave function with gradient zero (horizontal tangent).

Figure 9.2 illustrates the conditions for resonance. The two conditions may be summarized such that both logarithmic derivatives \( L_0 (L_0 \) for pure s waves) at the nuclear radius are exactly zero. With the form of the wave function in the external region

\[ u_0(r) = e^{-ikr} - \eta_0 e^{ikr}, \quad r > a \]  

(9.12)

and the wave numbers in the external \( k \) in the nuclear interior \( \kappa \) we obtain

\[ L_0(E) = \left( \frac{a}{u_0} \frac{du_0}{dr} \right)_{r=a}, \]  

(9.13)
9.4 First Evidence of Resonant Nuclear Reactions

which leads to the scattering function \( \eta_0 \) as function of \( L_0 \):

\[
\eta_0 = \frac{L_0 + ika}{L_0 - ika} e^{-2ika}.
\] (9.14)

Inserting this scattering function into the known expressions for elastic scattering and absorption (and with \( L_0 = \text{Re}(L_0) + i\text{Im}(L_0) \)) the result is

\[
\sigma_{\text{el}} = \frac{\pi}{k^2} \left| 1 - \eta_0 \right|^2 = \frac{\pi}{k^2} \left| \left( e^{2ika} - 1 \right) - \frac{2ika}{\text{Re}L_0 + i(\text{Im}L_0 - ka)} \right|^2
\] (9.15)

and

\[
\sigma_{\text{abs}} = \frac{\pi}{k^2} \left( 1 - |\eta_0|^2 \right) = \frac{\pi}{k^2} \left[ -4k_\text{in} \alpha \text{Im}L_0 \right. \\
\left. \frac{1}{(\text{Re}L_0)^2 + (\text{Im}L_0 - ka)^2} \right].
\] (9.16)

By expanding \( \text{Re}L_0 \) in a Taylor series and terminating it after the first term, by comparison – besides obtaining the resonance-scattering amplitude (with \( A_{\text{pot}} \propto e^{2ika} - 1 \)) – one obtains the results:

\[
\sigma_{\text{el}} = \frac{\pi}{k^2} \left[ \left( e^{2ika} - 1 \right) + \frac{i\Gamma_{\alpha A}}{(E - E_R) + i\Gamma/2} \right]^2
\] (9.17)

\[
\sigma_{\text{abs}} = \frac{\pi}{k^2} \frac{\Gamma_{\alpha A} (\Gamma - \Gamma_{\alpha A})}{(E - E_R)^2 + \Gamma^2/4}
\] (9.18)

One sees that in agreement with our definition of absorption this encompasses all exit channels except the elastic channel.

9.4 First Evidence of Resonant Nuclear Reactions

In the years around 1934 Enrico Fermi and his collaborators (Amaldi, Pontecorvo et al.) performed experiments at Rome with neutrons, especially slow neutrons that were produced by slowing down fast neutrons in hydrogenous materials. The fast neutrons were produced in \( \alpha, n \) reactions on nuclei such as \( ^9\text{Be} \) with \( \alpha \)'s from Radium or Radon sources. Their very systematic studies of the elastic scattering, but also capture reactions revealed very different cross sections for different elements. Very high absorption was observed for B and Cd, but only for very slow neutrons, also on many other nuclei without any systematics between neighboring nuclei.

Leo Szilard [SZI35] concluded already that there must exist small energy regions with these high cross sections. Niels Bohr [BOH36] used these observations to formulate his model of formation of a compound nucleus CN explaining the properties of the high cross sections as resonances in the system of the target nuclei plus one neutron being captured to form a highly excited, rather long-lived nuclear state (lifetimes up to \( 10^6 \) times longer than the traversal time).

E. Wigner [WIG55] gives a good account on these early developments. His statement, cited from that text “Experimental work constituted, in my opinion, the most important step in the development” clearly points at Fermi’s studies as key experiments leading to the CN model and theoretical developments hereafter.
Bibliography

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Chapter 10
First Nuclear Reaction with an Accelerated Beam;
Cockroft-Walton Accelerator

In an early paper [COC30]) Cockroft and Walton very clearly outline the necessity of using higher energy/higher intensity beams of projectiles for the study of nuclear reactions. They refer to the use of α particles from radioactive sources and the limitations connected with it: low intensities (a radium source with an activity equivalent to a decent charged-particle beam would require hundreds of grams of radium!), very limited energy variability and limitation to α’s only etc. They knew that in order to initiate nuclear reactions between charged particles surmounting the Coulomb barrier would need energies of many MeV. But recently G. Gamow et al. had shown that – via quantum-mechanical tunneling – nuclear reactions might be possible at much lower energies, with, however, reduced intensities [GAM29, CON29].

With this knowledge they discuss in detail possibilities of creating high voltages for particle acceleration, among them a Tesla-coil device, an AC device, and a pulse generator, and conclude that some kind of DC voltage-multiplication scheme using a high-voltage transformer, vacuum-tube rectifiers, and capacitors could be suitable. In addition they also discuss methods to produce low-energy high current ion beams in canal-ray tubes such as used for mass-spectrometry.

In 1932 Cockroft and Walton [COC32a, COC32b] published the results of their efforts to produce sufficiently high DC voltages to initiate a nuclear reactions. They developed a multiplier circuit using rectifier diodes and capacitors that could withstand high voltages up to 200 kV. Such circuits had been devised elsewhere [GRE14, SCH19, GRE21, SLE28], but Cockroft and Walton adapted them to their special needs. Fig. 10.1 explains the principle of the circuit and the vacuum accelerating-tube design in several stages connected to different voltages from the rectifier (For a comprehensive survey see Ref. [BAL59]). Figs. 10.2 and 10.3 show a schematic and a photograph of the entire accelerator setup. The proton beam of up to 15 μA could be accelerated to about 720 keV and focussed only by the accelerating tube arrangement with no extra focussing elements. In a first series of experiments the proton beam, described as a visible “pencil” exiting the thin mica win-
Figure 10.1: Schematic of the voltage multiplication circuit used in the first “Cockroft-Walton” accelerator and accelerator tube setup.

dow into open air, was used to measure the proton range in different gases, see Fig. 10.4.

In a modified setup the protons impinged on a Li target under 45° in a chamber shown in Fig. 10.4. The reaction particles were observed as scintillations on a ZnS screen with a microscope and also in a cloud chamber where by the thickness of the tracks they were identified as α particles. Together with a second such microscope setup on the opposite side frequent coincident emissions of two α’s were registered such that the authors unambiguously concluded that the

\[ ^7\text{Li} + p \rightarrow 2\alpha + 17.347\text{ MeV} \]  

(10.1)

had taken place. This was thus the first nuclear reaction initiated by artificially accelerated projectiles (and also one of the first (the first?) coincidence experiments) [COC32b].

The authors varied the proton energy and saw a typical Coulomb threshold behavior. Then they subjected a remarkably large number of elements to the proton beam and registered the relative number of counts per unit time and beam current (i.e. essentially the cross sections for (p,α) reactions).
Figure 10.2: Schematic of the accelerator setup

Figure 10.3: Photograph of the accelerator complex. In the box covered by a black cloth the experimenter would sit and count scintillations on a fluorescent screen e.g. as function of the scattering angle
Figure 10.4: Reaction chamber setup with a vacuum pump system, a thin mica window allowing protons to exit or to impinge on a Li target. In this latter case (left) a microscope was used to count the reaction alphas as scintillations on a fluorescent screen. When using a second scintillator on the opposite side coincident events could be observed.
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Chapter 11

Key Experiments Connected to Van de Graaff, Cyclotron, Synchrotron
CHAPTER 11. KEY EXPERIMENTS CONNECTED TO VAN DE GRAAFF, CYCLOTRON, SYNCHROTRON
Chapter 12

Discovery of the Neutron (Nuclear Kinematics etc.)

Chadwick discovered the neutron in 1932 by correctly identifying the energetic radiation emitted from the reaction \( \alpha + ^{9}_{4}\text{Be} \rightarrow ^{12}_{6}\text{C} + ^{1}_{0}\text{n} \) (induced by \( \alpha \)'s from a Po source). The recoil energies transferred to the protons and \(^{14}\text{N}\) nuclei of the filling gas of the ionization chamber were measured to be 5.7 MeV and 1.6 MeV, respectively (masses: \(^{9}\text{Be}: 9.011348\) MeV; \(^{14}\text{N}: 14.002863\) u) The value of the neutron mass (in u) obtained by Chadwick was 1.0067. From range measurements of protons ejected as recoils from hydrogenous material only a crude estimate of the neutron energies and its mass could be deduced. However, using the same apparatus but observing the reaction \( \alpha + ^{11}_{5}\text{Be} \rightarrow ^{14}_{7}\text{N} + ^{1}_{0}\text{n} \) because of better-known mass values, he obtained the mass of the neutron within the limits between 1.005 and 1.008. Today’s best value is 1.008664904 ± 0.000000014 u.

Other nuclear physicists (among them the Curies) had erroneously interpreted the energetic radiation as \( \gamma \) radiation with energies up to 50 MeV, transferring recoil energy by some Compton-like scattering process to the protons or \(^{14}\text{N}\) nuclei. However, such energies of \( \gamma \) transitions cannot occur in nuclei. Only the assumption of a neutral particle with a mass near that of the proton appeared consistent with all observations. Chadwick received the Nobel prize in 1935.

Fig. 12.1 shows the extremely simple experimental setup used by Chadwick. Initially the measured mass of the neutron led to the assumption of the neutron being a (quasi-)bound proton-electron system, but very shortly this idea was dismissed in favor of the neutron being an (elementary) particle of its own.

Immediate consequences of the discovery:

- For the first time the model of nuclei as being composed of protons and neutrons, the existence and “construction” of isotopes, the periodic table, and the chart of nuclides became unambiguous.

- The neutron, due to its electric neutrality, proved to be an ideal projectile to penetrate nuclei and to perform all kinds of nuclear reactions at all energies, including the induced fission of heavy nuclei and creation
Figure 12.1: Apparatus used by Chadwick to discover the neutron, [CHA32b].

of heavier isotopes (in the laboratory and in nucleosynthesis). Neutron multiplication allowing a chain reaction after fission is the basis of nuclear reactors for energy production as well as in the atomic bomb.

- The similarity of the properties of protons and neutrons led W. Heisenberg to postulate the symmetry of Charge Independence and the term and conservation quantity of Isospin. Both have been very important in nuclear and particle physics, also because this symmetry later proved to be slightly broken.

- The $\beta$ decay
  \[ n \rightarrow p + e^- + \bar{\nu}_e \] (12.1)
  is a prototype process caused by the weak interaction. The idea of the existence of the particle class of Leptons, especially of neutrinos (antineutrinos) is intimately connected with this process. This decay was measured already in 1914 by Chadwick displaying a continuous $\beta$ spectrum and thus could not be a two-particle decay, provided the kinematics of energy and momentum conservation holds. Only in 1930 when Pauli formulated tentatively the Neutrino Hypothesis [PAU30] and 1956 when the neutrino was detected directly by F. Reines and C.L. Cowan [COW56] $\beta$ decays were satisfactorily explained.

- The neutron allows a large number of applications ranging from neutron-activation analysis, creation of medically-required isotopes by neutron-capture reactions in reactors, neutron radiography complementing X-ray studies in materials analysis, to the study of biological structures.
Bibliography


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Chapter 13

NN Interaction and Isospin
Chapter 14

Scattering of Identical Nuclei, Exchange Symmetry and Molecular Resonances

First observation of interference in the scattering of identical nuclei:

Exchange Symmetry in Nuclear Reactions of Identical Particles
In addition to the forward-scattering Rutherford cross section there is a corresponding recoil Rutherford term plus an interference term between both. Fig. 14.2 shows this behavior (which is analogous to that of the light in Young’s double-slit experiment, but additionally shows the influence of spin and statistics).

In the scattering of identical particles a detector at the c.m. angle $\theta$ is unable to distinguish whether it registers forward-scattered ejectiles under $\theta$ or, under the angle $\pi - \theta$, backward-emitted recoils. This is shown in Fig. 14.1. The formal scattering theory (see below) shows that the angular distributions must be symmetric around $\pi/2$ and therefore must be described by even-order Legendre polynomials. Quantum-mechanically, in addition, it

Figure 14.1: Trajectories of identical particles in the c.m. system.
is to be expected that the forward- and backward-scattered particle waves interfere. In this case no classical description of the scattering process is possible. In addition, the details of the interference depend on the spin structure of the interacting particles: identical bosons behave differently from identical fermions, and when the particles have spin \(I \neq 0\) (i.e. always for fermions) the spin states must be coupled and superimposed in the cross section with their spin multiplicities as weighting factors. The following examples, which can be tested experimentally will explain this.

### 14.0.1 Identical Bosons with spin \(I = 0\)

Here

\[
[d\sigma/d\Omega(\theta)]_B = |f_1(\theta) + f_2(\pi - \theta)|^2. \tag{14.1}
\]

### 14.0.2 Identical Fermions with spin \(I = 1/2\)

For the fermions the spin singlet cross section

\[
[d\sigma/d\Omega(\theta)]_s = |f_1(\theta) - f_2(\pi - \theta)|^2 \tag{14.2}
\]

and the triplet cross section

\[
[d\sigma/d\Omega(\theta)]_t = |f_1(\theta) + f_2(\pi - \theta)|^2 \tag{14.3}
\]

in the total (integrated) cross section must be added incoherently, each weighted with their spin multiplicities:

\[
[d\sigma/d\Omega(\theta)]_F = \frac{1}{4} |f_1(\theta) + f_2(\pi - \theta)|^2 + \frac{3}{4} |f_1(\theta) - f_2(\pi - \theta)|^2. \tag{14.4}
\]

In these two cases the interference has opposite sign, which e.g. at \(\theta = \pi/2\) has the consequence that in the case of two bosons there is an interference maximum, for fermions a minimum. Under the special assumption that there is no spin-spin force acting \((f_s = f_t = f)\), and with \(f(\theta) = f(\pi - \theta)\) one obtains for identical fermions a decrease, for identical bosons an increase each by the factor 2 as compared to the classical cross section.

For pure (Sub-)Coulomb scattering (meaning: Coulomb scattering at energies below the Coulomb barrier) of identical particles the scattering amplitudes can be calculated explicitly (i.e. also summed over partial waves) since we deal with the Rutherford amplitude known from scattering theory, see Section ??:

\[
\left(\frac{d\sigma}{d\Omega}\right)_\text{Coul} = \left(\frac{Z^2 e^2}{4E_\infty}\right)^2 \left\{ \frac{1}{\sin^2 \frac{\theta}{2}} + \frac{1}{\cos^2 \frac{\theta}{2}} + \frac{2(-1)^{2s} \cos[\eta_S \ln \tan^2 \frac{\theta}{2}]}{(2s + 1) \sin^2 \frac{\theta}{2} \cos^2 \frac{\theta}{2}} \right\}. \tag{14.5}
\]

In addition to the forward-scattering Rutherford cross section there is a corresponding recoil Rutherford term plus an interference term between both. Fig. 14.2 shows this behavior (which is analogous to that of light in Young’s double-slit experiment, but additionally shows the influence of spin and statistics).
Figure 14.2: Experimental c.m. angular distributions of Coulomb scattering of two identical bosons ($^{12}\text{C}$) and fermions ($^{13}\text{C}$) as well as of two non-identical particles of nearly equal masses and theoretical cross sections at $E_{\text{lab}} = 7\text{ MeV}$. The angular distribution for the non-identical particles is obtained when the spectra of the forward and backward scattered particles cannot be separated by the detector, which is the case for (nearly) equal masses. Otherwise one would obtain a typical Rutherford distribution for each particle separately. The data were measured by students of an advanced lab. course at IKP Cologne in 2003.
CHAPTER 14. SCATTERING OF IDENTICAL NUCLEI, EXCHANGE SYMMETRY AND MOLECULAR RESONANCES
Bibliography


Nuclear fission – i.e. the disintegration of a heavy nucleus into two (sometimes three) lighter nuclei of roughly the mass and charge numbers of the original – can occur spontaneously in some heavy nuclei, but there are many instances of fission induced by a nuclear reaction. All kinds of combinations of projectile and heavy target nuclei may undergo fission. Nuclei, excited to very high (rotational) spin states could also fission.

After the discovery of the neutron by Chadwick in 1932 many groups started investigations of the interaction between neutrons and nuclei. The property of neutrality of the neutron made it very attractive as a probe of nuclei and nuclear reactions especially at low energies where no Coulomb barrier hindered the reactions. Even with the simple method of using radioactive sources such as Ra-Be and Hydrogen containing moderators (such as water or paraffin) many new results could be obtained. The practical use of accelerators after 1932 opened additional possibilities as did much later the use of nuclear reactors with high neutron fluxes.

In the 1930s E. Fermi and his collaborators tried to produce transuranic nuclei by adding a neutron to a known nucleus. They succeeded in the case of $^{239}_{93}$Np via the neutron capture of $^{238}_{92}$U and subsequent $\beta$ decay. At the suggestion of L. Meitner Hahn’s group started doing similar experiments with the goal of producing new elements. Only in 1938 O. Hahn and F. Strassmann (after L. Meitner had been forced to emigrate to Sweden) took the additionally produced activities, which looked like e.g. Ba and other medium-weight nuclei, seriously and identified these medium-weight products as fission products.

The nuclear chemists Otto Hahn and Fritz Strassmann published a series of papers in 1938/39, in which they clearly proved – with chemical means – that after the irradiation of uranium (and thorium) with neutrons not only transuranium elements were created, as was generally assumed, see Ref.[HAH38], but that unambiguously also medium-heavy isotopes of barium (and lanthanum and cerium) appeared [?].

The experimental setup was remarkably simple: Slow neutrons were produced by a radium-beryllium source, followed by a paraffin moderator and used to irradiate e.g. a uranium compound such as uranyl nitrate. The
captured neutrons left the reaction products in excited states and the decay activity was measured by a Geiger counter. A setup, which collects the different pieces of apparatus, is exhibited at the Deutsches Museum, Munich, see Fig. 15.1.

Figure 15.1: Museum exhibit at the Deutsches Museum, Munich: the famous Hahn-Meitner-Strassmann desk with the collected pieces of experimental equipment and Hahn’s laboratory logbook. Very probably the equipment has been used mostly by L. Meitner, she being the physicist in the group of nuclear chemists. The parts were actually used in different rooms.

Radium is chemically homologous to barium and thus both could appear together in chemical separations. Therefore, in the beginning, Hahn et al. assumed to have produced new isotopes of radium from $\alpha$ decays of transuranium nuclei. With more refined chemical methods (“fractionated crystallization”) barium and radium could be separated, and no trace of enrichment of radium (with its known half-life) could be found. Thus, they could only conclude that they had produced barium isotopes. It is interesting to read in the original paper, how hard this conclusion from the point of view of physics was for them, but as chemists they could not evade it. In translation: “As chemists in principle we should rename the above scheme and replace the symbols or Ra, Ac, and Th by Ba, La, and Ce. As ’nuclear chemists’ who are in a certain way close to physics we cannot yet make up our mind for this jump. Perhaps a number of strange accidents could still have simulated our results” [HAH39a]. But only little later [HAH39b] they wrote in the conclusion: “The generation of barium isotopes from uranium has finally been proven” and also isotopes of Sr and Y as fragments, and similarly for thorium. This hesitation shows how improbable the only possible conclusion of fission of heavy nuclei into approximately equal debris appeared. It was accepted knowledge that nuclei couldn’t fission. It is, however, hard to understand
that Fermi and his group at Rome didn’t conceive fission during their long series of experiments with neutrons produced from different sources such as Ra-Be, Po-Be etc. on many different elements, among them uranium and thorium. The results were published in ten short communications (letters) in Italian in the journal *Ricerca Scientifica* of the National Research Council and summarized in the Refs. [FER34a, FER34b, FER34c, FER34d]. The group was so fixated on producing “new” (transuranic) nuclei that even the idea of possible fission put forward by Ida Noddack already in 1934 [NOD34] was ignored (for a good account of this see e.g. Ref. [SEG70]). Part of this were energetic considerations and the lack of a plausible model (however, around that time the liquid-drop model just came in time for an explanation).

It took almost five years before the fission process was established. Very quickly at many places the experiments were confirmed and the possibility of fission was theoretically explained with the analogy of nuclei as liquid drops, first by Lise Meitner and Otto Frisch [MEI39], then by N. Bohr and A. Wheeler [BOH39].

The first physical, as compared to chemical, proof of the fission process was published by Otto Frisch in Ref. [FRI39]. He (at Copenhagen) used an ionization chamber lined with uranium irradiated with neutrons produced by an Ra-Be $\alpha$ source with and without paraffin moderator. In the first case the number of fission events was approximately twice that of the unmoderated case. When using thorium as “target” fission fragments, but no dependence on moderation were observed. The medium-mass fission fragments with their high energies and high charge states (estimated to about 20) could travel a few mm into the ionization volume and caused high pulses in the connected counter whereas “transuranes” would not be able to leave the uranium lining.

Also very quickly two important consequences of the phenomenon of nuclear fission were discussed.

- The fission process liberates a large amount of energy ($\approx 200$ MeV per fission), immediately inciting the idea of technical (and military?) useability.

- The fission products are nuclei with high neutron excess. So, besides the slower process of $\beta$ decay the emission of fast neutrons brought up the idea of a *chain reaction* that was later implemented in nuclear reactors and nuclear weapons. Already in 1933 L. Szilard had conceived of a chain reaction but applied for patents instead of publishing the idea, probably of fear of German developments. Proof of fission neutrons was first found in 1939 by Halban et al. [DOD39]. As an example an interesting paper on these considerations was published in June of 1939 by S. Flügge [FLU39] (with the title: “Kann der Energieinhalt der Atomkerne technisch nutzbar gemacht werden?”).
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Chapter 16

First Double Scattering and Polarization in p-$^4$He and the $(\ell \cdot s)$ Force

In 1949 the single-particle shell model of nuclear states was formulated by Haxel, Jensen, and Suess [HAX49], and M. Goeppert-Mayer [GOE49]. It became the basis of nuclear-structure models up to today. The essential ingredient in this model is the non-central spin-orbit force term in the nucleon-nucleus potential. Orbitals with a given orbital angular momentum $\ell$ can be split by the spin-orbit force into doublets corresponding to states with $J = \ell \pm 1/2$. Depending on the sign of this force the sequence of the levels could be “normal”, i.e. (like in atomic physics) the level with the higher could be raised in energy, the other lowered or vice versa, i.e. “inverted”. It turned out that the level ordering in nuclei is different from that in atomic states, meaning that the origin of the $(\ell \cdot s)$ force was probably not electromagnetic. The splitting increases with increasing $\ell$ such that increasingly the spin-orbit force lowers the energy of the lower state so much that it is now in the next-lower harmonic-oscillator shell with opposite parity explaining isomeric states, and creates a large energy gap that is characteristic for the shell model.

Up to 1949 the level ordering in $^5$Li$^*$ and $^5$He$^*$ was unsettled. The excited states at xxx MeV and yyy MeV, resp., could be studied as resonances in $^4$He + proton or $^4$He + neutron scattering. Fig. 16.1 shows the level schemes for $^5$Li and $^5$He.

For low-energy neutrons polarization effects in the interaction of the neutron’s magnetic moment and magnetized material (i.e. the polarized electrons) had been investigated earlier. Theoretical investigations of the effects of an $(\ell \cdot s)$ force (see Refs. [SCH46, SCH48, WOL49]) on nuclear reactions, e.g. production and measurements of spin polarization, for fast neutrons or protons predicted that

• the scattered particles would acquire spin polarization, and

• by time-reversal invariance (TRI), the scattering if done with polarized projectiles would experience a left-right asymmetry (i.e. an azimuthal
CHAPTER 16. FIRST DOUBLE SCATTERING AND POLARIZATION IN P-4HE AND THE $(\ell \cdot S)$ FORCE

Figure 16.1: Simplified level schemes of $^5$Li and $^5$He as of 1966 [AIZ66] showing the P-wave splitting

dependence of the cross sections) : The reaction would show a non-zero analyzing power. TRI states that the polarization produced in a forward reaction is equal to the analyzing power in the corresponding backward reaction, provided the reactions are measured at the same c.m. energies and scattering angles. For elastic scattering both are the same.

- Combining both – a first scattering producing a polarization which in turn may be determined by a second scattering of the same kind, a double scattering experiment – would be unique in showing the existence of the $(\ell \cdot S)$ force.

- At the same time the polarization/analyzing power of the above reactions are predicted to be strongly dependent on the j value of the split shell-model orbital.

Heusinkveld and Freier [HEU50] undertook the first double-scattering experiment

$$^4\text{He} + p \rightarrow ^5\text{Li} \rightarrow ^4\text{He} + p.$$  \hspace{1cm} (16.1)

They used photoplates for detecting the doubly-scattered protons. Their setup is depicted in Fig. 16.2. Figs. 16.3 show the predictions and results of
Figure 16.2: Apparatus to measure protons doubly scattered from $^4\text{He}$ in a gas target chamber

the experiment. The conclusions from this experiment are:

- The P wave in $^5\text{Li}$ (and by isobar arguments also in $^5\text{He}$) is strongly split between $P_{1/2}$ and $P_{3/2}$ which is proof of a strong $(\ell \cdot s)$ force.

- The level sequence is inverted, i.e. the origin of the force is not electromagnetic but a property of the nuclear force itself.

- A scattering experiment $n + ^4\text{He}$ or $p + ^4\text{He}$ produces highly polarized nucleons. The polarization is measurable in a double-scattering experiment.
CHAPTER 16. FIRST DOUBLE SCATTERING AND POLARIZATION IN P-HE AND THE $(\ell \cdot S)$ FORCE

Figure 16.3: Predictions and results for the double-scattering experiment

Table I. Ratio of proton tracks in equivalent strips in the forward and backward plates.

<table>
<thead>
<tr>
<th>No. of tracks on backward plate</th>
<th>No. of tracks on forward plate</th>
<th>Ratio—backward to forward</th>
<th>Theoretical ratio for ideal ray</th>
<th>Inverted doublet</th>
<th>Normal doublet</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.25 Mev 1/4-in. slits</td>
<td>364</td>
<td>191</td>
<td>1.9</td>
<td>2.6</td>
<td>1/20.2</td>
</tr>
<tr>
<td>3.25 Mev 1/2-in. slits</td>
<td>220</td>
<td>156</td>
<td>1.4</td>
<td>2.6</td>
<td>1/20.2</td>
</tr>
<tr>
<td>3.50 Mev 3/8-in. slits</td>
<td>61</td>
<td>33</td>
<td>1.85</td>
<td>1.9</td>
<td>1/6.6</td>
</tr>
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</table>
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Chapter 17

First Nuclear Reaction of an Accelerated Polarized Beam from a Polarized-Ion Source (Basel)

Usually the quantities most measured in a nuclear reaction are differential or total cross sections. For particles with spin these quantities are – formally – averages over incident-particle spin states and sums over outgoing states or (for the total cross section) also sums over scattering angles (i.e. over orbital angular momenta). In this way much information about the details of the interaction may be lost that might be hidden in the transition amplitudes between the spin-substates, especially about the spin dependence of the nuclear interaction, which has, besides a central spin-independent term, several spin-dependent contributions such as $\vec{L} \cdot \vec{S}$, spin-spin, and tensor force contributions.

Up to 1960 the only way to study the spin dependence of nuclear reactions was to produce spin-polarized particles in nuclear reactions with such an $\vec{L} \cdot \vec{S}$ force acting or to measure the polarization of the outgoing particles with such a reaction. The term double scattering for these experiments implies the difficulties involved such as the dependence on energy and angle properties of these reactions and very small event rates.

Starting in 1956 the idea to use atomic properties (in this case the fact that the magnetic moment of the electrons is on the order of about 2000 times larger than that of the nuclei, see the values of the Bohr and the nuclear magneton) for a Stern-Gerlach type separation of spin states was first formulated and realized by G. Clausnitzer, Fleischmann, and H. Schopper [CLA56, CLA59] at Erlangen. The electronic polarization of atoms is transferred to the nuclei by the hyperfine interaction and –later– enhanced by suitable radiofrequency transitions between substates. The beam intensity was increased by separation of the Zeeman components in multipole fields with cylindrical symmetry (quadrupole or sextupole fields). For accelerator use suitable ionizers (electron-collision or ECR type ionizers) were developed.

The first nuclear reaction initiated by a polarized and accelerated beam from such a source was the $^3\text{H}(\vec{d},\text{n})^4\text{He}$ reaction on resonance at $E_d = 107\text{ keV}$
at Basel [RUD61] which was the occasion for the first polarization conference [BAS61]. The setup of this experiment with the atomic-beam polarized deuteron source connected to a *cascade generator* with 100 kV with a tritium target at high voltage and plastic scintillator neutron detectors on ground potential. Fig. 17.1 shows the experimental setup and Fig.17.2 the construction details of the atomic-beam source.
Figure 17.1: Schematic showing the first atomic-beam polarized ion source connected to an accelerator and used for the nuclear reaction $^3\text{H}(\vec{d},\text{n})^4\text{He}$ thereby testing the sensitivity of this reaction as a polarization analyzer and checking the theoretical assumptions of the 107 keV resonance in $^5\text{He}$ as a pure s-wave resonance allowing no vector polarization sensitivity. After [RUD61].
Figure 17.2: Construction of the atomic-beam polarized ion source using an rf-discharge dissociator, long quadrupole magnets for spin-state separation, and an electron-collision ionizer to produce a positive beam of partly tensor-polarized deuterons. After [RUD61].
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Chapter 18

Discovery of the (Electron) Neutrino

W. Pauli formulated the Neutrino Hypothesis in 1930 [PAU30] as answer to open questions concerning $\beta$ decay. The continuous electron spectrum and the spins of the particles called for a third decay particle. E. Fermi formulated his theory of the weak interaction in 1934. Nevertheless the first neutrino (in its electron flavor) was detected directly in experiments beginning in 1953 by F. Reines and C.L. Cowan [REI53, COW56, REI59, REI60]. Only then $\beta$ decay was satisfactorily explained and conservation laws like lepton-number conservation could be postulated.

The experiment was based on the reaction

$$\bar{\nu} + p \rightarrow n + \beta^+$$  \hspace{1cm} (18.1)

(an inverse $\beta$ decay). The two particles in the exit channel are a condition for sharp energies (line spectra) facilitating their detection. Fig. 18.1 depicts the scheme of the detection setup. Antineutrinos come in large numbers from power reactors. Due to the neutron excess of primary fission product nuclei these undergo $\beta$ decay with the emission of antineutrinos – according to baryon- and lepton-number conservation. Fluxes on the order of $10^{13}$ cm$^{-2}$s$^{-1}$ are available, which – even with the very small weak-interaction cross section of typically $10^{-43}$ cm$^2$ – provides for well-measurable event rates of $\approx 5 \cdot 10^{-3}$ s$^{-1}$. The different parts of the experiment are:

- The neutrons from the reaction 18.1 are moderated (time scale: several hundred $\mu$s) and captured in a liquid target consisting of CdCl$_2$ in H$_2$O thereby emitting $\gamma$ rays that are registered in two large liquid scintillator counters (in anticoincidence for background radiation). $^{113}$Cd (with an abundance of 12.26% in natural Cd) has a very large absorption cross section for thermal neutrons ($\sigma = 63,600$ b at $E_n = 0.18$ eV).

- The positrons lose energy, are finally stopped in the target, and annihilate into two 511 keV annihilation $\gamma$ quanta emitted in opposite directions in two scintillator tanks where they are registered in coincidence. The coincidence and anti-coin-cidence requirements led to an elaborate construction of large target and scintillator tanks.
CHAPTER 18. DISCOVERY OF THE (ELECTRON) NEUTRINO

Figure 18.1: Antineutrino reaction and detection scheme of the neutron-capture γ's from Cd and positron-annihilation 511 keV γ’s in a suitable liquid scintillator containing a neutron moderator (water) and a Cd compound (e.g. CdCl$_2$)

- The two (related) coincidence events are measured in an additional time-delayed coincidence with variable delay time around the moderation time of the neutrons in the target. With the “true” events an average cross section of several measurements was

$$\sigma = 11 \pm 2.6 \cdot 10^{44} \text{cm}^2.$$  \hspace{1cm} (18.2)

The authors announced the success of finding the neutrino on July 1959 in a telegram to W. Pauli at the ETH Zürich.

A later experiment that became famous for the measurement of solar neutrinos was based on the reaction

$$\nu_e + ^{37}\text{Cl} \rightarrow ^{37}\text{Ar} + e^-,$$  \hspace{1cm} (18.3)  

[DAV55, DAV64, CLE98]. In this experiment the small cross section was compensated by a very large detector containing many tons of chlorine-compounds such as CCl$_4$, see Fig. 18.2. The experiment ran for about 25 years continuously. It turned out that the $^{37}$Ar produced could be extracted from the target liquid quantitatively and identified by its EC β decay

$$^{37}\text{Ar} + e^- \rightarrow ^{37}\text{Cl} + \nu,$$  \hspace{1cm} (18.4)

which is accompanied by the emission of 3-5 atomic K-shell Auger electrons. Fig. 18.3 shows the miniature proportional counter used for measuring the activity of the $^{37}$Ar β decay. The signals were selected by a pulse rise-time condition imposed thus reducing background. Great care had been observed
in the selection of all materials to cut any background rate substantially below that of true solar neutrino events ($\approx 1$ event per week).

The setup was tested at the Brookhaven and a power-plant reactor and then ran for many years 1.5 kilometers underground in the Homestake mine in South-Dakota to avoid background events from terrestrial sources.

The results of these measurements were manifold:

- The experiment proved that antineutrinos are emitted from the fusion reactions in the sun’s interior.

- Neutrino and antineutrino are different particles: The cross section measured of $\sigma < 0.9 \times 10^{-45}$ cm$^2$ per Cl atom is incompatible with that expected if neutrino and antineutrino were indistinguishable (“Majorana neutrino”). A consequence would that the neutrinoless double-$\beta$ decay would be allowed and that neutrinos must have mass. However, in contrast to the double $\beta$ decay with emission of two antineutrinos, e.g. in the decay

$$^{106}_{48}\text{Cd} \rightarrow ^{106}_{46}\text{Pd} + 2e^+ + 2\bar{\nu}_e,$$

this process has not been found. Neutrino oscillations have been detected, however:

  - At the Super-Kamiokande detector in Japan evidence of oscillations of cosmic-ray neutrinos was found in 1998.
  - In 2002 oscillations of solar electron antineutrinos to another flavor were seen by the Sudbury Neutrino Observatory in Ontario two kilometers underground, thus solving completely the “solar-neutrino puzzle”.
  - Oscillations of antineutrinos from 22 different nuclear power plants at different distances were detected by the KamLAND experiment in Japan in 2002 [EGU03, ?]. The detector was a massive kiloton liquid scintillator viewed by nearly 2000 photomultiplier tubes in the Kamioka mine.
The intensity of the solar neutrino flux as measured by the Davis experiment was only about one third of that expected from model calculations of solar models using all known parameters of the sun as input [CLE98, BAH95, SAC90]. The solution of this “solar neutrino puzzle” was that electron neutrinos have mass and thus can “oscillate”, i.e. periodically transform into one or both of the other neutrino flavors $\nu_\mu$ and $\nu_\tau$ (the evidence for cosmological as well as particle-physics reasons is very strong that there exist only three “families” of leptons – likewise as for the hadron families of quarks). With the existence of these oscillations the finite mass of the neutrinos is a fact – it remains, however, open what the masses of each of the three neutrino flavors are.
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Chapter 19

Spin, Magnetic Moments of Nuclei etc.
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